

Rents and Intangible Capital: A $Q+$ Framework*

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Abstract

Recent work on capital accumulation emphasizes the potential roles of intangible capital and market power to explain investment dynamics since 2000. Firms may be shifting investment toward intangible, rather than physical, capital, and firms with market power have less incentive to increase scale at the margin. Empirical validation of these ideas requires both a benchmark and a structure to consider alternatives. We extend a traditional investment- Q model (Hayashi, 1982; Abel and Eberly, 1994) to include economic rents (market power) and intangible capital. The gap between observable "average Q " and the marginal q that drives investment can be characterized analytically as a function of these rents and intangible capital. In addition, these factors can be characterized empirically using available data. We calculate the roles of rents, intangibles, and (importantly) their interaction in explaining the investment gap. Both market power and intangibles play a role, but the dominant force is their interaction. This occurs because of the high return on intangible capital, on which a firm with market power also earns rents.

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1 Introduction

Recent research on capital accumulation emphasizes two medium-run facts about the US economy: returns to business capital, and corporate profits relative to assets more generally, have been either stable or growing (Gomme et al., 2011); and investment has been lackluster, in particular relative to corporate valuations (Gutiérrez and Philippon, 2017). These facts are particularly puzzling because over the same period of time, measures of the risk-free interest rate have been sharply decreasing (Caballero et al., 2008). In the face of an exogenous decline in risk-free rates, standard models would predict an increase in investment and a decline in the rate of return on assets — both at odds with the data.

A number of recent papers have argued that an explanation for these facts could be an increase in economic rents, evidenced by rising markups (Barkai, 2017; Gutiérrez and Philippon, 2018). A rising markup can account for a stable or rising rate of return on assets despite a falling user cost of capital. A rising markup can also distort investment downward by reducing the marginal return to additional capital. Additionally, recent evidence directly documents a rise in markups (De Loecker and Eeckhout, 2017; Hall, 2018), though the quantitative magnitude of this evidence is controversial.

Over the same period of time, another important shift has taken place: the growing importance of intangible capital (Corrado et al., 2005, 2009). A change in the share of intangible capital in production can also explain both of the facts highlighted above. Profits relative to assets could appear to be high because part of the stock of corporate assets — intangible capital — is omitted. Low physical investment relative to valuations could be due to the fact that typical measures of corporate valuations, such as Tobin’s Q , underestimate the true stock of assets, and thus overstate the incentive to invest.

The goal of this paper is to assess the importance of these two explanations — rents and intangibles — for the decline in investment and the rise in returns to capital. To do this, we add two features to the standard Q -theory model (Hayashi, 1982; Abel and Eberly, 1994): the presence of economic rents, and the accumulation of a stock of intangible assets. We call this model the “ $Q+$ ” model. We show that, viewed through the lens of this model, Tobin’s Q can be decomposed into a component capturing rents to physical capital, a component capturing intangible capital, and a component capturing rents to intangible capital. Moreover, these terms can be quantified using data on profits, investment, valuations, and estimates of the intangible stock. Our empirical applications indicate that the term capturing rents to intangible capital — novel to this analysis — can account for between one third and one half of the apparent gap between Q and physical investment.

We start by describing the “ $Q+$ ” framework. We mainly use it to derive a decomposition of the gap between the average Q of physical capital — which is observable — and its marginal q — which is not, but provides the correct measure of the incentive to invest. The gap between average Q and marginal q , which we call the “investment gap”, is the main focus of our analysis. We show that this gap can be decomposed into three terms: a term capturing rents to physical capital, a term for the direct effect of intangibles, and a term capturing rents to intangible capital. The first two terms would obtain, respectively, in a model without rents (but with intangibles), and in a model without intangibles (but with rents). When both are present in the model, a third term appears in the gap between average Q and marginal q . This term captures the economic rents earned by intangible

capital, which can be separated, in the model, from the standard, [Lindenberg and Ross \(1981\)](#) term capturing rents earned by physical capital. An implication of this finding is there is not a clear dichotomy between rents and intangibles when assessing their respective contributions to the trends described above — that is, their interaction matters. The result is independent of the specifics of exogenous processes and of capital adjustment cost and revenue functions, so long as they satisfy simple homogeneity assumptions.

We then discuss special analytical cases of the $Q+$ framework, in which each of the three terms contributing to the gap between average Q and marginal q can be solved in closed form. These analytical examples are useful to clarify the key forces driving the interaction term. In particular, we show that, when adjustment costs are small, rents on intangible capital are simply the present value of markups multiplied by the user cost of intangible capital. Because intangibles depreciate quickly, their user costs are large, foreshadowing our findings on the quantitative importance of the term capturing rents on intangibles.

We next apply our decomposition to the data. We show how to estimate the three components of the investment gap using specific moments of corporate profits, investment, valuations, and estimates of the intangible stock. Our method is simple and requires relatively little data. We next apply this method to US non-financial corporations, using two complementary data sources: the National Income and Product Accounts (NIPA) and Compustat, a firm-level accounting dataset covering the population of publicly traded US corporations. Throughout, we focus on firms in the non-financial corporate business (NFCB) sector.

National accounts data are broader in coverage, but provide a narrower definition of intangibles, as they focus primarily on R&D capital. In these data, which cover the post-WW2 era, two periods stand out as having a large investment gap: the 1965-1975 decade, and the post-1990 period. Most interestingly, the composition of the gap is different between these two periods. Where as the 1965-1975 gap is mostly due to an increase in rents, approximately 35% of the post-1990's gap is due to the intangibles-related terms. The term capturing rents to intangibles, in particular, becomes sizable: it accounts for 25% of the gap, with the direct intangibles effect making up the other 10%. Importantly, the increase in rents to intangibles is not only due a rising share of intangible capital. We also estimate implied user costs of intangibles to be much higher (by a factor of about three), and more stable after 1990, than those of physical capital. Since conceptually, the term capturing rents on intangible in the present value of markups earned over user costs, the high and stable user costs of intangible capital boost the contribution of this term to the investment gap, relative to the term capturing rents to physical capital. Additionally, we note that our adjustment for intangibles substantially lowers the estimated level of markups, as well as its trend upward in markups after 1990; the estimated markup in 2015 is approximately 1.20 without intangibles, but only 1.10 with intangibles.

We then repeat our analysis using aggregated accounting data for public firms, for the 1975-2017 period. Using that data, and measuring intangibles in analogous way to the NIPA — that is, focusing on the R&D capital stock — sample, we find results that are consistent with those described above, though with a somewhat larger estimated increase in markups, reflecting higher profit rates among public firms. However, the advantage of these data is that they can be used to construct estimates of intangibles beyond R&D. In particular, we repeat our analysis including estimates of the organization capital stock of firms based on capitalied expenditures on Sales, General and Administrative expenses

(SG&A), following [Eisfeldt and Papanikolaou \(2013\)](#) and [Peters and Taylor \(2017\)](#). Using this expanded measure of the intangible capital stock, the two intangibles terms account for two thirds of the total investment gap by 2015. Including SG&A has relatively little impact on estimated user costs of intangibles — they remain very elevated —, but it substantially increases the total stock of intangibles relative to physical capital, which increases both the direct effect of intangibles on the investment gap, and the interaction term. Interestingly, the inclusion of organization capital implies even lower markup trends than those estimated for the NFCB sector as a whole using NIPA data.

An additional advantage of the firm-level accounting data is that they can be used to construct our decomposition at different levels of aggregation. This is useful to understand how distribution of rents and intangibles across sectors or firms affects the aggregate investment gap. In the last part of the paper, we divide our sample into four broad sectors: Consumer, High-tech, Healthcare, and Manufacturing, and repeat our exercise separately for each. The results show substantial heterogeneity across sectors. The investment gap is small in the manufacturing sector, where we estimate that rents on both physical and intangible capital, and the intangible share, have been in decline since the early 2000’s. By contrast, in the High-tech and Healthcare sectors, the investment gap has been growing rapidly since the 2000’s. In both sectors, the primary driver is rents to intangible capital. Finally, in the consumer sector, we show that results depend on the measurement of the intangible capital stock. Using only capitalized R&D, the larger investment gap in that sector appears to be driven by traditional rents to physical assets. However, innovation in the consumer sector is not well-measured in R&D (see [Foster et al. \(2006\)](#) and [Crouzet and Eberly \(2018a\)](#)). When including organization capital, most of the gap is estimated to reflect the direct effect of large investment in intangibles in that sector — rents on either physical or intangible capital appear to have only modestly increased.

Most closely related to our analysis is the work of [Farhi and Gourio \(2018\)](#), who estimate the contribution of market power, risk premia, and intangible capital to recent macro trends. Relative to their work, our analysis focuses more specifically on investment, and the role that intangible capital plays in explaining its low level relative to valuations. Additionally, our work is related to [Belo et al. \(2018\)](#), who also provide decompositions of firm value across types of capital; relative to their work, we allow for the presence of market power, and moreover, we focus on the implications of the decomposition for understanding aggregate investment trends.

2 Q , rents and intangibles: theory

In this section, we derive a general decomposition of the gap between average Q and marginal q — which we call the “investment gap” — in terms of contributions of economic rents and contributions of intangible capital. We then provide analytical examples of this decomposition in a number of simple cases.

2.1 Framework

We first state the general version of the q -theory model in which we derive our main results.

Time t is discrete. A firm, indexed by j , uses $n = 1, \dots, N$ different capital inputs in order to produce output; capital is the only input in production. Firm j 's operating profits are given by $\Pi_{j,t}(K_{j,t})$, where $K_{j,t}$ is an aggregate of the different types of capital given by:

$$K_{j,t} = F_{j,t} \left(\{K_{n,j,t}\}_{n=1}^N \right). \quad (1)$$

Investment is subject to adjustment costs given by:

$$\tilde{\Phi}_{j,t} \left(\{K_{n,j,t}, K_{n,j,t+1}\}_{n=1}^N \right). \quad (2)$$

We use the notation $\Pi_{j,t}$, $F_{j,t}$ and $\tilde{\Phi}_{j,t}$ to mean that these functions can depend arbitrarily on idiosyncratic and aggregate exogenous variables, which we do not specify explicitly. Firms share a common discount factor $M_{t,t+1}$, and firm j solves the following problem:

$$\begin{aligned} V_{j,t} \left(\{K_{n,j,t}\}_{n=1}^N \right) &= \max_{\{K_{n,j,t+1}\}_{n=1}^N} \Pi_{j,t}(K_{j,t}) - \tilde{\Phi}_{j,t} \left(\{K_{n,j,t}, K_{n,j,t+1}\}_{n=1}^N \right) \\ &\quad + \mathbb{E}_t \left[M_{t,t+1} V_{j,t+1} \left(\{K_{n,j,t+1}\}_{n=1}^N \right) \right] \quad (3) \\ \text{s.t. } K_{j,t} &= F_{j,t} \left(\{K_{n,j,t}\}_{n=1}^N \right). \end{aligned}$$

We make the following two assumptions about the primitives of the problem, namely the functions $\Pi_{j,t}$, $F_{j,t}$, and $\tilde{\Phi}_{j,t}$.

Assumption 1. *The function $\Pi_{j,t}(K_{j,t})$ is increasing, concave, and homogeneous of degree $\frac{1}{\mu_j}$ in $K_{j,t}$, where $\mu_j \geq 1$. The function $F_{j,t} \left(\{K_{n,j,t}\}_{n=1}^N \right)$ is homogeneous of degree 1 in $\{K_{n,j,t}\}_{n=1}^N$.*

Assumption 2. *Adjustment costs are additively separable across types of capital goods, $\tilde{\Phi}_{j,t} \left(\{K_{n,j,t}, K_{n,j,t+1}\}_{n=1}^N \right) = \sum_{n=1}^N \Phi_{n,j,t}(K_{n,j,t}, K_{n,j,t+1})$. Moreover, each adjustment cost function is homogeneous of degree 1 with respect to $(K_{n,j,t}, K_{n,j,t+1})$, and increasing and convex in its second argument.*

Later in the section, we provide specific examples of environments in which firms ultimately solve a version of problem (3). For now, we focus on deriving some general properties of the solution to this problem.

2.2 A general decomposition of the investment gap

Our main result uses the following expression for firm value, which is proved in appendix A.

Lemma 1. *Let $q_{n,j,t}$ denote marginal q for capital of type n :*

$$q_{n,j,t} = \frac{\partial V_{j,t}}{\partial K_{n,j,t}},$$

and let $\Pi_{K_{n,j,t}}$ denote the marginal revenue product of capital of type n :

$$\Pi_{K_{n,j,t}} = \frac{\partial \Pi_{j,t}}{\partial K_{j,t}} \frac{\partial F_{j,t}}{\partial K_{n,j,t}}.$$

Then, firm value can be written as:

$$V_{j,t} = \sum_{n=1}^N q_{n,j,t} K_{n,j,t} + (\mu_j - 1) \sum_{n=1}^N \sum_{k \geq 0} \mathbb{E}_t [M_{t,t+k} \Pi_{K_{n,j,t+k}} K_{n,t+k}]. \quad (4)$$

This result decomposes firm value into two parts. The first part of equation (4) is a weighted sum of the different stocks of capital, with weights given by the marginal q of each type of capital. This part of firm value is non-zero even when $\mu_j = 1$. This is a general version of the standard [Hayashi \(1982\)](#) homogeneity result equating marginal q and average Q . Both may be different from one, so long as adjustment costs are strictly convex.

In order to interpret the second part of equation (4), note that when there is only one type of capital, the expression boils down to the discounted sum of the terms $(\mu_j - 1)\Pi_{K_{n,j,t+k}} K_{n,t+k}$. Given assumption 1, these terms capture the gap between average and marginal products:

$$(\mu_j - 1)\Pi_{K_{j,t+k}} = \frac{\Pi_{j,t+k}}{K_{j,t+k}} - \Pi_{K_{j,t+k}}. \quad (5)$$

In the one-capital case, the second term in decomposition (4) is simply the present value of the gap between average and marginal products, which can be interpreted as the present value of economic rents. This gap is positive only when $\mu_j > 1$, a point first noted by [Lindenberg and Ross \(1981\)](#).

When there are multiple types of capital, the second term in equation (4) is the sum of terms of the form:

$$(\mu_j - 1)\Pi_{K_{n,j,t+k}} = \left(\frac{\Pi_{j,t+k}}{K_{j,t+k}} - \Pi_{K_{j,t+k}} \right) \frac{\partial F_{j,t+k}}{\partial K_{n,j,t+k}}. \quad (6)$$

These terms capture the marginal contribution of capital of type n to overall rents earned by the firm. These rents themselves depend on the gap between the average and marginal product of capital of type n , as in the one-capital case. Total rents are then the sum of the rents attributable to each type of capital, weighted by the marginal contribution to total capital, $\frac{\partial F_{j,t+k}}{\partial K_{n,j,t+k}}$. The intuition from the one-capital case thus carries through with multiple types of capital.

Note that this decomposition of firm value is general, and does not require further assumptions other than those stated above. In particular, no specific assumptions about the stochastic processes governing shocks are needed.

Corollary 1. Define average Q for capital of type n , $Q_{n,j,t}$, as:

$$Q_{n,j,t} = \frac{V_{j,t}}{K_{n,j,t}}, \quad (7)$$

and define the investment gap for capital of type n as:

$$G_{n,j,t} = Q_{n,j,t} - q_{n,j,t}. \quad (8)$$

Then, the investment gap for capital of type n can be written as:

$$G_{n,j,t} = (\mu_j - 1) \sum_{k \geq 0} \mathbb{E}_t [M_{t,t+k} \Pi_{K_{n,t,t+k}} (1 + g_{n,j,t,t+k})] \quad (9)$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^N S_{m,n,j,t} q_{m,j,t} \quad (10)$$

$$+ (\mu_j - 1) \sum_{\substack{m=1 \\ m \neq n}}^N S_{m,n,j,t} \sum_{k \geq 0} \mathbb{E}_t [M_{t,t+k} \Pi_{K_{m,t,t+k}} (1 + g_{m,j,t,t+k})], \quad (11)$$

where $1 + g_{n,j,t,t+k} \equiv \frac{K_{n,j,t+k}}{K_{n,j,t}}$, and $S_{m,n,j,t} \equiv \frac{K_{n,j,t}}{K_{m,j,t}}$.

This corollary provides a simple decomposition of the investment gap into three terms, (9), (10) and (11). They can be thought of as follows.

When there are no rents and a single type of capital ($\mu_j = 1$ and $N = 1$), average Q and marginal q are equal, as in the standard model of Hayashi (1982). In this case, the terms (9), (10) and (11) are zero, and the investment gap is zero.

If there are rents but only one type of capital ($\mu_j > 1$ and $N = 1$), average Q will overstate marginal q . The positive investment gap is then equal to the present value of the difference between average and marginal products of capital, that is, the term (9). This is the Lindenberg and Ross (1981) effect.

If there are no rents but several types of capital ($\mu_j = 1$ and $N > 1$), then for each type of capital, average Q will still overstate marginal q . Average Q for a specific type of capital reflects, in part, the value of other types of capital used by the firm. It therefore overstates the true incentive to invest — the marginal q — of that type of capital. This case is studied by Hayashi and Inoue (1991) and Crouzet and Eberly (2018b); it is captured by the term (10) in the expression of the investment gap.

If there are both economic rents and several types of capital ($\mu_j > 1$ and $N > 1$), the rents term (9) and the omitted capital term (10) are still non-zero. But additionally, the term (11) is non-zero. This term represents the interaction between the rents and the omitted capital effects: it captures how the present value of the rents accruing to *other* types of capital affects total firm value and, through the omitted capital effect described above, add to the gap between average Q and marginal q . This interaction term is larger, the higher the relative importance of other types of capital, and the higher the rents generated by other types of capital.

Before fleshing out this decomposition more precisely using analytical examples, it is worth saying a word about our chosen terminology of “investment gap” for the gap between average Q and marginal q . We borrow this terminology from Gutiérrez and Philippon (2017)

and [Alexander and Eberly \(2018\)](#), but it can be justified in the context of this class of models as follows. The first-order condition for investment in each type of capital can be written as:

$$g_{n,j,t,t+1} = \Psi_{n,j,t} (\mathbb{E}_t [M_{t,t+1} q_{n,j,t+1}] - 1),$$

where $g_{n,j,t,t+1}$ is the net investment rate in capital n , and $\Psi_{n,j,t}(y) \equiv (\Phi'_{n,j,t})^{-1} (1 + y) - 1$ is a (strictly increasing) function capturing the magnitude of investment adjustment costs. Whenever $Q_{n,j,t+1} > q_{n,j,t+1}$, that is, whenever the investment gap is positive, we have:

$$g_{n,j,t,t+1} < \Psi_{n,j,t} (\mathbb{E}_t [M_{t,t+1} Q_{n,j,t+1}] - 1).$$

Therefore, investment predicted using average Q — which is observable — instead of using marginal q — which is not — will always exceed actual investment. That is, there will appear to be a “gap” between actual investment and observed Q values. Moreover, the extent to which investment will appear to be low, relative to Q , will be driven by the gap between Q and q . In the standard, quadratic adjustment cost case, for instance, the relationship above can be written as:

$$i_{n,j,t} = \delta_n + \frac{1}{\gamma_n} (\mathbb{E}_t [M_{t,t+1} Q_{n,j,t+1}] - 1) - \frac{1}{\gamma_n} \mathbb{E} [M_{t,t+1} G_{n,j,t+1}],$$

where $i_{n,j,t}$ is gross investment, δ_n is the depreciation rate and γ_n the quadratic cost slope for capital of type n . The higher the value of the (discounted) gap, the bigger the discrepancy between average Q and observed investment.¹

2.3 Analytical examples

We next provide explicit expressions for the decomposition of the investment gap derived in corollary 1 in versions of the model with analytical solutions. These analytical expressions help build intuition for the decomposition of the investment gap, and also prepare for our empirical applications. For the rest of the section, we assume that the profit function is given by:

$$\Pi_{j,t} = A_{j,t}^{1-\frac{1}{\mu_j}} K_{j,t}^{\frac{1}{\mu_j}}. \quad (12)$$

The exogenous process $A_{j,t}$ can be thought of as reflecting a number of factors, both firm- and industry-specific.² Additionally, and without loss of generality, we assume that firms use two types of capital in production. They are combined via a CES aggregator:

$$K_{j,t} = \left((1 - \eta) K_{1,j,t}^\rho + \eta (K_{2,j,t})^\rho \right)^{\frac{1}{\rho}},$$

where $\rho \in]-\infty, 1]$ is the elasticity of substitution across capital types.³ We will refer to K_1 as “physical capital,” and to capital K_2 as “intangible capital.”

¹Appendix A.2 discusses the implications of the model for total Q , defined as $V_{j,t} / \sum_{n=1}^N K_{n,j,t}$ ([Peters and Taylor, 2017](#)), and the assumptions under which it provides a correct signal of the true incentive to invest for total investment. The absence of rents, $\mu_j = 1$, is necessary but not sufficient.

²Appendix A.3 gives an explicit example in a standard model where firms are monopolistic suppliers of substitutable varieties, and $\mu_j = \mu$ for all j . In this case, $A_{j,t}$ reflects a mix of firm-level productivity, and industry-level demand, prices and wages.

³Specifically, $\rho = -\infty$ is the case of perfect complements, $\rho = 0$ is the Cobb-Douglas case (with shares $1 - \eta$ and η), and $\rho = 1$ is the case of perfect substitutes.

Constant growth We start with the case where the discount rate, $M_{t,t+1}$, and the growth rate of the fundamentals, process, are constant:

$$\frac{A_{j,t+1}}{A_{j,t}} = 1 + g_j, \quad M_{t,t+1} = (1 + r)^{-1}, \quad \forall t.$$

The simplest expressions obtain when adjustment costs are linear, that is:

$$\Phi_{n,j,t}(1 + x) = x + \delta_n, \quad n = 1, 2.$$

In this case, the total capital stock, $K_{n,j,t}$, grows at rate g_j , and so does each type of capital. The first-order condition for investment, $\Phi'_{n,j,t} = \mathbb{E}_t [M_{t,t+1} q_{n,j,t+1}]$, simplifies to:

$$q_{n,j,t} = q_{n,j} = 1 + r.$$

That is, in the absence of convexity in adjustment costs, (discounted) marginal q is equal to 1. Finally, the envelope condition for each type of capital simplifies to:

$$\Pi_{K_{n,j,t}} = r + \delta_n, \quad n = 1, 2,$$

that is, the firm equates the marginal product of each type of capital to its [Jorgenson \(1963\)](#) user cost. Using these facts, we can rewrite our expression for the total value of the firm as:

$$V_{j,t} = (1 + r)K_{1,j,t} + (1 + r)K_{2,j,t} + \frac{1 + r}{r - g_j}(\mu - 1)(r + \delta_1)K_{1,j,t} + \frac{1 + r}{r - g_j}(\mu - 1)(r + \delta_2)K_{2,j,t}.$$

Dividing through by $K_{1,j,t}$ and subtracting $q_{1,j} = 1 + r$, we obtain the following expression for the physical investment gap:

$$G_{1,j} = \frac{\mu_j - 1}{r - g_j}(r + \delta_1) + S_j + \frac{\mu_j - 1}{r - g_j}(r + \delta_2)S_j. \quad (13)$$

where $S_j = \frac{K_{2,t}}{K_{1,t}}$ is the (constant) ratio of intangible to physical capital.⁴

Following corollary 1, the investment gap for physical capital has three components: the rents earned by physical capital; the omitted capital term, which is now simply captured by the ratio of intangible to physical capital, S_j ; and the rents earned by intangible capital. The closed form expressions for the two rents terms also provides some insight. In the frictionless model, the firm behaves as though it were renting capital on perfectly competitive markets, paying them their user costs. The two rents terms can be then be thought of as a markup on the marginal (user) cost of each of these two inputs, discounted by the Gordon growth term $(r - g_j)^{-1}$. Their relative magnitude will then depend on how intangible-intensive the firm is (that is, on S_j), and on how high intangible user costs are, relative to physical user costs.

These insights also hold, with some small modifications, when adjustment costs are convex. In order to simplify analytical expressions, we assume that adjustment costs are given by:

$$\Phi_{n,j,t}(1 + x) = \Phi_n(1 + x) = x + \delta_n + \gamma_n r \left(x + (r - x) \log \left(\frac{r - x}{r} \right) \right). \quad (14)$$

⁴Here, we expressed the investment gap in discounted terms, i.e. dividing by $(1 + r)$.

This particular adjustment cost function is increasing and strictly convex, and satisfies $\Phi_n(1) = \delta_n$, $\Phi'_n(1) = 1$ and $\Phi''_n(1) = \gamma_n$. Appendix A shows that the (discounted) investment gap for physical capital is then given by:

$$G_{1,j} = \frac{\mu_j - 1}{r - g_j}(r + \delta_1 + \gamma_1 r g_j) + S_j q_{2,j}^d + \frac{\mu_j - 1}{r - g_j}(r + \delta_2 + \gamma_2 r g_j) S_j. \quad (15)$$

where all variables are defined as before, and (discounted) marginal q for intangibles is given by $q_{2,j}^d = 1 + \gamma_2 g_j$. The only notable difference, with respect to the case of linear adjustment costs, is in the expression of user costs. The typical Jorgensonian user cost $r + \delta_n$ which appeared in decomposition (13) is replaced with $r + \delta_n + \gamma_n r g_j$, reflecting the added costs of continuously adjusting capital along the firm's growth path.

Stochastic growth Finally, similar closed-form expressions can be obtained when growth is stochastic. Assume that the fundamentals process follows:

$$\frac{A_{j,t+1}}{A_{j,t}} = 1 + g_{j,t} = \begin{cases} 1 + g_{j,t-1} & \text{w.p. } (1 - \lambda) \\ 1 + \tilde{g} & \text{w.p. } \lambda \end{cases}$$

Here, \tilde{g} is drawn, at time t , from a distribution $F(\cdot)$, which is time-invariant, and the draw is independent of past realizations of $g_{j,t}$. In this case, appendix A shows that, when the discount rate is constant, and adjustment costs are linear, the (discounted) investment gap for physical capital is given by:

$$G_{1,j,t} = \frac{\mu_j - 1}{r - \nu(g_{j,t})}(r + \delta_1) + S_j + \frac{\mu_j - 1}{r - \nu(g_{j,t})}(r + \delta_2) S_j. \quad (16)$$

The function $\nu(g_{j,t})$, the expression of which is reported in appendix A, depends on the parameters λ and $F(\cdot)$.⁵ The term $\frac{1}{r - \nu(g_{j,t})}$ is analogous to the standard Gordon growth formula, but the function $\nu(\cdot)$ adjusts for the possibility of future “regime changes” in firm-level growth. Moreover, when growth is i.i.d., these expressions extend to the case of convex adjustment costs, with similar change to those in equation (15). Thus, the key insights from the previous discussion survive. In particular, even with stochastic growth, the two rents terms can be thought of as the present value of markups over the user costs of physical and intangible capital, respectively.

3 An application to aggregate data

We now use our framework to analyze the investment gap among US non-financial corporations over the past several decades. We show that the gap roughly tripled from 1985 to 2015. Moreover, the contribution of the term capturing rents earned by intangibles grew substantially over the same period, and now accounts for one quarter to one half of the gap, depending on the data source used.

⁵ When $\lambda = 0$, the firm's growth rate is constant, and $\nu(x) = x$. When $\lambda = 1$, the growth rate of the firm is *i.i.d.* In this case, letting $\bar{g} = \mathbb{E}[\tilde{g}]$, $\nu(x)$ satisfies $\frac{1}{r - \nu(x)} = \frac{1}{r - \bar{g}} \left(1 + \frac{g_{j,t} - \bar{g}}{1+r}\right)$.

3.1 Methodology

Throughout this section, we use the constant growth version of the model. Moreover, we treat the model as describing one representative firm; we allow for heterogeneity in the following section. Finally, our baseline analysis assumes linear adjustment costs, but we relax this later in the section.

Since we are using the model with constant growth, we drop time subscripts for all variables that are constant along the firm's growth path. Recall that we defined the (discounted) investment gap as the gap between marginal Q and average q , $G_1 = Q_1 - q_1$. As discussed above, with linear adjustment costs, marginal (discounted) q for physical capital is $q_1 = 1$, so that the investment gap for physical capital is simply $Q_1 - q_1 = Q_1 - 1$. Following the derivations of section 2.1, the decomposition of this gap is given by:

$$G_1 = \frac{\mu - 1}{r - g}(r + \delta_1) + S + \frac{\mu - 1}{r - g}(r + \delta_2)S. \quad (17)$$

where $n = 1$ indexes physical capital and $n = 2$ indexes intangibles. The first term captures rents earned by physical capital; the second term captures the omitted capital effect of intangibles on the physical investment gap; and the third term captures rents earned by intangibles.

We construct the empirical counterpart of this model decomposition as follows. First, appendix A shows that with linear adjustment costs:

$$\mu = \frac{ROA_1}{R_1 + SR_2}, \quad (18)$$

where:

$$ROA_1 \equiv \frac{\Pi_t}{K_{1,t}}, \quad R_1 \equiv r + \delta_1, \quad R_2 \equiv r + \delta_2. \quad (19)$$

Intuitively, the parameter μ captures market power or pure economic rents. These are reflected in the gap between average returns to physical capital and a weighted average of the user costs of capital. In particular, in the absence of rents, the firm equates the average return on physical capital to this weighted average user cost measure. Thus, given empirical measures for the average return on physical capital, ROA_1 , the user costs of each type of capital, R_1 and R_2 , and the ratio of intangible to physical capital at replacement cost, S , a value for μ can be inferred.

As described below, we measure ROA_1 and S directly from either national or firm-level accounting data. For the user cost of each type of capital, following Farhi and Gourio (2018), we note that in the model with constant growth,

$$R_n = r + \delta_n = r - g + g + \delta_n = r - g + i_n, \quad n = 1, 2, \quad (20)$$

where $i_n = g + \delta_n$ is the gross investment rate in capital of type n . We obtain measures of these gross investment rates from the same sources as our measure of S . Finally, in order to measure $r - g$, we use the fact that in the model with constant growth:

$$r - g = \frac{ROA_1 - (i_1 + Si_2)}{Q_1}, \quad (21)$$

a relationship which can be obtained by substituting the user cost expressions (20) and the markup expression (18) into equation (17).

Therefore, only five data moments are needed to construct the decomposition of the physical investment gap using this approach: the average return on physical capital, ROA_1 ; gross physical and intangible investment rates, i_1 and i_2 ; the ratio of intangible to physical capital at replacement cost, S ; and average Tobin’s Q for physical capital, Q_1 .

Note that this approach matches, by construction, the empirical value of Q_1 . More specifically, it infers Gordon growth term $r - g$ which, given other data moments, ensures that the model will produce a value of Q_1 consistent with the data. Our robustness section discusses the results when using empirical measures of $r - g$ instead.

Finally, the goal of this section is construct changes in the investment gap over time. In order to do this, we compute moving averages of each of the five data moments over 7-year centered rolling windows. We then use the set of average moments for each window to construct the decomposition in a particular year, the center of the window, and report the resulting annual time series. This approach treats each successive window as being generated by a different version of the constant growth model, thus allowing us to capture gradual changes in the investment gap.⁶

3.2 Results for the non-financial corporate business sector

Data sources Our sample period is 1947-2017. We construct time series for the five ratios used in the decomposition, $\{i_{1,t}, i_{2,t}, S_t, ROA_{1,t}, Q_{1,t}\}$, using six times series in levels, $\{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\}$. They are, respectively, the operating surplus of the NFCB sector, the stock of physical capital at replacement cost, investment in physical capital, the stock of intangibles at replacement cost, investment in intangibles, and the market value of claims on the NFCB sector.

We obtain measures of $K_{1,t}, I_{1,t}, K_{2,t}$ and $I_{2,t}$ from BEA fixed asset tables 4.1 and 4.7. The BEA fixed asset tables construct the intangible capital stock using perpetual inventory methods applied to three categories of intangible investment: R&D; the creation of own-account software; and artistic originals. Below, we discuss broader measures of intangible capital which can be constructed using firm-level accounting data, but only for a subset of NFCB sector firms, namely, publicly traded firms.

Our measure of the operating surplus Π_t is obtained from NIPA Table 1.14; appendix B reports the details. Consistent with the model, this measure of operating surplus does not include expenditures categorized by the BEA as intangible investment; moreover, it lines up very closely with the Flow of Funds series for NFCB profits (after adding back depreciation expenses), which themselves are adjusted for expenditures categorized by the BEA as intangible investment.

Finally, we construct a measure of V_t using Flow of Funds data for the NFCB sector, specifically, tables L.103 and F.103. In the model, V_t represents the market value of all net claims on the NFCB sector, both debt and equity. The Flow of Funds data provide an estimate of the market value of equity for the NFCB sector, but not for debt. We follow

⁶Using alternative window sizes from 3 to 9 years gives quantitatively similar results, while constructing the decomposition on mutually exclusive subsamples shows that approximately 25% of the post-1984 rise in the investment gap is due to the term capturing rents on intangibles.

an approach analogous to Hall (2001) to estimate the latter. The methodology and data sources are described in appendix B, and the resulting time series for V_t is reported in appendix Figure C.1. Importantly, different from Hall (2001), we compute our time series for V_t by netting out only liquid financial assets from estimates of the market values of equity and debt. We do this primarily because the non-liquid financial assets reported in the Flow of Funds are a residual constructed to match estimates of aggregate book assets for the NFCB, themselves obtained from two other sources the Statistics of Income and the Quarterly Financial Report. In section 3.4, we discuss the results when using a value for V_t obtained when netting out all financial assets reported in the Flow of Funds, as in Hall (2001). This affects the overall level of the investment gap, but not the relative importance of rents on intangibles.

We then construct $ROA_{1,t} = \Pi_t/K_{1,t}$, $i_{1,t} = I_{1,t}/K_{1,t}$, $i_{2,t} = I_{2,t}/K_{2,t}$, $S_t = K_{2,t}/K_{1,t}$, and $Q_{1,t} = V_t/K_{1,t}$. The time series of these moments are reported in Figure 1, and averages across sub-samples are reported in Table 1. The key macroeconomic and financial trends discussed in the introduction are visible in that figure. The average return to physical capital increases after 1984, while the physical investment rate declines. The intangible investment rate decline after 2000, but remains elevated even after 2000. As a result, the ratio of intangible to physical capital increases throughout the sample period, and in particular after 1985. Tobin’s Q for physical capital rises sharply after 1985, and after a peak in 2000, remains approximately twice higher than in the pre-1985 period.

Findings Figure 2 reports, graphically, the investment gap and its three components.

The figure shows that there are two periods during which the physical investment gap was high: the 1960-1970 period, and the post-1985 period. The first important finding is that, while in both periods, the majority of the increase in the gap is due to an increase in rents to physical capital, the importance of the two intangible-related terms is higher in the post-1985 period. In the 1960-1970 period, at the physical investment gap peak, direct rents on physical capital account for 77% of the total gap, and the two intangibles-related terms account for the remaining 23%. By contrast, in 2015, market power term accounts for 64% of the total investment gap, and the two intangibles-related terms account for the remaining 36%.

The second important finding is that rents to intangibles — the third term in decomposition (17) — become a much larger contributor to the physical investment gap after 1985. In 1985, they only account for 13% of the physical investment gap, while by 2015, they account for 26% of it. The rising importance of these rents seems to be a distinctive feature of the post-1985 period; between 1960 and 1966, the contribution of rents on intangibles term only increased from 9% to 11%.

The rising contribution of rents earned on intangibles is the reflection of three distinct phenomena in the post-1985 period: the rise in markups; the rise in the intangible share; and the rise in user costs of intangible capital relative to physical capital. Figure 3 reports the time-series for the user costs, markups, and intangible shares implied by the model.

For the markup, in order to facilitate comparisons with existing estimates, we report values that adjust for labor income.⁷ Two points are worth noting regarding the markups

⁷Recall that we have derived our decomposition in a model where capital is the only factor in production.

reported in Figure 3. First, the model without intangibles implies the same sort of upward trend in markups after 1980 as has been documented in a number of recent papers, and in particular by Barkai (2017). Second, markups estimated using the model with intangibles are both lower on average, and also display a slightly smaller trend after 1980. This is because the markup is inferred using the gap between returns to physical capital and the user cost of capital, the latter of which are higher once intangibles are taken into account. After 1980, markups rise from 0 to 10% in the model with intangibles, versus 5% to 25% in the model without intangibles.

Figure 3 also shows that the share of intangible capital in production roughly doubled after 1985, from 0.15 to 0.30.⁸ The behavior of the share approximately mimics the behavior of the measured ratio of intangible to physical capital at replacement cost, which increases continuously through the sample, but particularly so after 1985. The direct contribution of the increase in the intangible share to the increase in the investment gap — the second term, in decomposition (17) — remains relatively small, given the magnitude of the increase in the investment gap. But its interaction with markups magnifies its total impact on the investment gap.

Finally, Figure 3 reports the behavior of implied user costs. The main point to note is that while both implied user cost series decline sharply after 1985, the relative magnitude of the decline is larger for physical user costs. They fall by about 25% between 1985 and 2015, while intangible capital user costs fall by 11%. As discussed in section 2.1, the relative user costs of intangible and physical capital matter in determining the relative importance of the two rents terms, since rents for each type of capital are simply the present value of markups earned over user costs. The lower decline in intangible user costs thus magnified the relative contribution of rents on intangibles to the physical investment gap. Empirically, intangible investment rates are substantially higher — about twice as high, on average — than physical investment rates, implying high depreciation rates and thus high user costs. Moreover, the mild post-2000 decline in intangible investment rates relative to physical investment rates (as reported in Figure 2) implies, in our approach, a mild decline in user costs, since we infer user costs from $R_2 = r + \delta_2 = r - g + g + \delta_2 = r - g + i_2$.

Summarizing, the analysis of the NFCB data shows both a large increase in the physical investment gap after 1985, and a growing contribution of rents earned by intangibles to this gap. By 2015, 26% of the gap is due to these rents, capturing the combined effects of a high intangible share, high user costs of intangibles, and rising market power.

3.3 Results for publicly traded non-financial firms

As mentioned above, the NIPA data omit some potentially important forms of intangibles, in particular organization capital and brand capital. However, accounting data can be used

The adjustment for payments to a flexible labor input is straightforward and described in Figure 3 and appendix A; it does not affect our other results.

⁸This figure reports the implied share of intangible capital in production, η , when the two forms of capital are assumed to be Cobb-Douglas complements. In general, we have $S = (\frac{\eta}{1-\eta}(R_1/R_2))^{1/(1-\rho)}$, and so ρ and η cannot be separately identified from data on S , R_1 and R_2 only. We report $\eta = (R_2/R_1)S/(1 + (R_2/R_1)S)$ in Figure 3.

to construct proxies for these forms of intangibles. The drawback is that this data is only available for publicly traded firms. Appendix B discussed in detail the differences between the national accounts data and our source for publicly traded firms, the non-financial segment of Compustat. Here we use the latter to construct an alternative decomposition of the physical investment gap. Results indicate that the gap is comparable among publicly traded non-financial firms and in the NFCB sector overall, but that more of it — about half — is due to rents on intangible capital.

Data sources Our sample is annual, from 1975 to 2017, and our data source are the Compustat-CRSP merged annual files. Appendix B describes our sample selection procedure; we focus on domestically incorporated, publicly traded US firms not in the financial or utilities sectors. As in the case of the NFCB sector, we start by constructing five time series in levels, $\{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\}$, using annual aggregated data from our sample. We consider two definitions of intangibles in this data: R&D capital, and organization capital. We measure R&D investment using reported R&D expenditures at the firm level. For investment in organization capital, we follow [Eisfeldt and Papanikolaou \(2013\)](#) and use 30% of SG&A expenditures net of R&D investment. For the stocks, we used the capitalized values of the two investment measures, provided by [Peters and Taylor \(2017\)](#).⁹ For Π_t , we use operating income before depreciation. Importantly, consistent with our model, we adjust operating income for the expensing of intangible investment in accounting data (that is, we add estimates of intangible investment expenditures to actual measures of operating income in order to obtain an adjusted gross surplus measure, our data equivalent for Π_t in the model.) Finally, for V_t , we use the sum of the market value of common stock and the book value of debt, net of cash and liquid securities. Appendix B reports more details on data sources and data construction.

Figure 4 and Table 1 report, for this data source, the key ratios $ROA_{1,t} = \Pi_t/K_{1,t}$, $i_{1,t} = I_{1,t}/K_{1,t}$, $i_{2,t} = I_{2,t}/K_{2,t}$, $S_t = K_{2,t}/K_{1,t}$, and $Q_{1,t} = V_t/K_{1,t}$ used in our analysis. We report these data when intangibles are measured using either R&D capital, or R&D plus organization capital. The same key macro trends as in the NFCB sector are visible, in particular with elevated rates of returns to physical capital, high Q_1 , and declining physical investment rates after 1990. When measuring intangibles using only R&D capital, the time series for the NFCB sector as a whole, and for publicly traded firms only, are essentially identical. This is reassuring, since the NIPA definition of intangibles (and the one used in our prior analysis of the NFCB) is primarily a measure of R&D capital. The one notable difference is that returns to capital are higher among publicly traded firms, by 3 to 5 p.p. over the sample. Expanding the definition of intangibles to include organization capital leads to substantially higher intangible investment rates and ratios of physical to intangible capital (about double). Additionally, it produces an upward revision of returns to physical capital, of 5 to 7 p.p., since operating surplus must now be adjusted for the expensing of intangible investment in organization capital.

⁹A limitation of this approach is that it does not allow for either rates of depreciation, or for the imputation of investment in organization capital, to vary across industries. This can be done for R&D capital by using the industry-level depreciation rate produced by the BEA for the fixed asset tables, as in [Belo et al. \(2018\)](#). No similar data source exists for organization capital, however.

Findings Figure 5 reports the physical investment gap decomposition (17) for publicly traded non-financial firms. Two findings are worth highlighting.

First, when intangibles are measured using only R&D expenditures in Compustat, the decomposition points to a similar, if only slightly higher, contribution of the term capturing rents to physical assets (67% of the gap in 2015, as opposed to 64% in the NFCB data.) This may be surprising, since average return on physical capital is higher among publicly traded firms. The reason why this does not translate into substantially higher markups is that physical investment rates are also, on average, higher among publicly traded firms, relative to the NFCB sector as a whole, as indicated in Table 1. Markups, though, are slightly higher in this approach relative to the NFCB — by about 2 p.p. in 2015, as reported in figure 6.

Second, the inclusion of organization capital substantially increases both the direct effect of intangibles, and rents on intangibles. These terms together account for between one half and two thirds of the implied investment gap. In 2015, for instance, the two intangibles-related terms account for 54% of the total investment gap, as opposed to 36% in our baseline analysis. The direct effect of intangibles is 18%, due to the large estimated intangible share of capital in production; and rents on intangibles account for the remaining 36%.

Figure 6 reports model-implied moments for publicly traded non-financial firms, along with the model-implied moments for the NFCB sector obtained in our earlier analysis. The notable differences between the NFCB sector as a whole and publicly traded firms are in the user cost of physical capital and the intangible share. In particular, user costs are initially higher, but fall more rapidly among publicly traded firms, and moreover, the intangible share of capital inclusive of organization capital is much higher among these firms. Both of these effects amplify the term capturing rents to intangibles in the physical investment gap. Markups, on the other hand, are comparable to those obtained for the NFCB sector as a whole. If anything, the increase in markups is more muted once organization capital is taken into account: they rise by only approximately 2 p.p. from 1995 to 2015, instead of 4p.p. when only including R&D capital.

Overall, the analysis of publicly traded non-financial firms largely confirms the findings for the NFCB sector; moreover, they indicate that including a broader measure of intangible capital — organization capital — leads to a larger estimate of the contribution of rents earned by intangibles to the overall physical investment gap.

3.4 Robustness

Alternative measures of V_t As discussed earlier, our measure of V_t does not net out all financial assets from our estimate of the market value of liabilities of the NFCB sector, but instead only nets out those financial assets that the Flow of Funds tables identify explicitly as liquid. These include cash and cash-like securities, mutual fund shares, and corporate equities.¹⁰ We proceed this way for three reasons. First, this corresponds most closely to the definition of V_t which can be derived from balance sheet data for non-financial firms in Compustat. Second, the main other line item for financial assets in the Flow of Funds,

¹⁰This item is constructed by Flow of Funds from Compustat data, and represents non-consolidated investments in other firms.

Miscellaneous assets, is primarily derived as a residual which reconciles the Flow of Funds data with two other sources, for total balance sheet assets of non-financial corporations, the Statistics of Income and the Quarterly Financial Report. Conceptually, financial assets are netted out in the computation of V_t in order to obtain a measure of net debt claims. Because it is unclear whether the residual financial assets estimated in the Flow of Funds are liquid, and can therefore be used to pay down gross debt, we choose to leave them out of the computation.

Our approach thus differs from the computation of V_t in a number of papers, and in particular, Hall (2001), where all Flow of Funds financial assets are subtracted from liabilities in the computation of V_t . Appendix Figure C.1 reports our estimate of V_t against the estimate obtained using the Hall (2001) approach. The gap between the two is substantial; the latter approach estimates \$25trn of net claims on the NFCB sector in 2015, or approximately the market value of equities of S&P500 firms at that time. As a result, the values of Q_1 obtained in this approach, and reported in appendix Figure C.3, are lower than in our baseline, though they display approximately the same medium and long-run trends. In particular, according to this measure, Q_1 is below one prior to the 1960's, as well as from 1973 to 1990.

Appendix Figure C.4 reports the physical investment gap decomposition obtained using this alternative measure of Q_1 . The main difference with our baseline estimates is in the overall level of the physical investment gap; it is about half as large. However, our main conclusions remain the same. First, the investment gap is positive in the 1960 to 1970 period, and after 1990. Second, after 1990, the contribution of the two intangibles terms to the gap is substantially larger; together, they account for about half of the gap during the period. Moreover, the contribution of rents to intangibles (as opposed to the direct intangibles effect) is larger in the post-1990's period, accounting for 20% of the total gap in 2015, as opposed to 26% in our baseline approach.

Targeting the PD ratio As discussed in section 3.1, our baseline approach infers the value of the Gordon growth term, $r - g$, from the empirical value of Q_1 . An alternative way of inferring the value of this term is to use measures of the price-dividend (PD) ratio. Indeed, in the model with constant growth and linear adjustment costs,

$$PD = \frac{1}{1+r} \frac{V_t}{D_t} = \frac{1}{r-g},$$

where $D_t = \Pi_t - I_{1,t} - I_{2,t}$ denotes total distributions to owners of the firm.

In principle, one could therefore construct our decomposition using a measure of the PD ratio for the NFCB sector, as opposed to Q_1 , in order to infer the value of $r - g$. Directly measuring total distributions for the NFCB sector is however challenging, in particular for debt distributions. Proxies such as changes in the estimated market value of debt outstanding, plus interest payments, lead to estimates that are both very volatile and persistently negative for a large part of the post-2000's data. We therefore use a simpler approach. We estimate a price-dividend ratio for equity, using cum- and ex-dividend returns on the S&P500, and adjust these estimates for leverage (at market values) and interest payments, estimated from Flow of Funds data. The details of this variable construction are reported in appendix B.

Appendix Figure C.5 reports the resulting empirical measure of the PD ratio, along with the model-implied PD ratio obtained using our baseline approach. The model-implied PD ratio is lower overall, and increases less during the 1960 and post-1990 periods, than its two empirical counterparts. This indicates that the decline in $r - g$ implied by our measure of Q_1 in the NFCB data is smaller than what is implied by our empirical PD ratio measure, which could potentially affect our estimates.

We therefore construct an alternative version of decomposition (17), using our empirical measure of the PD ratio — the solid line in appendix Figure C.5 — instead of our measure of Q_1 for NFCB as an input.¹¹ Appendix A contains the details for the construction of the decomposition of the physical investment gap in this case. Figure C.6 reports the implied investment gap and its decomposition in this alternative approach.

A number differences are worth highlighting. First, targeting the PD ratio implies a much larger estimated investment gap in the first half of the sample, prior to 1980. Intuitively, this comes from the fact that empirical measures of the PD ratio are about twice as large than implied by our baseline approach in that part of the sample. This translates into lower measures of user costs during that part of the sample, as reported in Figure C.6. In 1955, for instance, user costs for physical capital were 16% in our baseline approach, whereas they are 12% when targeting the PD ratio. The lower user costs, in turn, imply that estimates of the markups are substantially larger for the first half of the sample when targeting the PD ratio, as indicated by the bottom left panel of Figure C.6. In fact, in this alternative approach, the post-1985 increase in markups does not look large in comparison to markup levels in the mid-1960's. The high inferred markup in the first half of the sample means that the investment gap is larger during that time period. Note, however, that user costs and markups coincide in the two approaches in the latter part of the sample, and that inferred intangible shares are largely identical in the two approaches.)

Second, the PD ratio approach leads to model-implied values of Q_1 that are also larger than our empirical measure constructed using Flow of Funds data. Appendix Figure C.10 shows that the model-implied measure of Q_1 in this approach captures medium-run trends in the empirical measure of Q_1 well (which is to be expected, since both the empirical measures of PD and Q_1 are partially driven by movements in the market value of equities of non-financials), but that its overall level is higher by about 40% than its empirical counterpart.

These discrepancies may, at least in part, be driven by differences the sample underlying our measure of Q_1 (the NFCB sector) and the sample underlying our measure of the PD ratio (the S&P 500). Indeed, our estimates of the physical investment gap using the Compustat non-financial sample lead to somewhat higher estimates of Q_1 , and as a result of the physical investment gap, than for the NFCB sector as a whole; moreover, these estimates were derived using data on all publicly traded non-financial firms, as opposed to the S&P500, which consists of larger, potentially higher- Q firms. An additional reason for the discrepancy may

¹¹Note that the model cannot simultaneously match separately constructed measures of the PD ratio and of Q_1 . Indeed, the definition of distributions in the model implies that $\Pi_t = I_{1,t} + I_{2,t} + (PD)^{-1}Q_1K_{1,t}$; moreover, $\Pi_t, I_{1,t}, I_{2,t}$ and $K_{1,t}$ are all inputs into our measurement strategy. Of course, our baseline decomposition approaches produces a PD ratio that is, by construction, consistent with the PD ratio measured as the ratio of the market value of net claims on the NFCB, V_t , to the gap between the surplus Π_t and investment $I_{1,t} + I_{2,t}$. But this gap may not coincide with measures of distributions for the NFCB derived from other sources, in particular from the Flow of Funds.

be mis-measurement in distribution to equityholders; in particular, the PD ratio we use is based on cash distributions, and excludes share repurchases, which became more common after the early 2000's.

However, targeting the PD ratio still leads to the two same basic insights as the baseline approach: first, there are two important periods during which the investment gap was elevated, the 1960-1970 period, and the post-1985 period; and second, the contribution of the interaction between intangibles and markups is muted prior to 1985, but becomes important after 1985. By 2015, the interaction term accounts for 28% of the total investment gap in the PD ratio approach (compared to 26% in our baseline approach).

Adjustment costs Our baseline exercise uses the model with linear adjustment costs but, as discussed in section 2.3, decomposition (17) can be extended to the case of convex adjustment costs, as follows:

$$G_1 = \frac{\mu_j - 1}{r - g} R_1 + S q_2^d + \frac{\mu - 1}{r - g} R_2 S, \quad (22)$$

where $q_2^d = 1 + \gamma_2 g$, and the adjusted user costs R_1 and R_2 are given by $R_1 = r + \delta_1 + \gamma_1 r g$, $R_2 = r + \delta_2 + \gamma_2 r g$. Note that, in this case, the physical investment gap is $G_1 = Q_1 - (1 + \gamma_1 g)$.

Appendix A shows how to construct the different elements of the decomposition above, given values for γ_1 , γ_2 , and g . The method effectively backs out the discount rate r that is consistent with given adjustment costs and a measure for the growth rate of the capital stock, g . We apply this approach to the data by measuring g as the growth rate of the sum of the two capital stocks, $(K_{2,t+1} + K_{1,t+1}) / (K_{1,t} + K_{2,t}) - 1$, consistent with the model, in which both grow at the same rate. Moreover, we use estimates of the adjustment cost to the physical and knowledge (R&D) capital stock produced by [Belo et al. \(2018\)](#).¹²

Results are reported in appendix Figure 15. Different from the baseline model, the investment gap is now somewhat negative in the late 1970's, a period during our empirical proxy for g is high. But results after 1985 are consistent with our baseline approach; in particular, the term capturing rents to intangibles still accounts for approximately one quarter of the total gap. Thus, our approach is robust to the introduction of convex adjustment costs.

4 An application to sectoral data

We finally use our framework to compare trends in the physical investment gap across sectors. We focus particularly on differences across sectors in the contribution of the term capturing rents to intangibles.

¹²These authors estimate a model with different types of intangible capital and adjustment costs to each, but no market power. We use their estimates of $\gamma_1 = 4.73$ and $\gamma_2 = 6.21$, from Table 3, column (4) of their paper, which is the version of their model with only physical and R&D capital. Consistent with our adjustment cost function, which is approximately quadratic, their estimation cannot reject the null that adjustment costs are quadratic for both types of capital.

4.1 Data sources

Our data source for the sectoral analysis is the Compustat non-financial sample, from 1975 to 2017. We use the Compustat non-financial sample primarily because, to our knowledge, there is no comprehensive data sectoral source that can be used to measure gross surplus at the annual frequency in a sufficiently long sample in order to construct our decomposition of the physical investment gap.¹³

We split the Compustat sample into four broad sectors: the Consumer sector, which primarily captures retail and wholesale trade; the High-tech sector, which contains firms in the Software and IT sectors; the Healthcare sector, which contains producers of medical devices, drug companies, and healthcare services companies; and the Manufacturing sector. This industry classification is similar to the Fama-French 5 classification. We omit the fifth group, which primarily contains service industries, including professional and business services, and hospitality services. The details of the classification are reported in appendix B. The rest of our variables are constructed as in the previous section.

For each of our four sectors, we report, in appendix Figures C.11 to C.14, the time series for the five data moments that are used in the analysis. The High-tech and Healthcare sectors are characterized by the same combination of high asset returns and high valuation, declining physical investment, and increasing intangible share. The Consumer sector also features high returns and low physical investment, but the trends in the intangible share are more muted there. Finally, the manufacturing sector features declining returns, declining physical investment, and a declining intangible share after the 2000's.

4.2 Results

Figure 8 reports the decomposition of the physical investment gap obtained using the model with constant growth — equation (17) — for the four sectors of our analysis, when using only R&D capital as a measure of intangibles in each sector. The main message from this figure is that there is substantial sectoral heterogeneity both in the level and in the composition of the investment gap.

One extreme is the Manufacturing sector. In that sector, the investment gap is, overall, low. Moreover, little of it is explained by either the direct effect of intangible capital, or by rents earned on intangible capital. This is consistent with the fact that in that sector, returns have been declining since the late 1990's, and the stock of R&D capital has been declining, relative to the stock of physical capital, since the early 2000's, as reported in appendix Figure C.14. As a result, implied markups in the manufacturing sector are low, and the implied intangible share of capital in production has also been either stable or declining, as reported in Figure 9.

The other extreme is the Consumer sector. There, the investment gap is large, in particular after 1990, reflecting rising asset returns and valuations. However, the quasi-totality of it is explained by rents to physical capital. This is due to the fact that, in this sector, measured R&D capital — our metric for intangibles in this baseline exercise — is very small. It rises slightly after the mid-2000's, driven primarily by Amazon's reported R&D expenditures, but remains overall too small to account for the physical investment gap. As a result,

¹³The closest data source is the KLEMS database, but these data only cover the post-1990 period.

the model-implied markups for that sector, reported in Figure 9, are both high and rising after the late 1990’s. Using R&D as a measure of intangible capital, the investment gap in the Consumer sector thus primarily appears to be the result of growing rents to existing physical assets.

The Healthcare and High-tech sectors are intermediate cases. Both have experienced a large increase in the physical investment gap after the late 1990’s. In both cases, the decomposition indicates that rents on physical capital have also increased. However, they only account for about one-half — in the High-tech sector — and one-third — in the Healthcare sector — of the investment gap overall. The key change in the composition of the investment gap after 1990 seems to have been, for both sectors, a substantial increase in rents to intangible capital. For the Healthcare sector, for instance, they account, alone, for approximately half of the total investment gap. Figure 9 shows that this is the effect of two combined trends: a rising intangible share; and a rise in markups overall. Markups in both sectors rise by approximately 10 p.p. between 1995 and 2015, in line with the estimated increase in markup for the entire Compustat. The intangible share in both sectors also increases substantially, particularly for the Healthcare sector, where it roughly doubles.

These baseline results only use R&D as a measure of the intangible capital stock. When adding organization capital to the measurement of total intangibles, as in section 3, results are qualitatively unchanged for the High-tech, Healthcare, and Manufacturing sectors.¹⁴ However, they do change substantially for Consumer sector. Figure 10 shows that including organization capital, the direct effect of intangibles becomes the most important contributor to the investment gap for the Consumer sector. Our estimates of organization capital indeed imply a very high ratio of intangible to physical assets for that sector, as reported in Figure 11. Estimated markups for the retail sector also decline substantially in levels, and somewhat in trend. Prior work (Foster et al., 2006; Crouzet and Eberly, 2018a) has indeed argued that the Consumer sector relies extensively on intangible capital, particularly brand capital and, in more recent years, innovations to supply chain and logistics. Investment in these intangibles are not recorded as R&D expenditures, but instead expensed as SG&A, and so they are picked up by our measure of organization capital. However, this measure of intangibles shows no substantial trend upward after the 1990’s. Thus, our results are consistent with the view that the overall level of the investment gap in the Consumer sector is attributable to the effect of intangibles, the coarseness of our intangibles measure for this sector makes it difficult to capture more specific medium-run trends.

Overall, application of our decomposition of the investment gap to sectoral data reveals large sectoral heterogeneity in the importance of the key underlying forces documented in section 3. In particular, the rise in rents to intangible capital seems to be the key driver of the investment gap in both the High-tech and Healthcare sectors.

5 Conclusion

We show that the gap between average Q , as observed, and marginal q , which measures the incentive to invest, can be decomposed into terms capturing economic rents and intangible capital. Viewed through the lens of this decomposition, aggregate and sectoral data on US

¹⁴Appendix Figures C.15 to C.17 report these results.

non-financial firms suggest that, while the rise in economic rents on traditional, physical capital is a central explanation of recent investment trends, it is far from the whole story. We highlighted the growing contribution of rents to intangible capital, driven by a growing intangible share and elevated user costs of intangibles. Additionally, we showed that taking intangibles into account lowers substantially both the level and the trend in estimated markups. Finally, we showed that aggregate trends mask considerable heterogeneity across sectors, with rents to intangible capital appearing to be the dominant contributor to the physical investment gap in the High-tech and Healthcare sectors, while less so in other sectors.

Our work could naturally be expanded in two directions. First, the "Q+" framework could be expanded to allow for risk premia, which [Farhi and Gourio \(2018\)](#) (among others) have argued are important in understanding investment trends. Second, the framework could be used to analyze the distribution of rents and intangible capital within sectors, and ask, in particular, whether the rise in the investment gap and in rents to intangibles is driven by within-firm changes, or by a reallocation of activity toward certain firms. We explore these questions in future research.

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Tables and figures

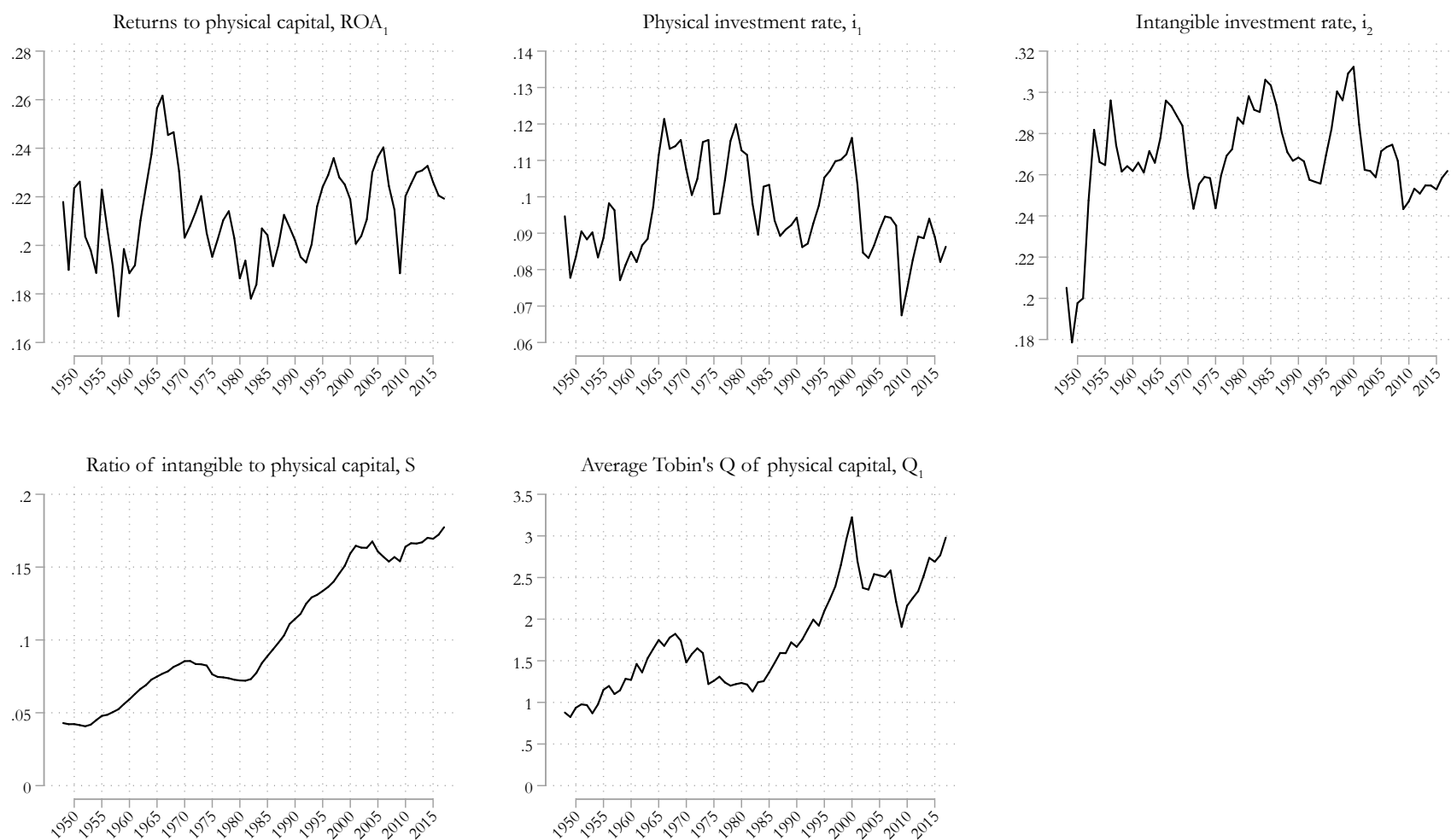


Figure 1: Time series for the moments used in the construction of the physical investment gap decomposition (17). Returns to physical capital are defined as $\Pi_t/K_{1,t}$, where Π_t is operating surplus and $K_{1,t}$ the stock of physical capital at current (replication) cost. Investment rates are defined as $i_{n,t} = I_{n,t}/K_{n,t}$, $n = 1, 2$, where $n = 1$ indexes physical capital and $n = 2$ indexes intangible capital, $K_{2,t}$ is the stock of intangible capital at current (replication) cost, and $I_{n,t}$ are investment expenditures for each type of capital. The ratio of intangible to physical capital is $S_t = K_{2,t}/K_{1,t}$. Average Tobin's Q of physical capital is defined as $Q_{1,t} = V_t/K_{1,t}$, where V_t is an estimate of the total market value of net claims on the sector. The time series are the raw data; in particular, they are not averaged over seven-year windows. Data sources and variables are described in the main text and in appendix B.

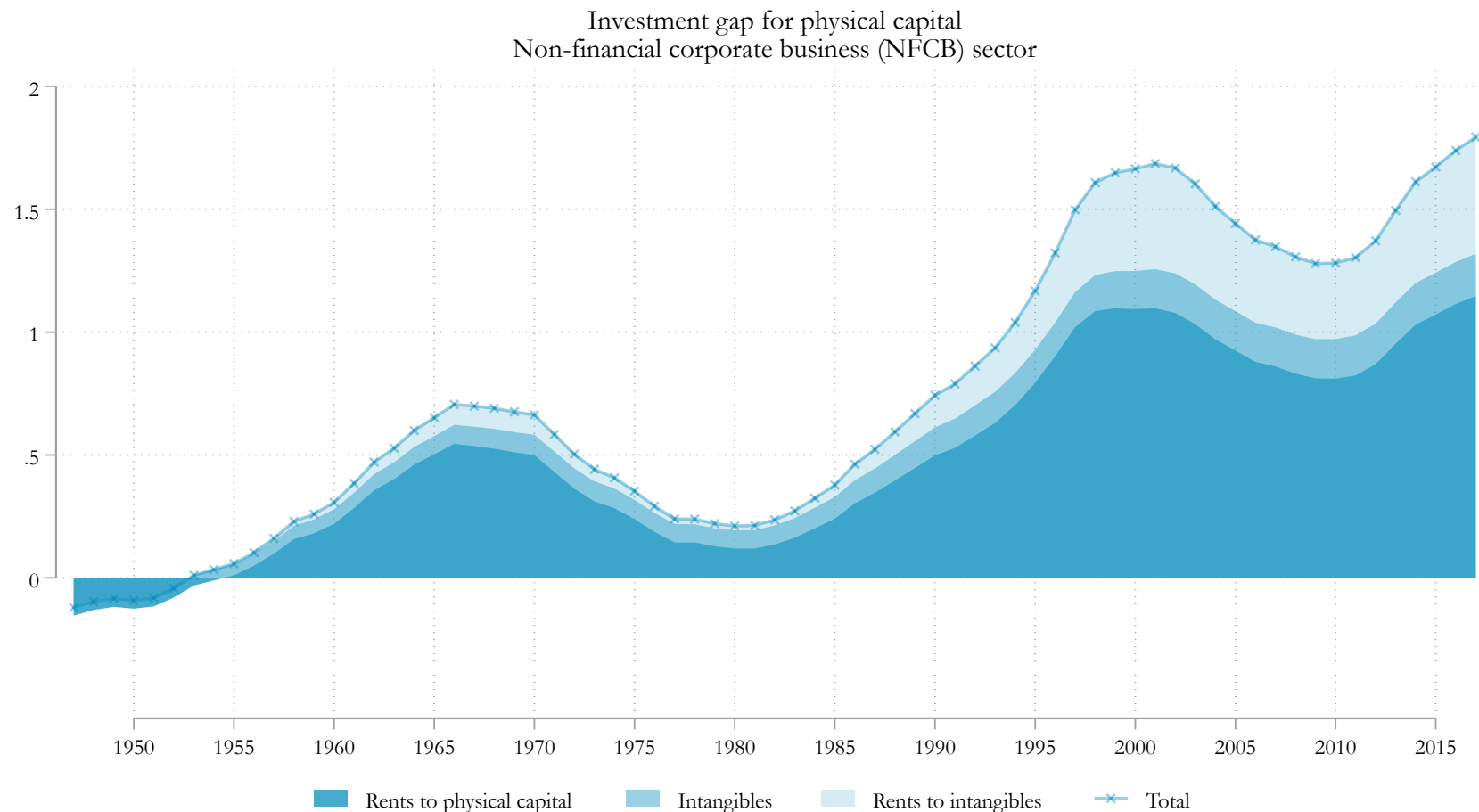


Figure 2: The physical investment gap for the NFCB sector, in the constant growth model with linear adjustment costs. The physical investment gap is defined as the gap between the average Q of physical capital, Q_1 , and its marginal q ; in the model with linear adjustment costs, marginal q is equal to 1 and so the investment gap is simply $Q_1 - 1$. The crossed blue line is an estimate of Q_1 constructed using data from the Flow of Funds (for the numerator) and from the BEA fixed asset tables (for the denominator). The shaded areas present the decomposition of the physical investment gap into three terms, corresponding to the effects of rents on physical capital, intangibles, and rents on intangibles. The decomposition is described in equation (17). The time series for the moments used to construct the decomposition are reported in figure 1; section 3 and appendix B discuss data sources and variables in detail.

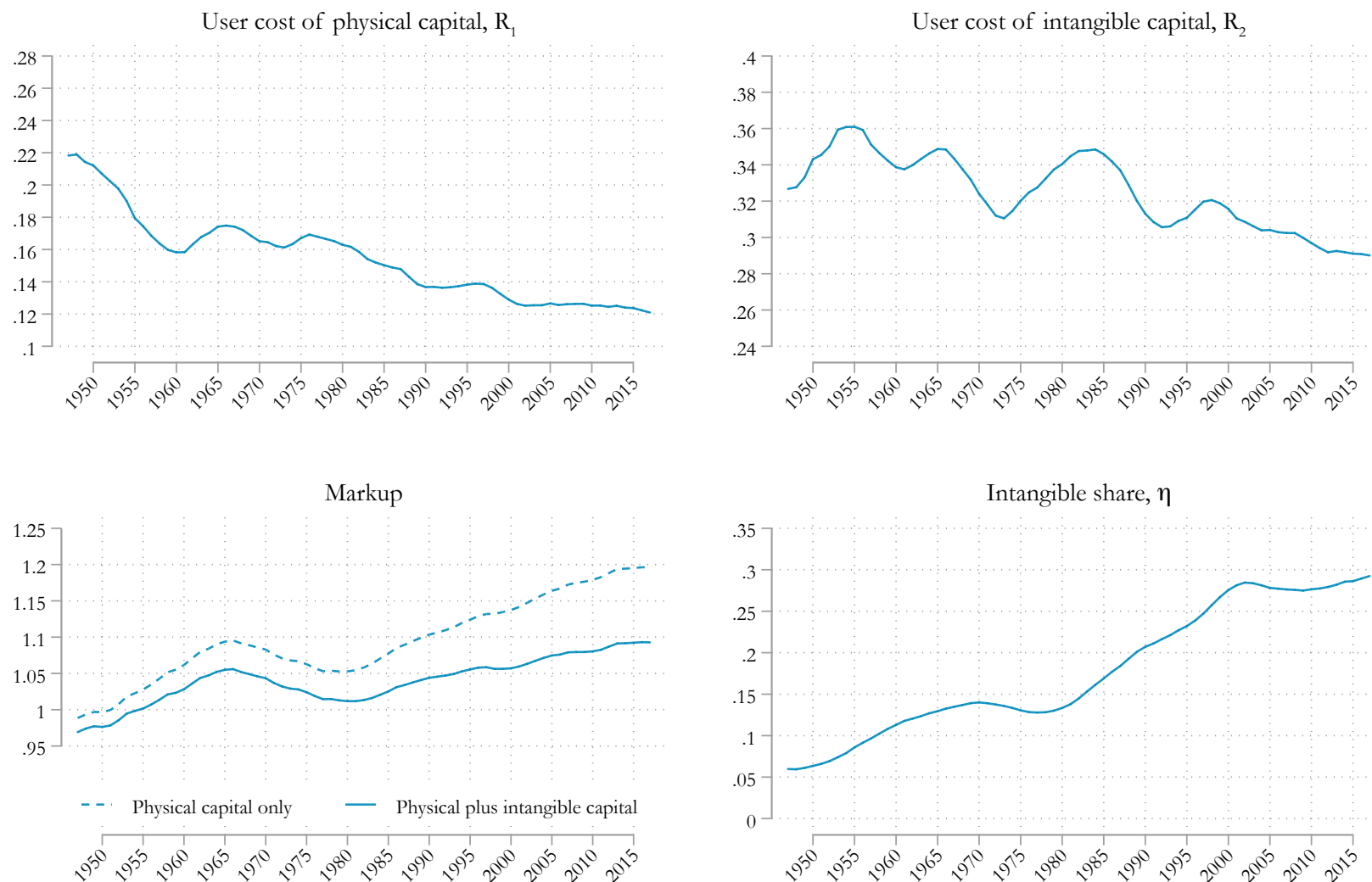


Figure 3: Model-implied real moments for the NFCB sector, in the constant growth model with linear adjustment costs. The top panels report the estimated path of the user costs of physical and intangible capital. The bottom left panel reports implied markups. So as to make them comparable to existing estimates, markups reported this figure include the contribution of labor income, as explained in appendix A.3. Specifically, they are given by $\tilde{\mu} = \frac{\mu}{LS\mu + (1-LS)}$, where μ is the market power index, measured in the data using $\mu = \frac{ROA_1}{R_1 + SR_2}$. We measure the share of labor income LS using NIPA Table 1.14; the corresponding time series is reported in appendix Figure C.2. The bottom left panel also reports, in the dashed line, the markup implied by a model without intangible capital. The bottom right panel reports the implied share of intangible capital in production assuming $\rho = 0$, i.e. Cobb-Douglas substitutability between intangible and physical capital.

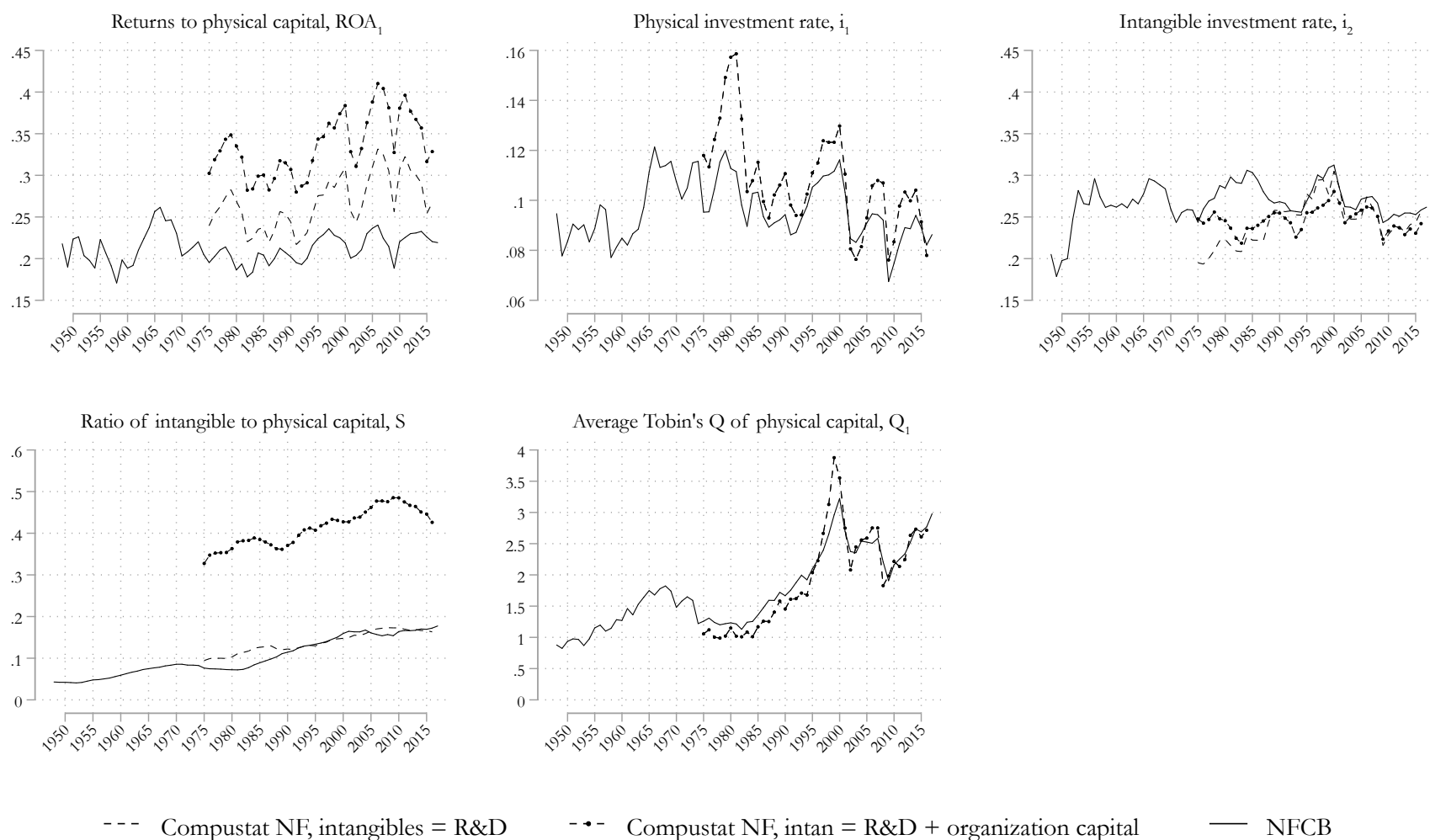


Figure 4: Time series for the moments used in the construction of the physical investment gap decomposition (17), for the sample of publicly traded non-financial firms (Compustat NF.) Returns to physical capital are defined as $\Pi_t/K_{1,t}$, where Π_t is operating surplus and $K_{1,t}$ the stock of physical capital at current (replication) cost. Investment rates are defined as $i_{n,t} = I_{n,t}/K_{n,t}$, $n = 1, 2$, where $n = 1$ indexes physical capital and $n = 2$ indexes intangible capital, $K_{2,t}$ is the stock of intangible capital at current (replication) cost, and $I_{n,t}$ are investment expenditures for each type of capital. The ratio of intangible to physical capital is $S_t = K_{2,t}/K_{1,t}$. We report times series for two alternative measures of the intangible capital stock: R&D capital, and the sum of R&D and organization capital, with the latter measured following Eisfeldt and Papanikolaou (2013) and Peters and Taylor (2017). Average Tobin's Q of physical capital is defined as $Q_{1,t} = V_t/K_{1,t}$, where V_t is an estimate of the market value of claims. The time series reported are the raw data; in particular, they are not averaged over seven-year windows. We also report the same moments, for the NFCB sector, for comparison; these are the same moments as those reported in figure 1. Data sources and variables are described in more detail the main text and in appendix B.

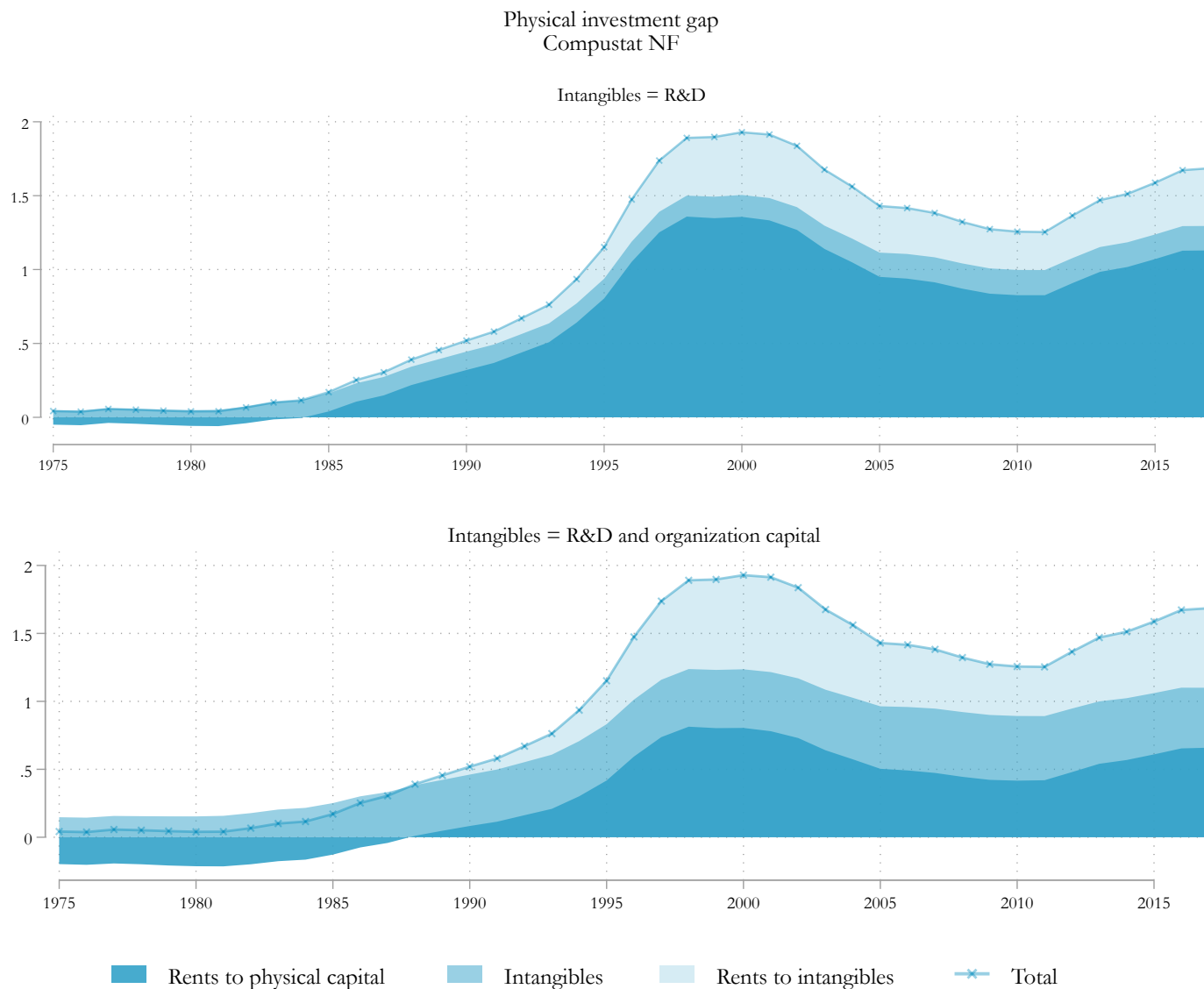


Figure 5: The physical investment gap among publicly traded non-financial firms (Compustat NF), in the constant growth model with linear adjustment costs. The physical investment gap is defined as the gap between the average Q of physical capital, Q_1 , and its marginal q ; in the model with linear adjustment costs, marginal q is equal to 1 and so the investment gap is simply $Q_1 - 1$. The crossed blue line is an estimate of Q_1 constructed using accounting data on the value of equity and debt claims, the value of liquid securities, and the replacement cost of physical capital. The shaded areas present the decomposition of the physical investment gap into three terms, corresponding to the contribution of rents on physical capital, intangibles, and rents on intangibles. The top panel uses only R&D capital as the measure of intangibles, while the bottom panel uses the sum of R&D and organization capital. The decomposition is described in equation (17). The time series for the moments used to construct the decomposition are reported in figure 4; section 3 and appendix B discuss data sources and variables in detail.

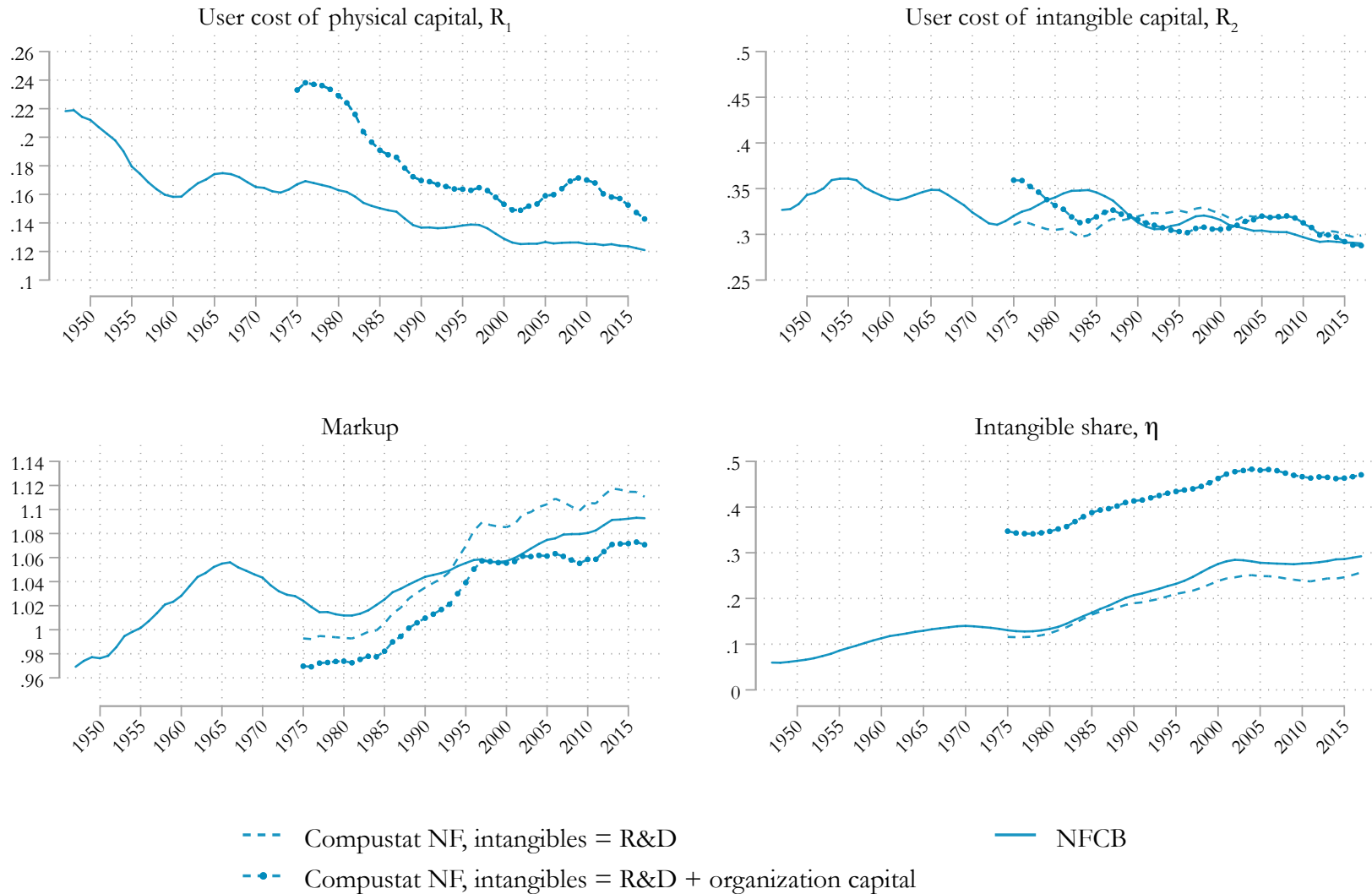


Figure 6: Model-implied real moments for publicly traded non-financial firms, in the constant growth model with linear adjustment costs. The top panels report the estimated path of the user costs of physical and intangible capital. The bottom left panel reports implied markups. So as to make them comparable to existing estimates, markups reported in this figure include the contribution of labor income, as explained in appendix A.3. Specifically, they are given by $\tilde{\mu} = \frac{\mu}{LS\mu + (1-LS)}$, where μ is the market power index, measured in the data using $\mu = \frac{ROA_1}{R_1 + SR_2}$. We measure the share of labor income LS using NIPA Table 1.14; the corresponding time series is reported in appendix Figure C.2. The bottom right panel reports the implied share of intangible capital in production assuming $\rho = 0$, i.e. Cobb-Douglas substitutability between intangible and physical capital. In each panel, we report model-implied moments when measuring intangibles as either R&D capital, or the sum of R&D capital and organization capital. The implied moments for the NFCB sector, which are the same as in figure 3, are also reported.

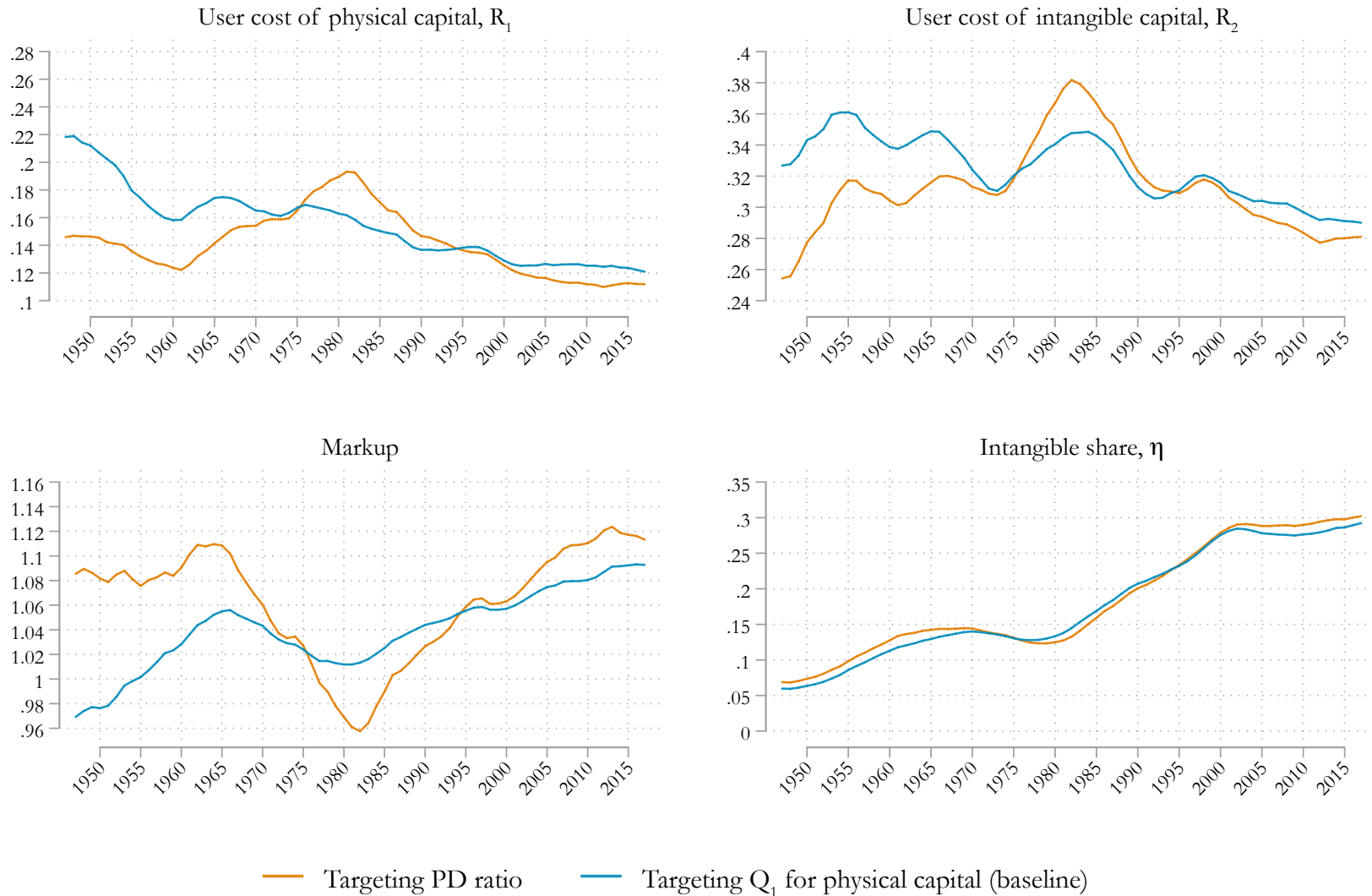


Figure 7: Model-implied real moments for the NFCB sector, in the constant growth model with linear adjustment costs, when targeting the PD ratio instead of average Q . The top panel reports the estimated path of the user costs of physical and intangible capital. The bottom left panel reports implied markups. So as to make them comparable to existing estimates, markups reported in this figure include the contribution of labor income. Specifically, they are given by $\tilde{\mu} = \frac{\mu}{LS\mu + (1-LS)}$, where μ is the market power index, measured in the data using $\mu = \frac{ROA_1}{R_1 + SR_2}$. We measure the share of labor income LS using NIPA Table 1.14; the corresponding time series is reported in appendix Figure C.2. The bottom right panel reports the implied share of intangible capital in production assuming $\rho = 0$, i.e. Cobb-Douglas substitutability between intangible and physical capital. In all figures, the orange lines correspond to the estimates obtained when targeting the PD ratio, while the blue lines are identical to those reported in figure 3, that is, they are derived from targeting average Q_1 .

Physical investment gap by sector
Compustat NF, intangibles = R&D



Figure 8: The physical investment gap among in four sectors. The data source is the sample of non-financial publicly traded firms. We split this sample into four main sectors, according to the industry classification reported in appendix B. Variables are otherwise identical to the analysis of the aggregate sample of Compustat non-financial firms described in section 3. The time series for the moments used to construct the decomposition are reported in appendix Figures C.11 to C.14. For this figure, we only use R&D capital as our measure of intangibles.

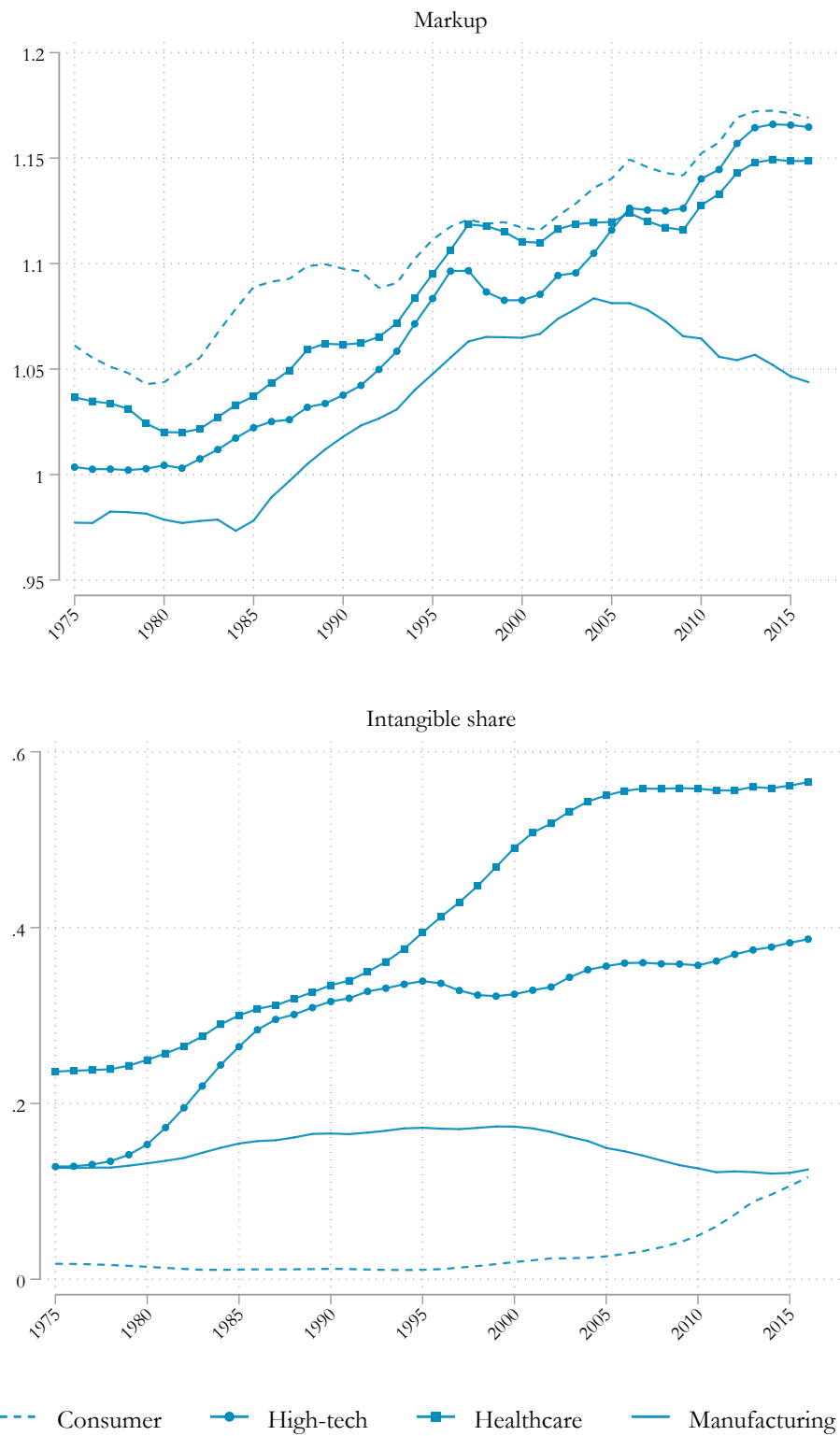


Figure 9: Model-implied real moments for publicly traded non-financial firms, by sector. The top panel reports implied markups. So as to make them comparable to existing estimates, markups reported in this figure include the contribution of labor income, as explained in appendix A.3. The bottom panel reports the implied share of intangible capital in production assuming $\rho = 0$, i.e. Cobb-Douglas substitutability between intangible and physical capital. For both panels, we use R&D as our measure of intangible capital.

Physical investment gap
Compustat NF, Consumer sector

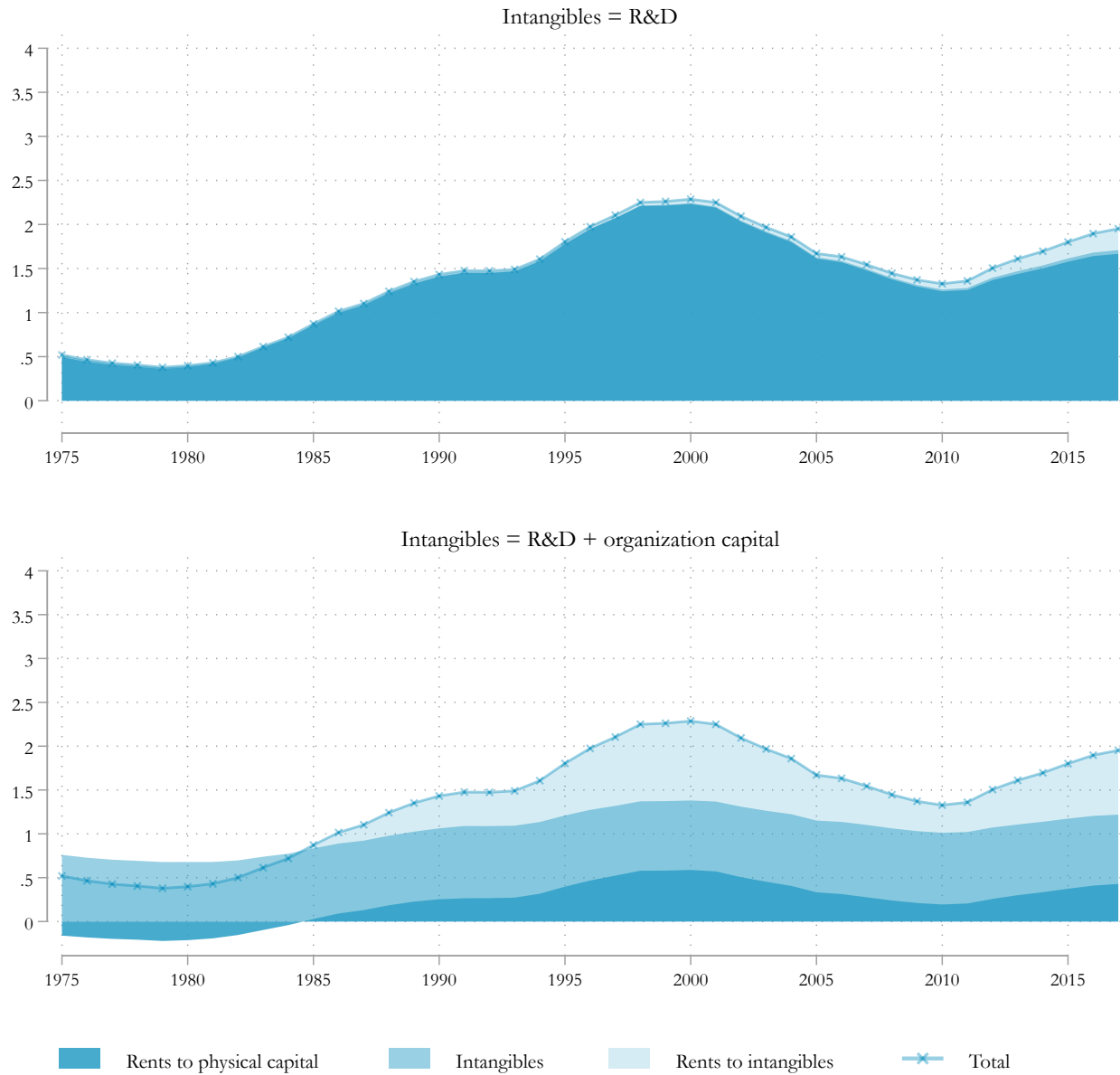


Figure 10: The physical investment gap among publicly traded non-financial firms belonging to the Consumer sector, with two different measures of the intangible capital. The top panel uses only R&D capital as the measure of intangibles, while the bottom panel uses the sum of R&D and organization capital. The time series for the moments used to construct the decomposition are reported in figure C.11.

Compustat NF, Consumer sector

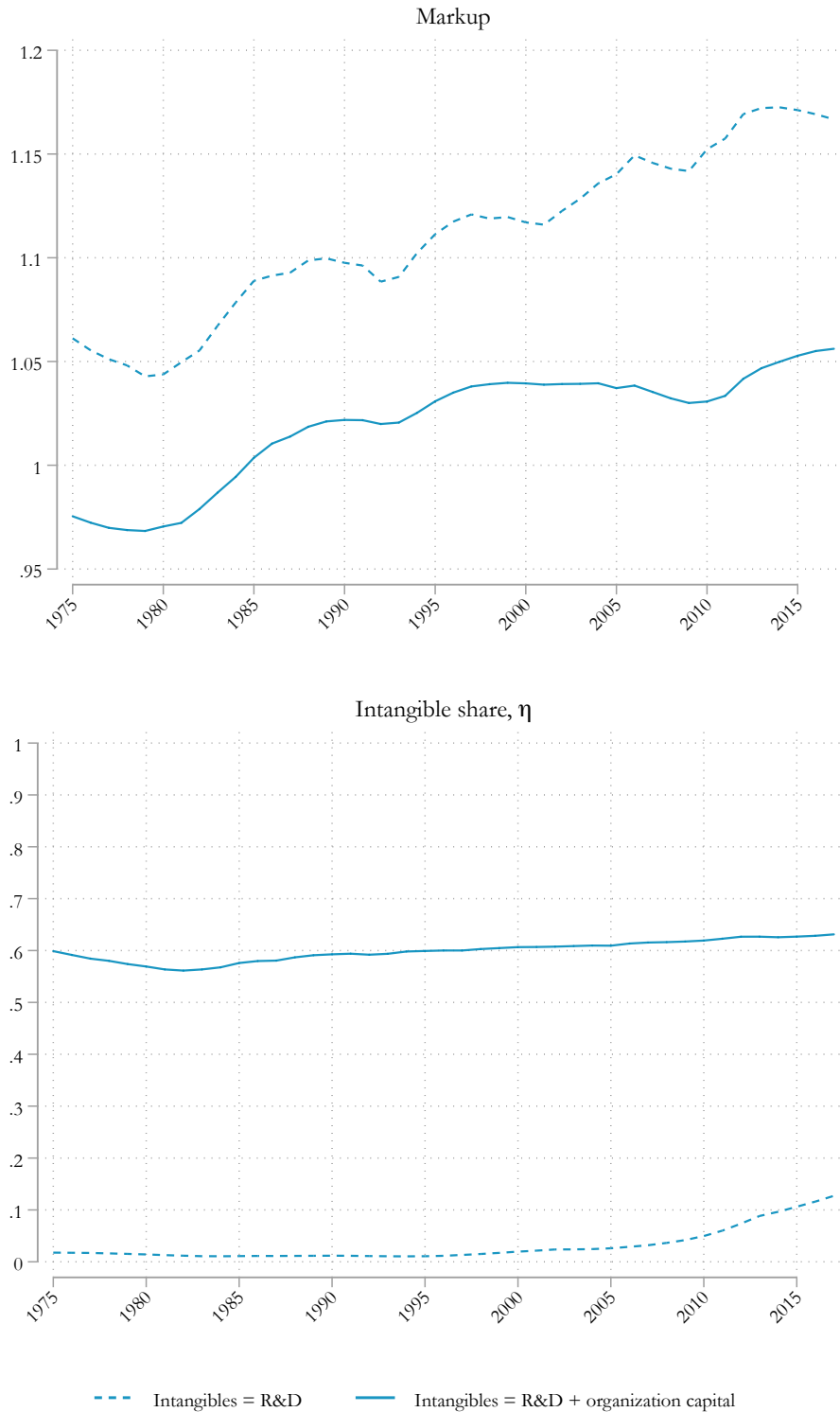


Figure 11: Model-implied real moments for publicly traded non-financial firms for the Consumer sector. The top panel reports implied markups. So as to make them comparable to existing estimates, markups reported this figure include the contribution of labor income, as explained in appendix A.3. The bottom panel reports the implied share of intangible capital in production assuming $\rho = 0$, i.e. Cobb-Douglas substitutability between intangible and physical capital. The two panels report both the results obtained using only R&D capital as the measure of intangibles, and the results obtained using the sum of R&D and organizational capital as the measure of intangibles.

		NFCB				Compustat NF (R&D)		Compustat NF (R&D+SG&A)	
Targeted moments		1947	1966	1985	2001	1985	2001	1985	2001
		1965	1984	2000	2017	2000	2017	2000	2017
i_1	Physical investment rate	0.089	0.107	0.098	0.088	0.107	0.095	0.107	0.095
i_2	Intangible investment rate	0.251	0.275	0.279	0.262	0.259	0.249	0.250	0.245
S	Intangible/physical capital	0.053	0.079	0.123	0.164	0.131	0.265	0.399	0.459
ROA_1	Return on physical capital	0.209	0.211	0.210	0.222	0.254	0.291	0.320	0.362
Q_1	Av. Q for physical capital	1.15	1.39	1.98	2.50	1.94	2.50	1.94	2.50
Non-targeted moments (model)		1947	1966	1985	2001	1985	2001	1985	2001
		1965	1984	2000	2017	2000	2017	2000	2017
R_1	User cost of physical capital	0.186	0.165	0.139	0.125	0.170	0.158	0.170	0.158
R_2	User cost of intangible capital	0.346	0.333	0.320	0.299	0.321	0.311	0.312	0.307
μ	Markup (excl. labor)	1.03	1.09	1.17	1.27	1.20	1.39	1.09	1.21
$\tilde{\mu}$	Markup (incl. labor)	1.01	1.02	1.05	1.08	1.05	1.10	1.02	1.06
PD	Price-dividend ratio	11.0	17.1	25.8	27.8	16.9	16.3	16.9	16.4
Non-targeted moments (data)		1947	1966	1985	2001	1985	2001	1985	2001
		1965	1984	2000	2017	2000	2017	2000	2017
PD^e	Price-dividend ratio (equity-only)	22.7	24.9	40.2	50.4	41.2	49.7	41.2	49.7
PD^f	Price-dividend ratio (adj. for leverage)	22.2	21.9	35.2	45.4	26.1	38.4	26.1	38.4

Table 1: Data moments used in the decomposition of the physical investment. The moments reported are for the NFCB sector (columns 3 to 5), for Compustat non-financials using capitalized R&D as a measure of intangible capital (columns 6 and 7), for Compustat non-financials using using capitalized R&D and capitalized SG&A as a measure of intangible capital (columns 8 and 9). The moments are averages over each sub-period. Sources and construction method for the targeted moments and the data counterparts to the non-targeted moments are described in section 3 and in appendix B.

A Appendix: theory

A.1 Derivation of lemma 4

The first-order necessary condition and the envelope theorem, for each capital type, are:

$$\begin{aligned}\Phi'_{n,j,t} &= \mathbb{E}_t [M_{t,t+1} q_{n,j,t+1}] \\ q_{n,j,t} &= \Pi_{K_{n,j,t}} - \Phi_{n,j,t} + \Phi'_{n,j,t} \frac{K_{n,j,t+1}}{K_{n,j,t}}\end{aligned}\quad (23)$$

Multiplying the latter by $K_{n,j,t}$ and combining it with the former, we obtain:

$$q_{n,j,t} K_{n,j,t} = \Pi_{K_{n,j,t}} K_{n,j,t} - \Phi_{n,j,t} K_{n,j,t} + \mathbb{E}_t [M_{t,t+1} q_{n,j,t+1} K_{n,j,t+1}],$$

so that, iterating forward, and summing across capital types, we have:

$$\sum_{n=1}^N q_{n,j,t} K_{n,j,t} = \sum_{n=1}^N \sum_{k \geq 0} \mathbb{E}_t [M_{t,t+k} \{ \Pi_{K_{n,j,t+k}} K_{n,j,t+k} - \Phi_{n,j,t+k} K_{n,j,t+k} \}]. \quad (24)$$

On the other hand, firm value is given by:

$$V_{j,t} = \sum_{k \geq 0} \mathbb{E}_t \left[M_{t,t+k} \left\{ \Pi_{j,t+k} K_{j,t+k} - \sum_{n=1}^N \Phi_{n,j,t+k} K_{n,j,t+k} \right\} \right].$$

Note that, given assumption 1:

$$\begin{aligned}\Pi_{j,t+k} &= \mu_j \Pi_{K_{j,t+k}} K_{j,t} \\ &= \mu_j \sum_{n=1}^N \Pi_{K_{n,j,t+k}} \frac{\partial F_{j,t+k}}{\partial K_{n,j,t+k}} K_{n,j,t+k} = \mu_j \sum_{n=1}^N \Pi_{K_{n,j,t+k}} K_{n,j,t+k},\end{aligned}$$

so that firm value can be rewritten as:

$$V_{j,t} = \sum_{n=1}^N \sum_{k \geq 0} \mathbb{E}_t [M_{t,t+k} \{ \mu_j \Pi_{K_{n,j,t+k}} K_{n,j,t+k} - \Phi_{n,j,t+k} K_{n,j,t+k} \}]. \quad (25)$$

Taking the difference between equations (25) and (24) gives the result.

A.2 Total Q

Finally, one might wonder whether, in a model with homogeneity like the one studied in this paper, average Q for some measure of the total stock of capital properly captures the overall incentive to invest. Define total Q (Peters and Taylor, 2017) and total net investment as:

$$Q_{j,t} \equiv \frac{V_{j,t}}{\sum_{n=1}^N K_{n,j,t}}, \quad g_{j,t,t+1} = \sum_{n=1}^N w_{n,j,t+1} g_{n,j,t,t+1}. \quad (26)$$

In general, we have:

$$Q_{j,t} = \sum_{n=1}^N w_{n,j,t} q_{n,j,t} + (\mu_j - 1) \sum_{n=1}^N w_{n,j,t} \sum_{k \geq 0} \mathbb{E}_t [M_{t,t+k} \Pi_{K_{n,t,t+k}} (1 + g_{n,j,t,t+k})]. \quad (27)$$

Thus, total Q is a weighted average of marginal q plus a weighted average of economics rents, across types of capital. Total investment is given by:

$$g_{j,t,t+1} = \sum_{n=1}^N w_{n,j,t+1} \Psi_{n,j,t} (\mathbb{E}_t [M_{t,t+1} q_{n,j,t+1}] - 1). \quad (28)$$

Unsurprisingly, in the presence of rents ($\mu_j > 1$), total Q does not provide a measure of the incentive to invest overall.

However, even in the absence of rents ($\mu_j = 1$), total Q may not provide a measure of the overall incentive to invest; knowledge of each marginal $q_{n,j,t,t+1}$ is required. An exception is when adjustment costs are identical across types of capital goods, that is: $\Psi_{n,j,t} = \Psi_{j,t}$, and when the function $\Psi_{j,t}$ is linear; in that case, the expression above simplifies to $g_{j,t,t+1} = \Psi_{j,t} (\mathbb{E}_t [M_{t,t+1} Q_{j,t+1}] - 1)$, and total Q is indeed a sufficient statistic for total investment. Otherwise, the presence of convex adjustment costs implies that total Q is a biased signal for total investment, though the sign of the bias is ambiguous.

A.3 An explicit example leading to the profit function (12)

Consider the following model. A representative household chooses consumption per capita, c_t , to solve:

$$U_t = \max \frac{(C_t)^{1-\sigma}}{1-\sigma} + \beta U_{t+1}, \quad (29)$$

which implies the discount rate $M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$. Labor supply is exogenous. Final goods are produced and sold by a perfectly competitive firm; total final output is given by:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{1}{\tilde{\mu}}} dj \right)^{\tilde{\mu}}, \quad (30)$$

with $\tilde{\mu} > 1$. This leads to the demand curve $Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\frac{\tilde{\mu}}{\tilde{\mu}-1}} Y_t$, and a price index is given by $P_t = \left(\int_0^1 P_{j,t}^{-\frac{1}{\tilde{\mu}-1}} dj \right)^{-(\tilde{\mu}-1)}$. Each firm j produces an intermediate variety, with a production function taking labor and total capital as inputs:

$$Y_{j,t} = Z_{j,t} K_{j,t}^\alpha L_{j,t}^{1-\alpha}, \quad (31)$$

where total capital is an aggregate of physical and intangible capital, as described in the main text. Here, $Z_{j,t}$ is an exogenous process capturing firm-level or aggregate total factor productivity. Finally, the goods market clears:

$$P_t Y_t = P_t (c_t L_t) + \sum_{n=1}^2 \int_0^1 \Phi \left(\frac{K_{n,j,t+1}}{K_{n,j,t}} \right) K_{n,j,t} dj. \quad (32)$$

In this model, labor is flexible in the short-run, so each firm solves:

$$\Pi_{j,t} = \max_{L_{j,t}} P_{j,t} \left(\frac{P_{j,t}}{P_t} \right)^{-\frac{\tilde{\mu}}{\tilde{\mu}-1}} Y_t - W_t L_{j,t}.$$

After some computation, the solution to this problem can be written as:

$$\Pi_{j,t} = A_{j,t}^{\frac{1}{\mu}-1} K_{j,t}^{\frac{1}{\mu}}$$

where:

$$\mu = 1 + \frac{\tilde{\mu} - 1}{\alpha},$$

and:

$$A_{j,t} = (\alpha + \tilde{\mu} - 1)^{1 + \frac{\alpha}{\tilde{\mu}-1}} \tilde{\mu}^{-\frac{\tilde{\mu}}{\tilde{\mu}-1}} (1 - \alpha)^{\frac{1-\alpha}{\tilde{\mu}-1}} D_t W_t^{-\frac{1-\alpha}{\tilde{\mu}-1}} Z_{j,t}^{\frac{1}{\tilde{\mu}-1}},$$

$$D_t \equiv P_t^{\frac{\tilde{\mu}}{\tilde{\mu}-1}} Y_t.$$

Moreover, labor demand is given by:

$$P_{j,t} = \tilde{\mu} MC_{j,t}$$

$$MC_{j,t} = (1 - \alpha)^{-\frac{(1-\alpha)(\tilde{\mu}-1)}{\tilde{\mu}-1+\alpha}} \tilde{\mu}^{-\frac{\alpha\tilde{\mu}}{\tilde{\mu}-1+\alpha}} D_t^{\frac{\alpha(\tilde{\mu}-1)}{\alpha+\tilde{\mu}-1}} W_t^{\frac{(1-\alpha)(\tilde{\mu}-1)}{\tilde{\mu}-1+\alpha}} Z_j^{-\frac{\tilde{\mu}-1}{\tilde{\mu}-1+\alpha}} K_{j,t}^{-\frac{(\tilde{\mu}-1)\alpha}{\tilde{\mu}-1+\alpha}}$$

$$L_{j,t} = \left(\frac{(1 - \alpha) MC_{j,t} Z_j}{W_t} \right)^{\frac{1}{\alpha}} K_{j,t}$$

where:

$$MC_{j,t} = (1 - \alpha)^{-\frac{(1-\alpha)(\tilde{\mu}-1)}{\tilde{\mu}-1+\alpha}} \tilde{\mu}^{-\frac{\alpha\tilde{\mu}}{\tilde{\mu}-1+\alpha}} D_t^{\frac{\alpha(\tilde{\mu}-1)}{\alpha+\tilde{\mu}-1}} W_t^{\frac{(1-\alpha)(\tilde{\mu}-1)}{\tilde{\mu}-1+\alpha}} Z_j^{-\frac{\tilde{\mu}-1}{\tilde{\mu}-1+\alpha}} K_{j,t}^{-\frac{(\tilde{\mu}-1)\alpha}{\tilde{\mu}-1+\alpha}}.$$

Since prices are given by: $P_{j,t} = \tilde{\mu} MC_{j,t}$, the labor demand curve implies that:

$$LS_{j,t} \equiv \frac{W_t L_{j,t}}{P_{j,t} Y_t} = \frac{1 - \alpha}{\tilde{\mu}}.$$

For given values of μ and $LS_{j,t}$, we can recover the corresponding value of $\tilde{\mu}$, the markup earned by the firm net of labor costs. (Instead, μ is the total markup earned by the firm, or alternatively, the markup earned by capital when there are no labor costs, i.e. $\alpha = 1$.) We have:

$$\tilde{\mu} = \alpha(\mu - 1) + 1 = (1 - \tilde{\mu} LS_{j,t})(\mu - 1) + 1,$$

and so, solving for $\tilde{\mu}$:

$$\tilde{\mu} = \frac{\mu}{\mu LS_{j,t} + (1 - LS_{j,t})}.$$

A.4 Derivation in specific cases with analytical solutions

In the following derivations, we omit the index j to simplify notation.

A.4.1 Constant growth

We assume that the adjustment cost function, for each type of capital, is given by::

$$\Phi_n(1+x) = x + \delta_n + \gamma_n r \left(x + (r-x) \log \left(\frac{r-x}{r} \right) \right), \quad n = 1, 2.$$

It can be checked that this cost function is strictly convex and satisfies the standard conditions $\Phi_n(1) = \delta_n$, $\Phi'_n(1) = 1$ and $\Phi''_n(1) = \gamma_n$, $n = 1, 2$. Additionally, these functions satisfy the relationship:

$$(r-x)\Phi'(1+x) + \Phi(1+x) = r + \delta_n + \gamma_n r x.$$

The case $\gamma_n = 0$, $n = 1, 2$, is the case of linear adjustment costs. The necessary first-order conditions, for each type of capital, are given by:

$$\begin{aligned} \Phi'_{n,t} &= \mathbb{E}_t [M_{t,t+1} q_{n,t+1}] = M_{t,t+1} q_{n,t+1}, \\ q_{n,t} &= \Pi_{K_{n,t}} - \Phi_{n,t} + \Phi'_{n,t} \frac{K_{n,t+1}}{K_{n,t}}. \end{aligned} \tag{33}$$

Moreover, recall that $M_{t,t+1} = (1+r)^{-1}$, and that A_t is growing at the exogenous rate g . We can use this to rewrite the necessary two first-order conditions as:

$$(1+r)\Phi'_n(1+g_{n,t-1}) = \Pi_{K_{n,t}} - \Phi_n(1+g_{n,t}) + \Phi'_n(1+g_{n,t})(1+g_{n,t}),$$

where $g_{n,t} \equiv \frac{K_{n,t+1}}{K_{n,t}} - 1$. We next guess and verify that $g_{n,t} = g$ for $n = 1, 2$ is a solution. Substituting into the condition above, and re-arranging, we obtain:

$$(r-g)\Phi'_n(1+g) + \Phi_n(1+g) = \Pi_{K_{n,t}}.$$

Using the functional form for Φ_n , we can rewrite this as:

$$r + \delta_n + \gamma_n r x = \Pi_{K_{n,t}}. \tag{34}$$

Moreover:

$$\Pi_{K_{n,t}} = \frac{1}{\mu} A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}-1} \frac{\partial K_t}{\partial K_{n,t}}.$$

Given our guess and the linear homogeneity of the capital aggregator, K_t also grows at rate g . Moreover, each partial derivative $\frac{\partial K_t}{\partial K_{n,t}}$ is homogeneous of degree 0 in each of its arguments, implying that it only depends on the ratio $S_t = \frac{K_{2,t}}{K_{1,t}}$ of the two capital stocks. This ratio, given our guess, is constant. Hence, the right-hand side of (34), verifying our guess. The ratio of the two capital stocks, as well as the ratio K_t/A_t , adjust so that the two first-order conditions (34) hold. In particular, taking the ratio of these two first-order conditions, we obtain:

$$\frac{r + \delta_2 + \gamma_2 r g}{r + \delta_1 + \gamma_1 r g} = \frac{\partial K_{2,t}}{\partial K_{1,t}} = \frac{1-\eta}{\eta} \left(\frac{K_{2,t}}{K_{1,t}} \right)^{\rho-1},$$

which allows to solve for the stationary ratio of intangible to physical capital, $S = \frac{K_{2,t}}{K_{1,t}}$. Finally, note that in this model, the investment- q relationship $\Phi'_n(1+g_{n,t}) = (1+r)^{-1} q_n$, can be approximated, up to first order, by:

$$1 + \gamma_n g = 1 + \gamma_n (i_n - \delta_n) = (1+r)^{-1} q_n,$$

given the fact that $\Phi'_n(1+x) = 1 + \gamma_n x + o(x^2)$.

A.4.2 Stochastic growth

In this version of the model, we assume linear adjustment costs, $\Phi_n(1+x) = x + \delta_n$, and a constant discount factor, $M_{t,t+1} = (1+r)^{-1}$. Moreover, we assume the following process for A_t :

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 1 + g_{t-1} & \text{w.p. } (1-\lambda) \\ 1 + \tilde{g} & \text{w.p. } \lambda \end{cases}$$

Here, g_t is known at the beginning of period t . Moreover, \tilde{g} is drawn from a distribution $F(\cdot)$, which is time-invariant, and the draw is independent of past realizations of g_t . We use the following result, which is a version of the result used in [Abel and Eberly \(2011\)](#).

Lemma 2. *We have:*

$$\mathbb{E}_t \left[\sum_{k \geq 0} (1+r)^{-k} \frac{A_{t+k}}{A_t} \right] = \frac{1+r + \lambda(1+g_t)\zeta^*}{r - g_t + \lambda(1+g_t)},$$

where:

$$\zeta^* = \mathbb{E} \left[\frac{1+r}{r - \tilde{g} + \lambda(1+\tilde{g})} \right] \mathbb{E} \left[\frac{r - \tilde{g}}{r - \tilde{g} + \lambda(1+\tilde{g})} \right]^{-1}.$$

Proof. Let:

$$\zeta(g_t) \equiv \mathbb{E}_t \left[\sum_{k \geq 0} (1+r)^{-k} \frac{A_{t+k}}{A_t} \right],$$

then $H(g_t)$ satisfies:

$$\zeta(g_t) = 1 + \frac{1+g_t}{1+r} [\lambda\zeta^* + (1-\lambda)\zeta(g_t)],$$

where we used the law of iterated expectations, and the fact that $\frac{A_{t+1}}{A_t} = 1 + g_t$ is known at time t . Here, we have denoted:

$$\zeta^* = \int_{\tilde{g}} \zeta(\tilde{g}) dF(\tilde{g}).$$

Solving for $\zeta(g_t)$, we obtain:

$$\zeta(g_t) = \frac{1+r + \lambda(1+g_t)\zeta^*}{r - g_t + \lambda(1+g_t)}.$$

Taking expectations on both sides,

$$\zeta^* = \mathbb{E} \left[\frac{1+r + \lambda(1+\tilde{g})\zeta^*}{r - \tilde{g} + \lambda(1+\tilde{g})} \right].$$

Re-arranging,

$$\mathbb{E} \left[\frac{r - \tilde{g}}{r - \tilde{g} + \lambda(1+\tilde{g})} \right] \zeta^* = \mathbb{E} \left[\frac{1+r}{r - \tilde{g} + \lambda(1+\tilde{g})} \right],$$

which gives the result. □

The necessary first-order conditions to the firm's problem are:

$$\begin{aligned} q_{n,t} &= \frac{1}{\mu} A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}-1} \frac{\partial K_t}{\partial K_{n,t}} + 1 - \delta_n, \\ 1 &= \frac{1}{1+r} \mathbb{E}_t [q_{n,t+1}], \end{aligned}$$

for $n = 1, 2$, which we can rewrite (taking expectations at time $t - 1$, and using the fact that A_t is known at $t - 1$) as:

$$\mu(r + \delta_n) = A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}-1} \frac{\partial K_t}{\partial K_{n,t}}. \quad (35)$$

In particular, this implies that $q_{n,t} = 1 + r$ is constant. Tedious computation shows that these first-order conditions, in combination with the aggregator defining K_t , can be written as:

$$\begin{aligned} K_{1,t} &= \left(\frac{r + \bar{\delta}}{r + \delta_1} \right)^{\frac{1}{1-\rho}} \eta^{\frac{1}{1-\rho}} [\mu(r + \bar{\delta})]^{-\frac{\mu}{\mu-1}} A_t, \\ K_{2,t} &= \left(\frac{r + \bar{\delta}}{r + \delta_2} \right)^{\frac{1}{1-\rho}} (1 - \eta)^{\frac{1}{1-\rho}} [\mu(r + \bar{\delta})]^{-\frac{\mu}{\mu-1}} A_t, \\ K_t &= [\mu(r + \bar{\delta})]^{-\frac{\mu}{\mu-1}} A_t, \end{aligned}$$

where:

$$r + \bar{\delta} \equiv \left(\eta^{\frac{1}{1-\rho}} (r + \delta_1)^{-\frac{\rho}{1-\rho}} + (1 - \eta)^{\frac{1}{1-\rho}} (r + \delta_2)^{-\frac{\rho}{1-\rho}} \right)^{-\frac{1-\rho}{\rho}}.$$

The expression for the value of the firm reported in the main text, (4), can be written, in the case of this model, as:

$$\begin{aligned} V_t &= \sum_{n=1}^2 q_{n,t} K_{n,t} + (\mu - 1) \sum_{n=1}^2 \mathbb{E}_t \left[\sum_{k \geq 0} (1+r)^{-k} \frac{1}{\mu} A_{t+k}^{1-\frac{1}{\mu}} K_{t+k}^{\frac{1}{\mu}-1} \frac{\partial K_{t+k}}{\partial K_{n,t+k}} K_{n,t+k} \right] \\ &= \sum_{n=1}^2 q_{n,t} K_{n,t} + (\mu - 1) \sum_{n=1}^2 \mathbb{E}_t \left[\sum_{k \geq 0} (1+r)^{-k} (r + \delta_n) K_{n,t+k} \right] \\ &= \sum_{n=1}^2 q_{n,t} K_{n,t} + (\mu - 1) \sum_{n=1}^2 \alpha_n (r + \delta_n) \mathbb{E}_t \left[\sum_{k \geq 0} (1+r)^{-k} A_{t+k} \right] \\ &= \sum_{n=1}^2 q_{n,t} K_{n,t} + (\mu - 1) \sum_{n=1}^2 \alpha_n (r + \delta_n) A_t \zeta(g_t) \\ &= \sum_{n=1}^2 q_{n,t} K_{n,t} + (\mu - 1) \sum_{n=1}^2 \frac{1+r}{r - \nu(g_t)} (r + \delta_n) \alpha_n A_t \\ &= \sum_{n=1}^2 q_{n,t} K_{n,t} + (\mu - 1) \sum_{n=1}^2 \frac{1+r}{r - \nu(g_t)} (r + \delta_n) K_{n,t}. \end{aligned}$$

Here, using conditions (35), we defined α_1 and α_2 such that $K_{1,t} = \alpha_1 A_t$ and $K_{2,t} = \alpha_2 A_t$, and we used lemma 2. Additionally, by analogy with the case of constant growth rates, we

define:

$$\zeta(g_t) = \frac{1+r}{r-\nu(g_t)},$$

or equivalently:

$$\nu(g_t) = g_t + \lambda(1+g_t) \frac{(r-g_t)\zeta^* - (1+r)}{(1+r) + \lambda(1+g_t)\zeta^*}.$$

Finally, note that the conditions (35) imply that:

$$\mu(R_1K_{1,t} + R_2K_{2,t}) = \Pi_t,$$

where $R_1 = r + \delta_1$, $R_2 = r + \delta_2$. Using the expressions for $K_{1,t}$ and $K_{2,t}$, this in turn implies that:

$$\mu = \frac{ROA_1}{R_1 + SR_2},$$

where $S = \frac{K_{2,t}}{K_{1,t}}$, and $ROA_1 = \frac{\Pi_t}{K_{1,t}}$.

A.5 Constructing the decomposition of the investment gap

Our baseline empirical approach for the aggregate and sectoral decomposition proceeds as follows (omitting the time subscripts). Targeted moments are:

$$\{Q_1, S, i_1, i_2, ROA_1\}. \quad (36)$$

Model-implied moments are:

$$\begin{aligned} R_1 &= \frac{ROA_1 - (i_1 + Si_2)}{Q_1} + i_1 \\ R_2 &= \frac{ROA_1 - (i_1 + Si_2)}{Q_1} + i_2 \\ PD &= \frac{Q_1}{ROA_1 - (i_1 + Si_2)} \\ \mu &= \frac{ROA_1}{(i_1 + Si_2) + \frac{1+S}{Q_1}(ROA_1 - i_1 - i_2S)} \end{aligned} \quad (37)$$

Moreover, a necessary condition for the implied PD ratio to be positive is:

$$ROA_1 > i_1 + Si_2. \quad (38)$$

while a necessary condition for the implied μ to be larger than 1 is:

$$Q_1 > 1 + S. \quad (39)$$

This approach will lead to implied values of the PD ratio that are not necessarily consistent with the data. By construction, in this approach, the total firm value used in the numerator of the computation of Q will be consistent with the model-implied value, i.e. $V = Q_1K_1$.

But the PD ratio could use a different number than V in the numerator, or, the model-implied value for dividends, $D = \Pi - I_1 - I_2$, could be inconsistent with the values used in the denominator of the computation of the PD ratio. (Alternatively, this approach can be viewed as producing a time series for D for the entire *NFCB*, and hence an estimate of the PD ratio consistent with whatever measure of the market values of claims on the *NFCB* were used to compute Q_1 .)

The alternative approach targets values for the PD ratio. It can be summarized as follows. Targeted moments are:

$$\{PD, S, i_1, i_2, ROA_1\}. \quad (40)$$

Model-implied moments are:

$$\begin{aligned} R_1 &= PD^{-1} + i_1 \\ R_2 &= PD^{-1} + i_2 \\ Q_1 &= PD(ROA_1 - (i_1 + Si_2)) \\ \mu &= \frac{ROA_1}{(i_1 + Si_2) + PD^{-1}(1 + S)} \end{aligned} \quad (41)$$

Moreover, in this approach, a necessary condition for the implied Q_1 to be larger than 1 is:

$$ROA_1 > PD^{-1} + (i_1 + Si_2). \quad (42)$$

while a necessary condition for the implied μ to be larger than 1 is:

$$ROA_1 > (1 + S)PD^{-1} + (i_1 + Si_2). \quad (43)$$

This approach will lead to implied values of Q_1 that are not necessarily consistent with the data. If K_1 that is used in the denominator for the computation of Q_1 in the data is the same as the one used in the computation of the ratios above, the discrepancy comes from the fact that in the model:

$$V = PD(\Pi - I_1 - I_2),$$

where I_1 , I_2 and Π are empirical measures of the operating surplus. This model-implied value of the firm could be different from the value of V used to construct Q_1 in the data.

Finally, we briefly discuss how to construct the decomposition of the investment gap with convex adjustment costs, which is given by:

$$G_1 = \frac{\mu_j - 1}{r - g}(r + \delta_1 + \gamma_1 r g) + S q_2^d + \frac{\mu - 1}{r - g_j}(r + \delta_2 + \gamma_2 r g)S, \quad (44)$$

with $q_2^d = 1 + \gamma_2 g$. The discussion of the model with adjustment costs in this appendix shows that:

$$\mu = \frac{ROA_1}{R_1 + SR_2}, \quad R_1 \equiv r + \delta_1 + \gamma_1 r g, \quad R_2 = r + \delta_2 + \gamma_2 r g.$$

Substituting this into the expression of $G_1 = Q_1 - (1 + \gamma_1 g)$, and re-arranging, we obtain:

$$r = \frac{ROA_1 - i_1 - Si_2}{Q_1} + g - \frac{\gamma_1 + \gamma_2 S}{Q_1} g^2.$$

The rest of the moments needed for the decomposition can then be derived as described in the approach that targets Q_1 .

B Appendix: data

B.1 Data sources

B.1.1 National accounts

We use the following time series from NIPA, all for the non-financial corporate business sector (NFCB): NFCB gross value added ($Y^{(BEA)}$) (FRED series A455RC1Q027SBEA), NFCB compensation of employees ($WN^{(BEA)}$) (A460RC1Q027SBEA), NFCB taxes on production less subsidies ($T^{(BEA)}$) (FRED series W325RC1Q027SBEA), NFCB transfers ($Tr^{(BEA)}$) (FRED series W325RC1Q027SBEA). The data are annual. We use these data to compute the surplus of the NFCB sector as:

$$\Pi^{(BEA)} = Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}$$

and to compute the labor share of the NFCB sector as:

$$LS = WN^{(BEA)} / (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)}).$$

The labor share for the NFCB sector is reported in figure C.18. Additionally, we obtain current cost measures of the capital stock for the NFCB sector from the BEA fixed asset tables. We extract $K_{struct}^{(BEA)}$, $K_{equip}^{(BEA)}$ and $K_{intan}^{(BEA)}$, from BEA table 4.1.¹⁵ This table only contains estimates of non-residential fixed assets. We obtain residential fixed assets $K_{resid}^{(BEA)}$ as the difference between the sum of the three capital stocks above, and total fixed assets of NFCB sector report in BEA fixed asset table 6.1. As described below, the stock of residential fixed assets of the NFCB is small and does not affect any of our findings, so with the exception of a few graphs where it is explicitly mentioned, we only use information from table 4.1. We then define:

$$K_1^{(BEA)} = K_{struct}^{(BEA)} + K_{equip}^{(BEA)}, \quad K_2^{(BEA)} = K_{intan}^{(BEA)}.$$

We use tables 4.7 and 6.7 to obtain measures of current investment for the NFCB sector, and we define $I_1^{(BEA)}$ and $I_2^{(BEA)}$ analogously to $K_1^{(BEA)}$ and $K_2^{(BEA)}$. In some of the discussion that follows, we also use the following annual time series: current-cost net stock of fixed assets for the private sector (FRED series W327RC1A027NBEA), Nominal GDP (FRED series GDPA), Consumer Price Index (FRED series CPALTT01USA661S).

The top panel of figure C.18 reports time series for the various measures of the ratio $\Pi^{(BEA)} / K^{(BEA)}$, which we use in our calibration. The solid red and solid orange line use $\Pi^{(BEA)}$ as the numerator, and either the total capital stock $K = K_{struct}^{(BEA)} + K_{equip}^{(BEA)} + K_{intan}^{(BEA)} + K_{resid}^{(BEA)}$ or $K = K_{struct}^{(BEA)} + K_{equip}^{(BEA)} + K_{intan}^{(BEA)} + K_{resid}^{(BEA)}$; the two lines are almost identical. The dashed blue line report $\Pi / K_1^{(BEA)}$, the ratio of the surplus of the NFCB sector to the stock of physical assets (excluding residential assets.) This rate of return is approximately 5% points larger by the end of the sample than $\Pi / K^{(BEA)}$, reflecting the rising importance of intangibles.

Finally, the crossed blue line reports Π / K as measured in Farhi and Gourio (2018). The rate of return on capital measured by these authors is substantially lower than our measures of rates of return for the NFCB sector (by about 5-7% throughout the sample.) We discuss

¹⁵The tables are available at <https://apps.bea.gov/iTable/iTable.cfm?ReqID=10&step=2>.

differences here because they matter for inferences on the level of markups. The authors compute Π/K as:

$$\Pi/K = [(Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/(Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})] \times Y/K,$$

where Y is total nominal GDP (include other sectors than the NFCB) and K is the total private capital stock (at replacement cost). This adjustment is made in order to maintain comparability with other ratios in their analysis, which has a broader scope than the NFCB. By contrast, our measures of Π/K are:

$$\begin{aligned} \Pi/K &= [(Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/(K^{(BEA)})] \\ &= [(Y^{(BEA)} - WN^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/(Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})] \\ &\quad \times (Y^{(BEA)} - T^{(BEA)} - Tr^{(BEA)})/K^{(BEA)}. \end{aligned}$$

Thus the differences between our measures of Π/K and the measures in [Farhi and Gourio \(2018\)](#) is must be due to differences in the ratio of value added to capital between the NFCB sector and the economy as a whole. The bottom panel of figure [C.18](#) indeed shows that the NFCB sector has a substantially higher dollar of value added per dollar of capital at current cost. The most accurate comparison is between the crossed blue line of the bottom panel, and the orange solid line, which measures K for the NFCB sector as the sum of all types of capital (residential, non-residential physical, and non-residential intangible): the value added to capital ratio is approximately 10% points in the NFCB sector versus the economy as a whole.

B.1.2 Compustat Non-Financial

We use the annual version of the Compustat-CRSP merged files. We apply the standard screens (indfmt=INDL, popsrc=D, consol=C, datafmt=STD). We keep firm-year observations that satisfy the following criteria: fic=USA (domestically incorporated), 2-digit SIC code (first two digits of the variable sic) not equal to 49 (utilities), not between 60 and 69 (finance and real estate), and not between 90 to 99 (public administration); 2-digit SIC code not missing; variable sale (sales) and at (assets) not missing; variables emp, sale, at, act, lct, ppent, ppegt, che, and gdwl not negative. Finally, we drop any observation which we can identify as an American Depository Institution (ADR). We use only data from 1974 onward (included), as the data prior to 1974 has incomplete coverage (a jump in the number of firms in the sample occurs from 1973 to 1974.) Finally, we use the cum- and ex-price dividend ratio from CRSP for the S&P500 for the PD ratio. Specifically, we measure $R_t^{EX} = E_t[M_{t,t+1}V_{t+1}]/E_t[M_{t-1,t}V_t]$ $R_t^{CUM} = (D_t + E_t[M_{t,t+1}V_{t+1}])/E_t[M_{t-1,t}V_t]$, and use the fact that $PD_t = R_t^{EX}/(R_t^{CUM} - R_t^{EX})$.

B.1.3 Differences between NIPA and Compustat data for the NFCB sector

There are two potentially important differences between our main data sources. First, Compustat only includes publicly traded corporations. There may be systematic differences in

returns to capital and intangible intensity between privately held and publicly traded corporations. Second, rates of returns and capital stocks in Compustat may reflect both domestic and international activities. In the following section we explore these differences in more detail.

In Compustat, our baseline measure of surplus as the sum of EBITDA (variable `oibdp`) across all observations in our sample. (Missing observations are thus treated as zeros.) We use EBITDA because it is the financial statement measure most closely related to our model definition of Π ; it measures of operating income before depreciation, and does not deduct costs of capital, or non-operating income, which our model does not capture.¹⁶ The top right panel of figure C.19 report the NFCB sector surplus measured in this manner in Compustat, and the measure from the BEA. The two are highly correlated, but their levels differs substantially. This reflects the fact that the BEA NFCB sector data also includes private firms. The surplus of public firms (from Compustat) represents about two thirds of the total surplus of the NFCB sector (from the BEA).

The main difficulty in the Compustat data is in computing estimates of the current-cost total stock of physical capital. A natural definition would seem to be net property, plant and equipment (variable `ppent`.) However, measuring K_1 for the NFCB sector in Compustat leads to extremely elevated measures of Π/K_1 , as reported in the bottom right panel of figure C.19. These measures are almost double the BEA-derived measures. This is primarily because the aggregate value of `ppent` in Compustat is only about a third of physical capital in the NFCB sector according to BEA data (top left panel of figure C.19). The reason for this gap are unclear. One hypothesis is that the surplus of Compustat firms includes income from foreign subsidiaries, and so could overestimate the true surplus of public NFCB firms. Alternatively, it could be that private firms indeed have much lower rates of return on physical capital than public firms do (though the gap would have to be enormous, given the relative importance of public firms in total surplus, as indicated in the top right panel of figure C.19). The more likely reason is that the accounting treatment of depreciation may lead the (balance sheet) `ppent` stock to underestimate the true current cost stock of physical assets. The red line in the top left and bottom left panels of figure C.19 instead report measures of asset returns using aggregate `ppegt`, that is, property, plant and equipment at historical cost (appropriately deflated.) The bottom left panel shows that this estimate of K_1 leads to values of Π/K_1 that align more closely (in levels) with those provided by the BEA data on the NFCB. In what follows, in order to align our BEA and Compustat profitability moments as closely as possible, we therefore use total `ppegt` as our main measure of K_1 in Compustat data.

We measure (gross) investment in physical capital in Compustat using capital expenditures (variable `capx`) minus sales of property, plant and equipment (variable `sppe`). Figure C.20, top panel, shows that physical investment, computed in this manner, accounts for about two thirds of total physical investment in the BEA NFCB sector ($I_1^{(BEA)}$), with closely related cyclical movements. For investment rates (the bottom panel of figure C.20), the data again suggest a much higher investment rate in Compustat when K_1 is measured using `ppent`, but investment rates are closer in levels when the capital stock is measured in gross terms (using `ppegt`) in Compustat data.

¹⁶The inclusion of non-operating income makes little difference to the results.

C Appendix tables and figures

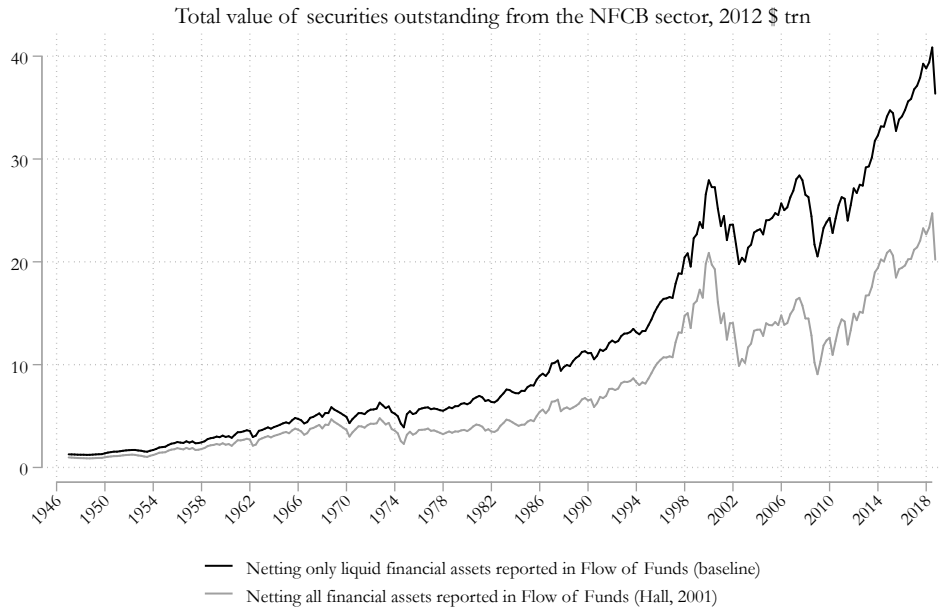


Figure C.1: Estimate of the market value of claims on the NFCB sector (V_t). This estimate uses data from the Flow of Funds tables F.103 and L.103. We use the Flow of Funds reported estimate of the market value of equities, and construct and estimate for the market value of debt using the approach of Hall (2001). The data series reported in this figure are quarterly, but we use the annualized (end-of-year) version in our main analysis. Details on methodology and data sources are reported in appendix B.

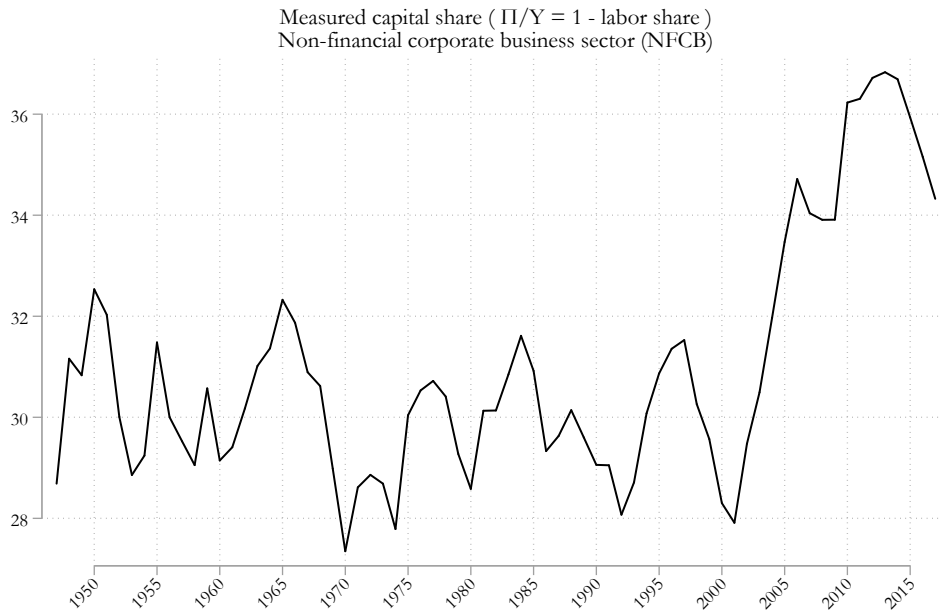


Figure C.2: Measured capital share in the non-financial corporate (NCFB) sector. The measured capital share is one minus the labor share, where the labor share is constructed using data from NIPA table 1.14. More details on data sources and variable construction are reported in appendix B.

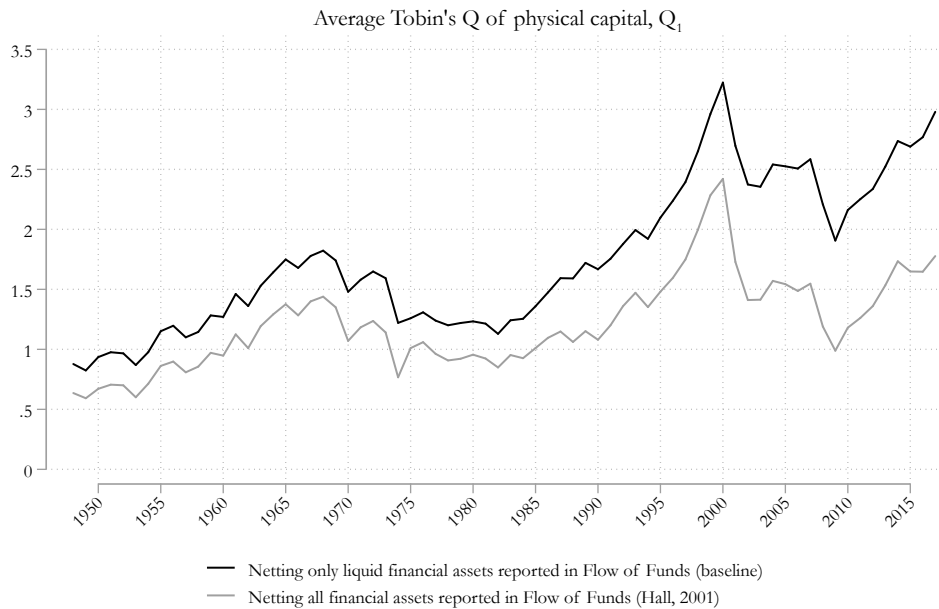


Figure C.3: Estimates of average Tobin's Q of physical capital, Q_1 , using two different measures of the value of claims on the NFCB sector. For the black line, we use our baseline measure of V_t , which only nets out liquid claims; the grey line uses a measure of V_t that nets out all financial assets reported in the Flow of Funds, as in Hall (2001). The two time series for V_t are reported in appendix Figure C.1. The numerator for both measures of Q_1 is the same, the NIPA estimate of the physical capital stock of the NFCB sector, used throughout the paper. The black line for Q_1 is the same as used in our baseline estimation and reported in figure 1. Details on methodology and data sources are reported in appendix B.

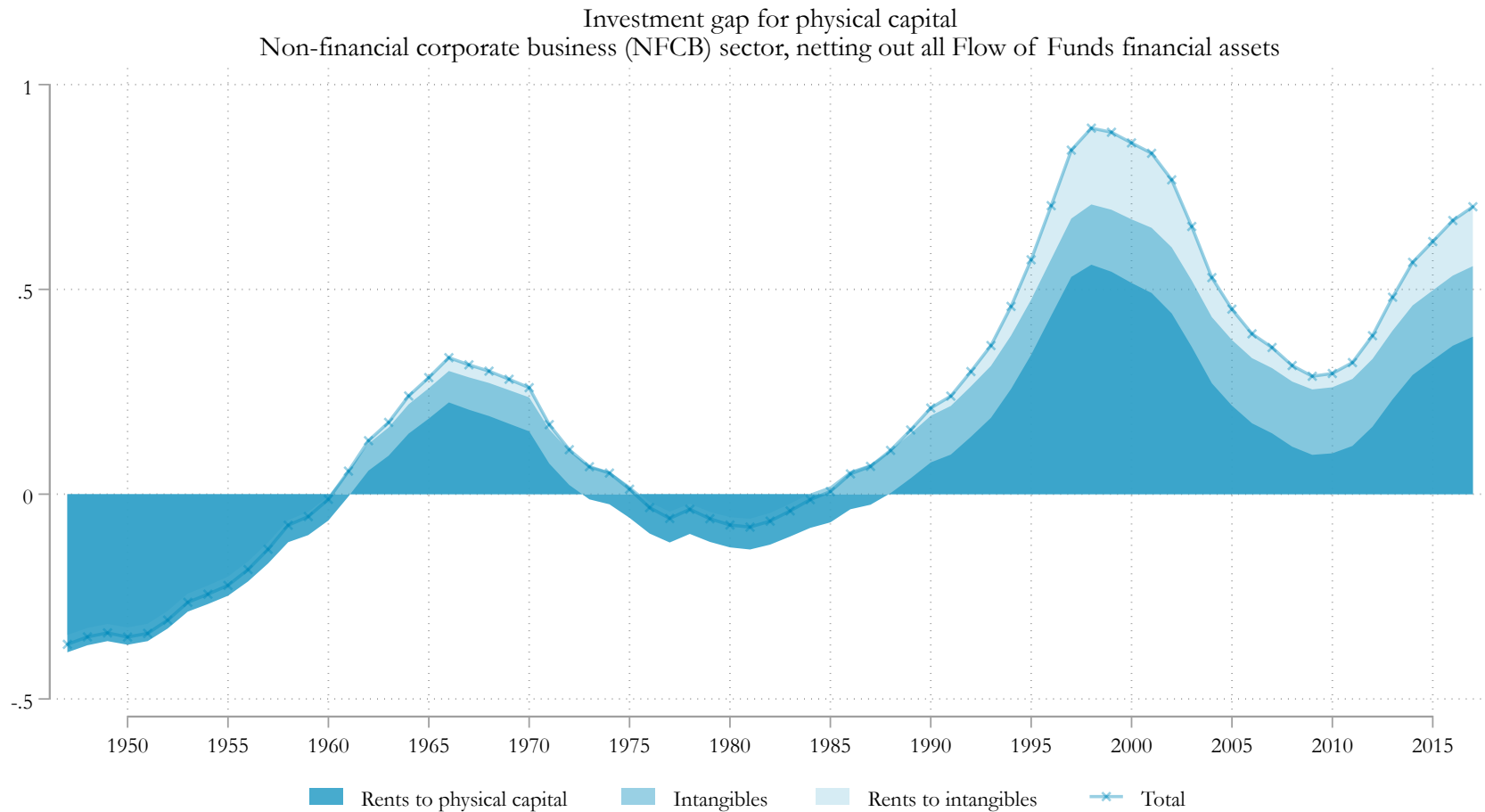


Figure C.4: The physical investment gap for the NFCB sector, in the constant growth model with linear adjustment costs, when netting out all Flow of Funds financial assets, instead of only those financial assets identified by the Flow of Funds as liquid. With respect to our baseline decomposition reported in Figure 1, all the moments we use are the same, except for the time series for $Q_{1,t}$. For the latter, we use the value of $Q_{1,t}$ computed the Hall (2001) estimate of the market value of claims on the NFCB sector, and reported in appendix Figure C.3. Section 3 and appendix B discuss data sources and variables in detail.

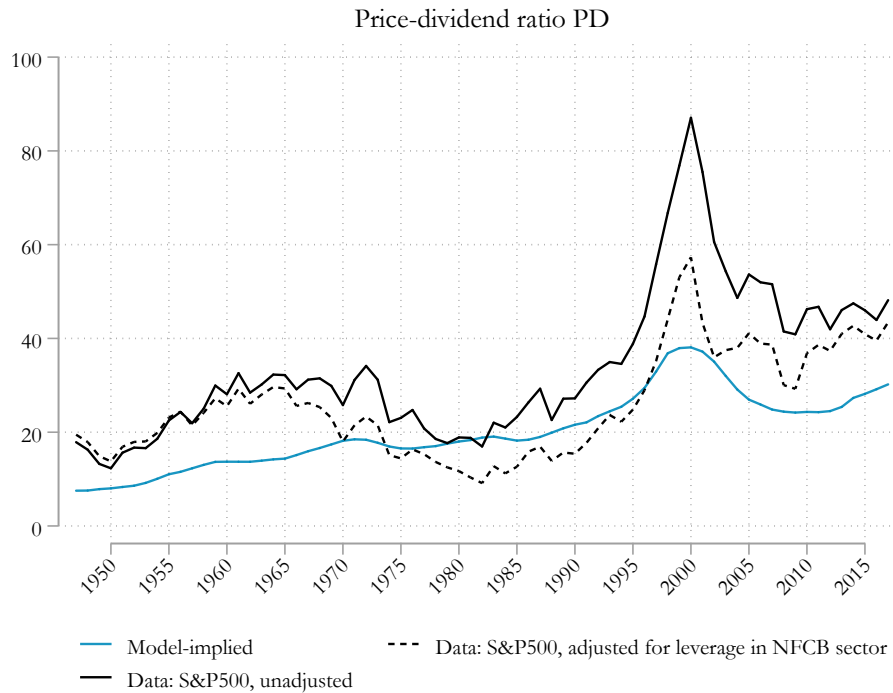


Figure C.5: Empirical and model-implied measures of the price-dividend ratio for the NFCB sector. The empirical measures use the cum- and ex-dividend returns on the S&P500. The dashed black line does not adjust for leverage, and the solid black line adjusts for leverage, using data from the Flow of Funds on the NFCB sector. The solid blue line is the price-dividend ratio implied by the baseline decomposition discussed in section 3. Data sources and methodology for the construction of the empirical price-dividend ratio measures are reported in appendix B.

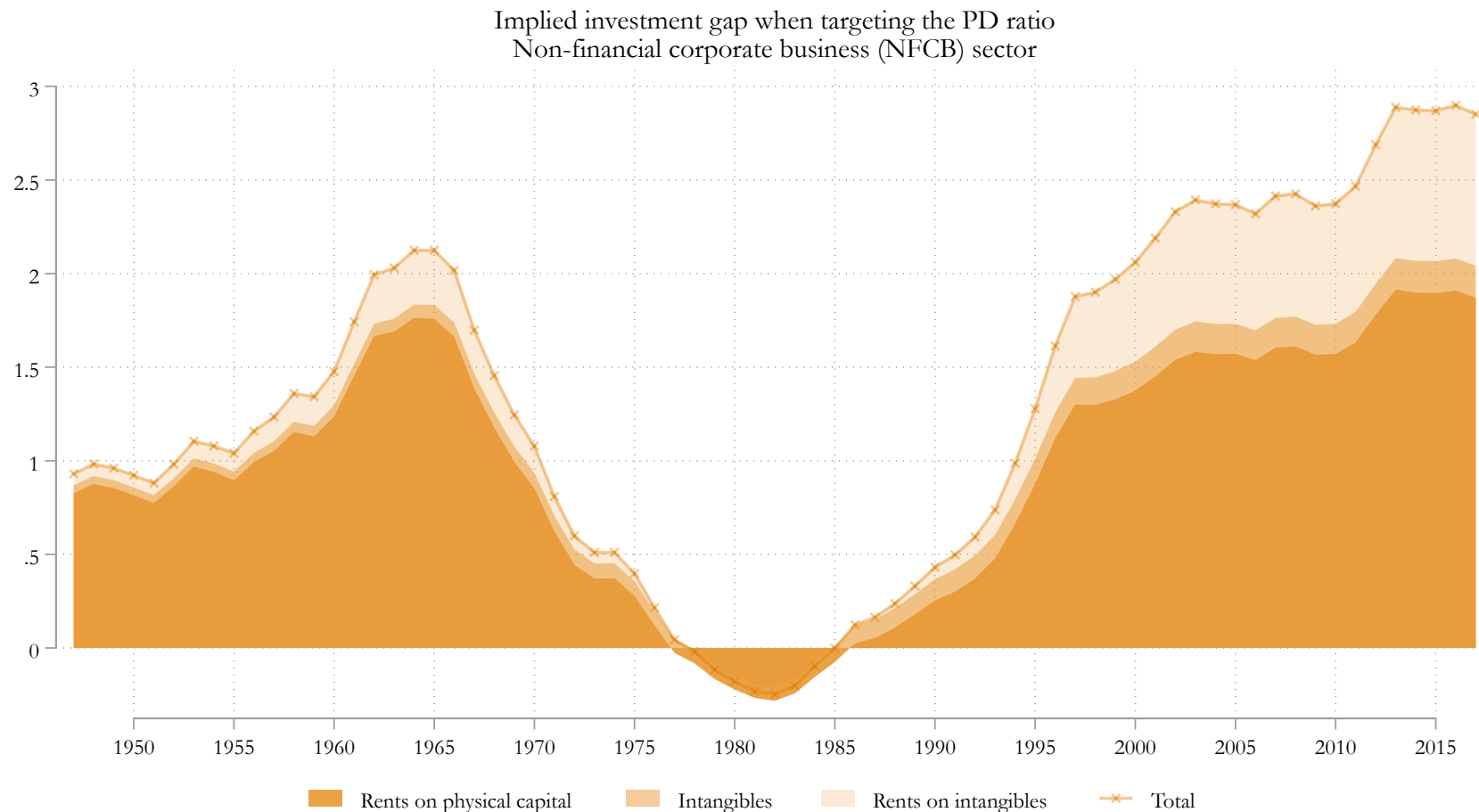


Figure C.6: The physical investment gap for the NFCB sector, in the constant growth model with linear adjustment costs, when targeting the PD ratio instead of average Q . The physical investment gap is defined as the gap between the average Q of physical capital, Q_1 , and its marginal q ; in the model with linear adjustment costs, marginal q is equal to 1 and so the investment gap is simply $Q_1 - 1$. The crossed orange line is the model-implied value of $Q_1 - 1$. The shaded areas present the decomposition of $Q_1 - 1$ into three terms, corresponding to the contribution of market power, intangibles, and the interaction between market power and intangibles. The decomposition is described in equation (17). The time series for the moments used to construct the decomposition are reported in figure 1, with the exception of average Q_1 , which is not used in this decomposition. Instead, we use the PD ratio time series reported in appendix Figure C.5 to measure the Gordon Growth term ($r - g$) in decomposition (17). Section 3 and appendix B discuss data sources and data construction in detail.

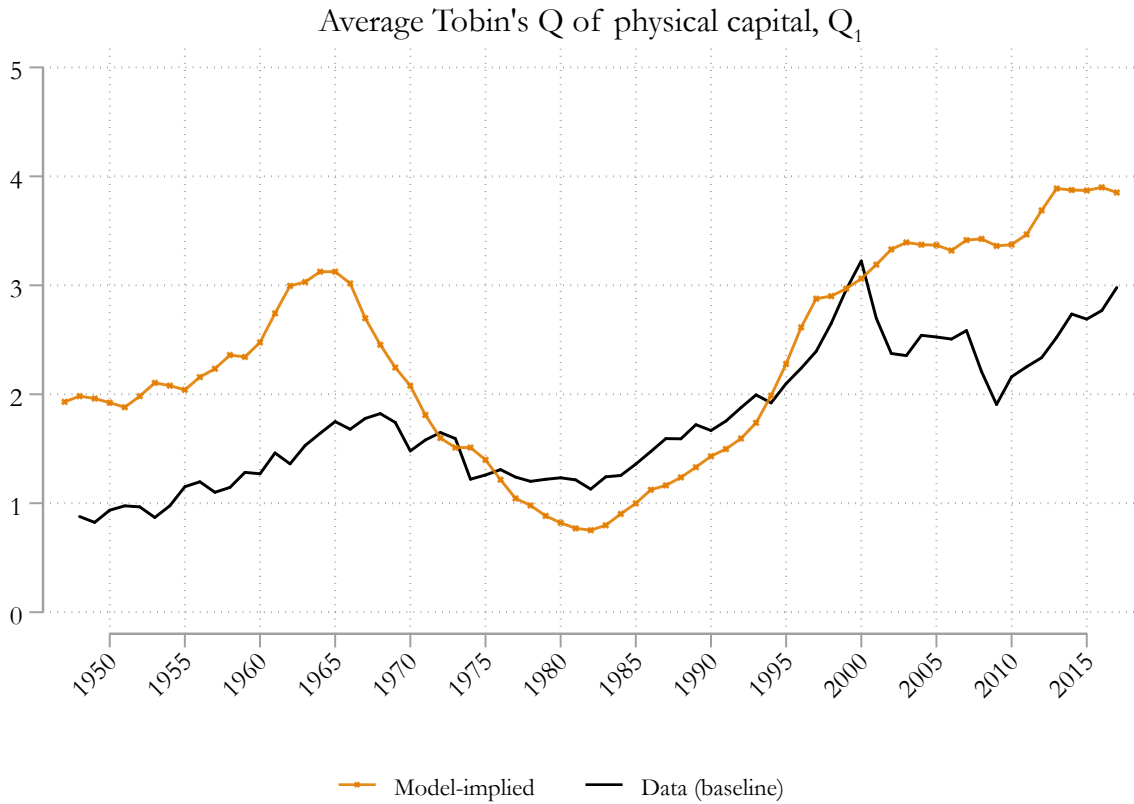


Figure C.7: Empirical and model-implied values of average Q for physical assets, Q_1 when targeting the PD ratio. The model-implied value is constructed using empirical measures of the PD ratio as an input, instead of empirical measures of Q_1 . The empirical measure of the PD ratio is derived from cum- and ex-dividend returns on the S&P500, adjusted for leverage in the NFCB; the time series is reported in Figure C.5 (solid black line). The empirical measure of Q_1 is the same as the one used in the baseline analysis for the NFCB sector. Data sources and methodology for both are reported in appendix B.

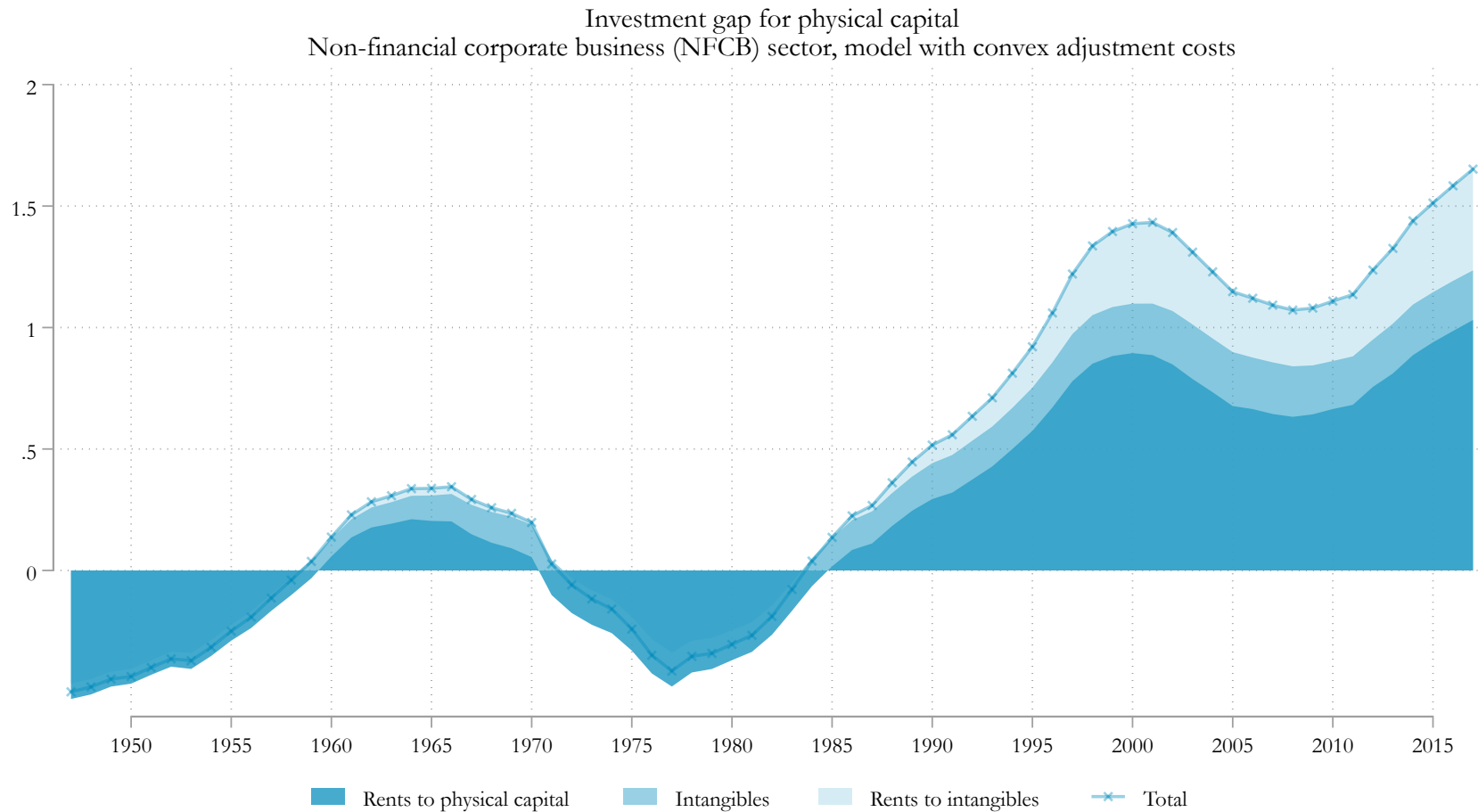


Figure C.8: The physical investment gap for the NFCB sector, in the constant growth model with convex adjustment costs. In this version of the model, marginal q for physical assets is not equal to 1. The shaded areas present the decomposition of $Q_1 - q_1$ into three terms, corresponding to the contribution of market power, intangibles, and the interaction between market power and intangibles. The decomposition is described in equation (15). A description of the derivation of the components of the decomposition is provided in appendix A. Section 3 and appendix B discuss data sources and data construction in detail.

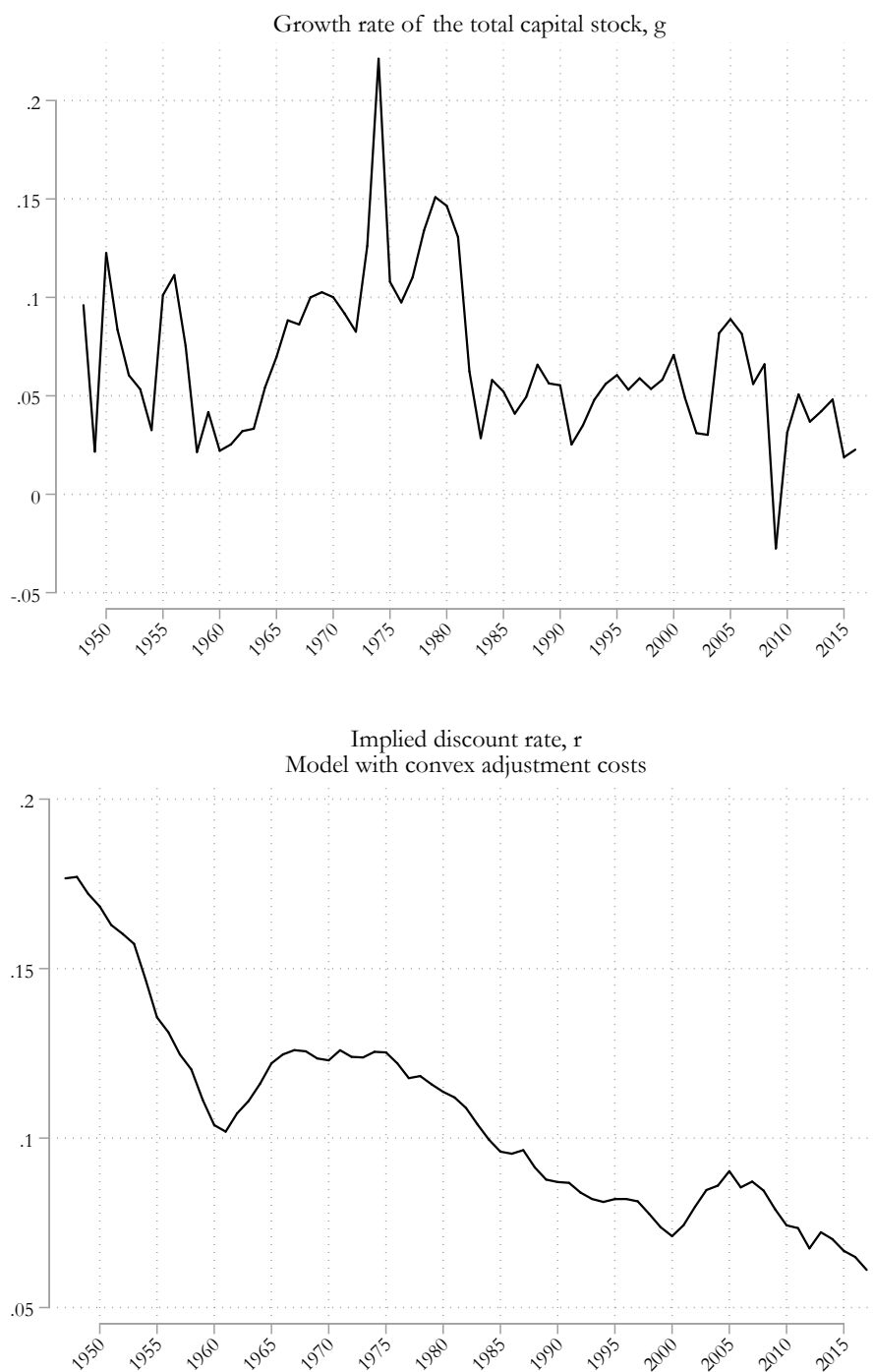


Figure C.9: Empirical proxy for g (top panel) and implied value for the discount rate r (bottom panel) in the model with convex adjustment costs. The empirical proxy for g is the growth rate of the total (physical plus intangible) capital stock. The top panel is the raw data. The derivation of the implied discount rate is reported in appendix [A](#).

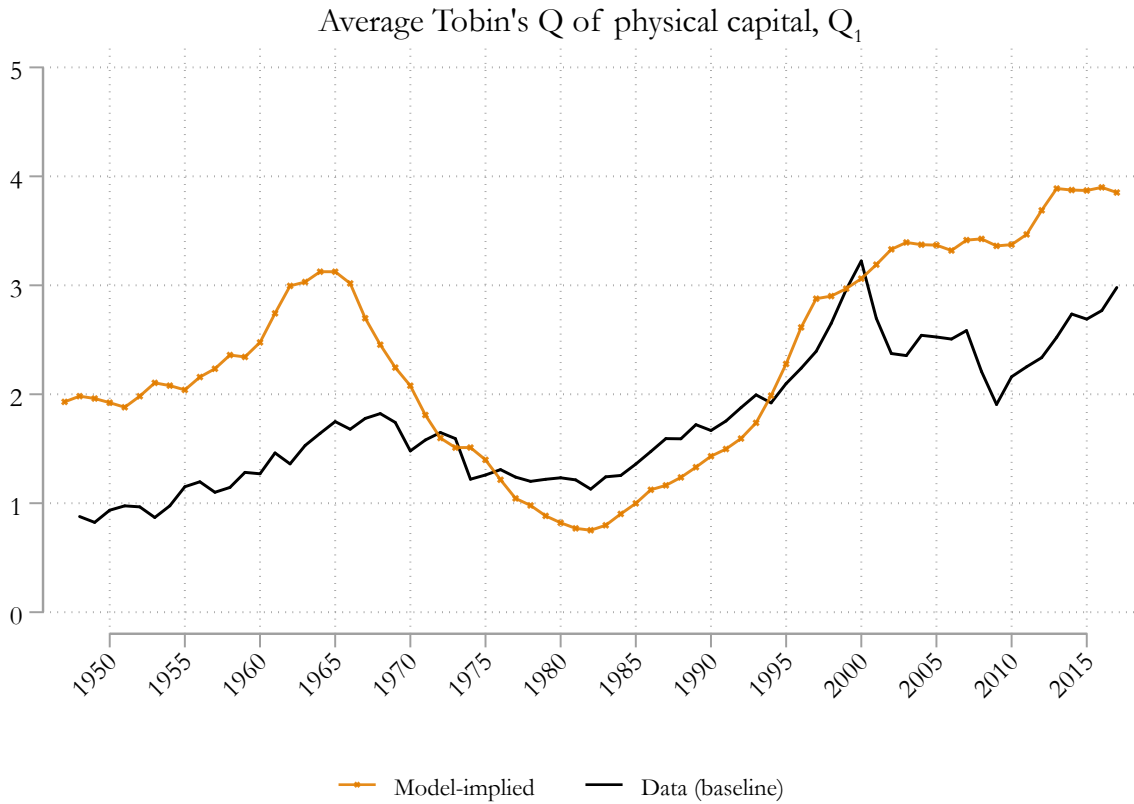


Figure C.10: Empirical and model-implied values of average Q for physical assets, Q_1 when targeting the PD ratio. The model-implied value is constructed using empirical measures of the PD ratio as an input, instead of empirical measures of Q_1 . The empirical measure of the PD ratio is derived from cum- and ex-dividend returns on the S&P500, adjusted for leverage in the NFCB; the time series is reported in Figure C.5 (solid black line). The empirical measure of Q_1 is the same as the one used in the baseline analysis for the NFCB sector. Data sources and methodology for both are reported in appendix B.

Compustat NF, Consumer sector

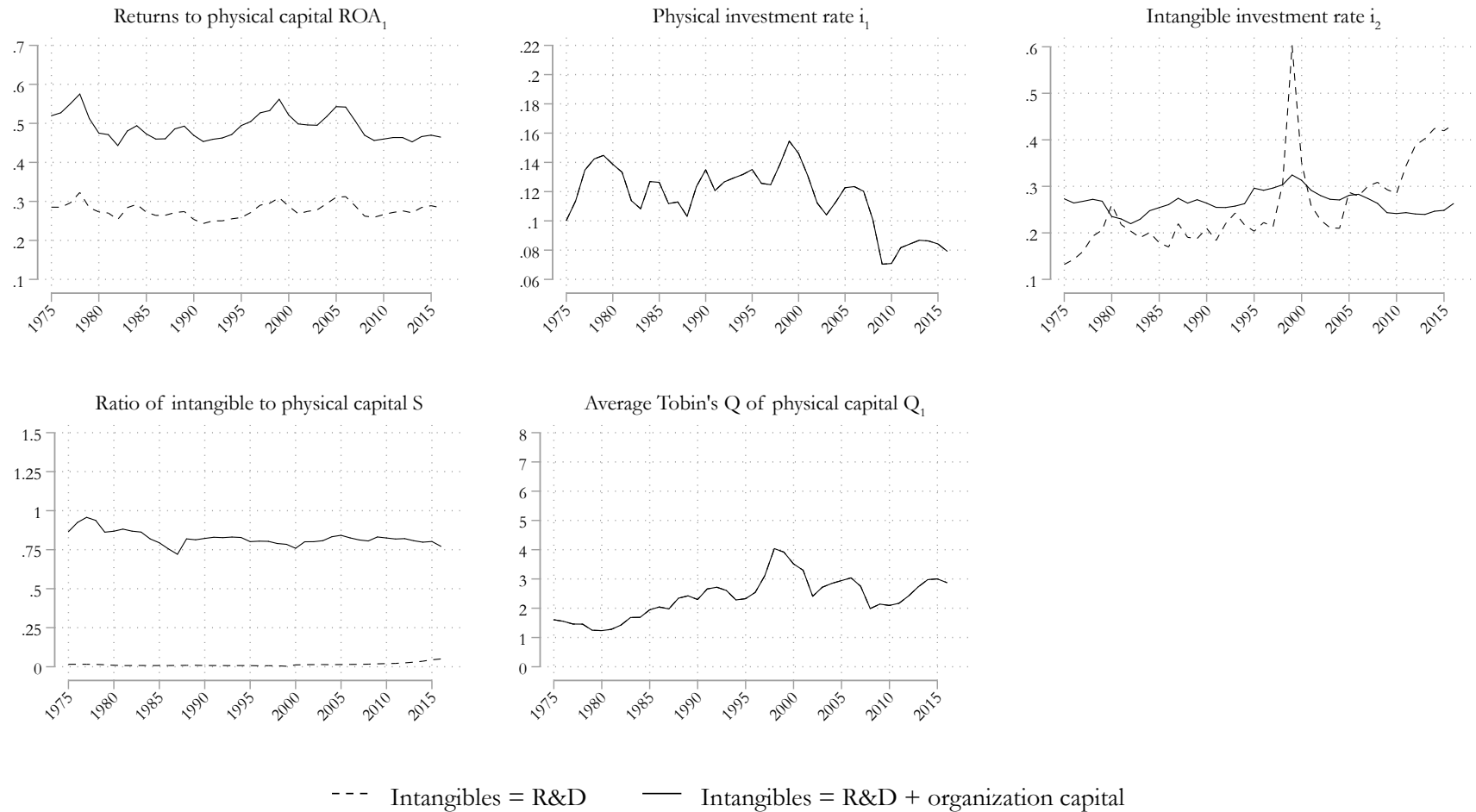


Figure C.11: Time series for the moments used in the construction of the physical investment gap decomposition (17), for the sample of publicly traded non-financial firms belonging to the Consumer sector. The variable definitions are the same as those described in section 3 and in the caption to figure 4. The time series reported are the raw data; in particular, they are not averaged over seven-year windows. Data sources and variables are described in more detail the main text and in appendix B.

Compustat NF, High-tech sector

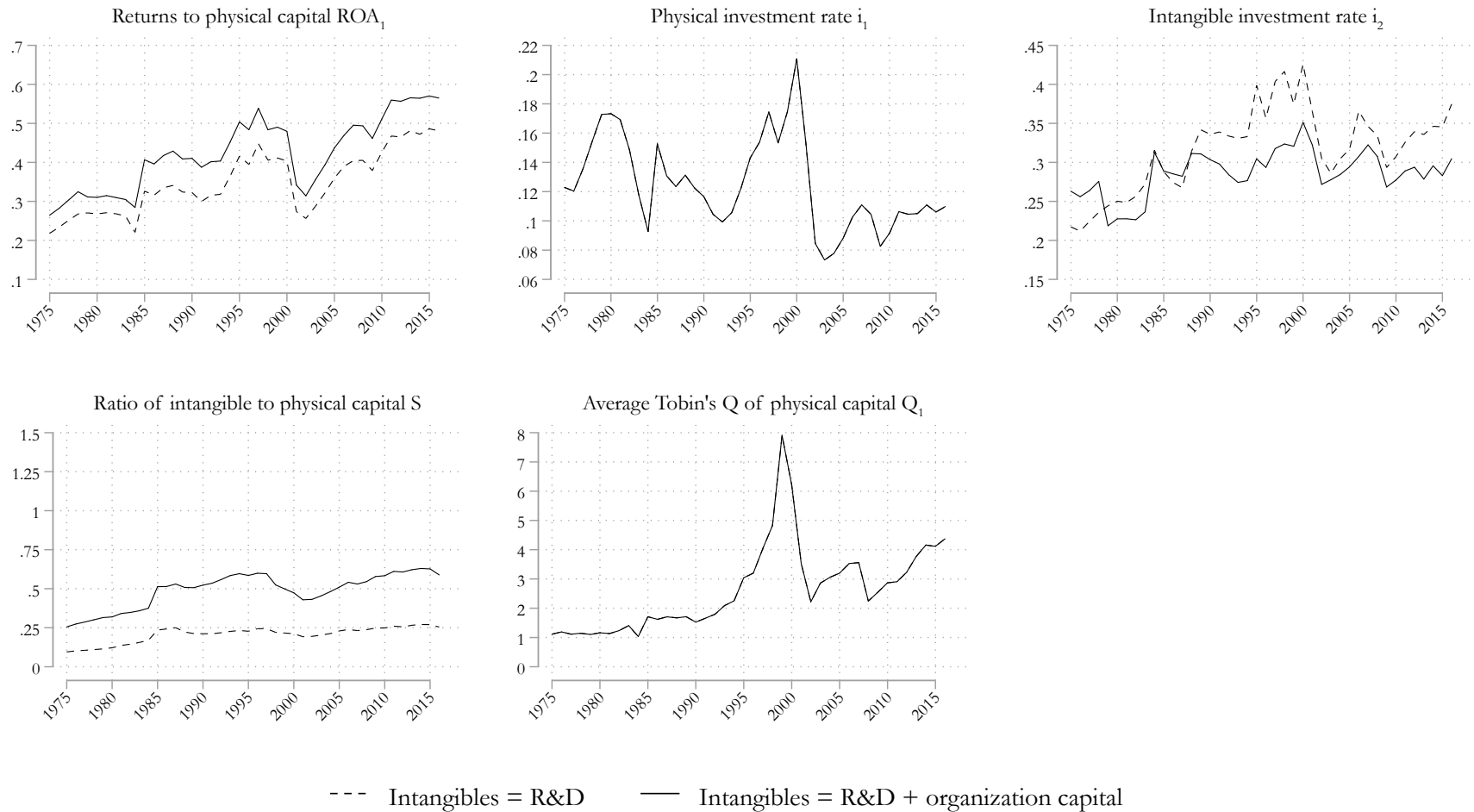


Figure C.12: Time series for the moments used in the construction of the physical investment gap decomposition (17), for the sample of publicly traded non-financial firms belonging to the High-tech sector. The variable definitions are the same as those described in section 3 and in the caption to figure 4. The time series reported are the raw data; in particular, they are not averaged over seven-year windows. Data sources and variables are described in more detail the main text and in appendix B.

Compustat NF, Healthcare sector

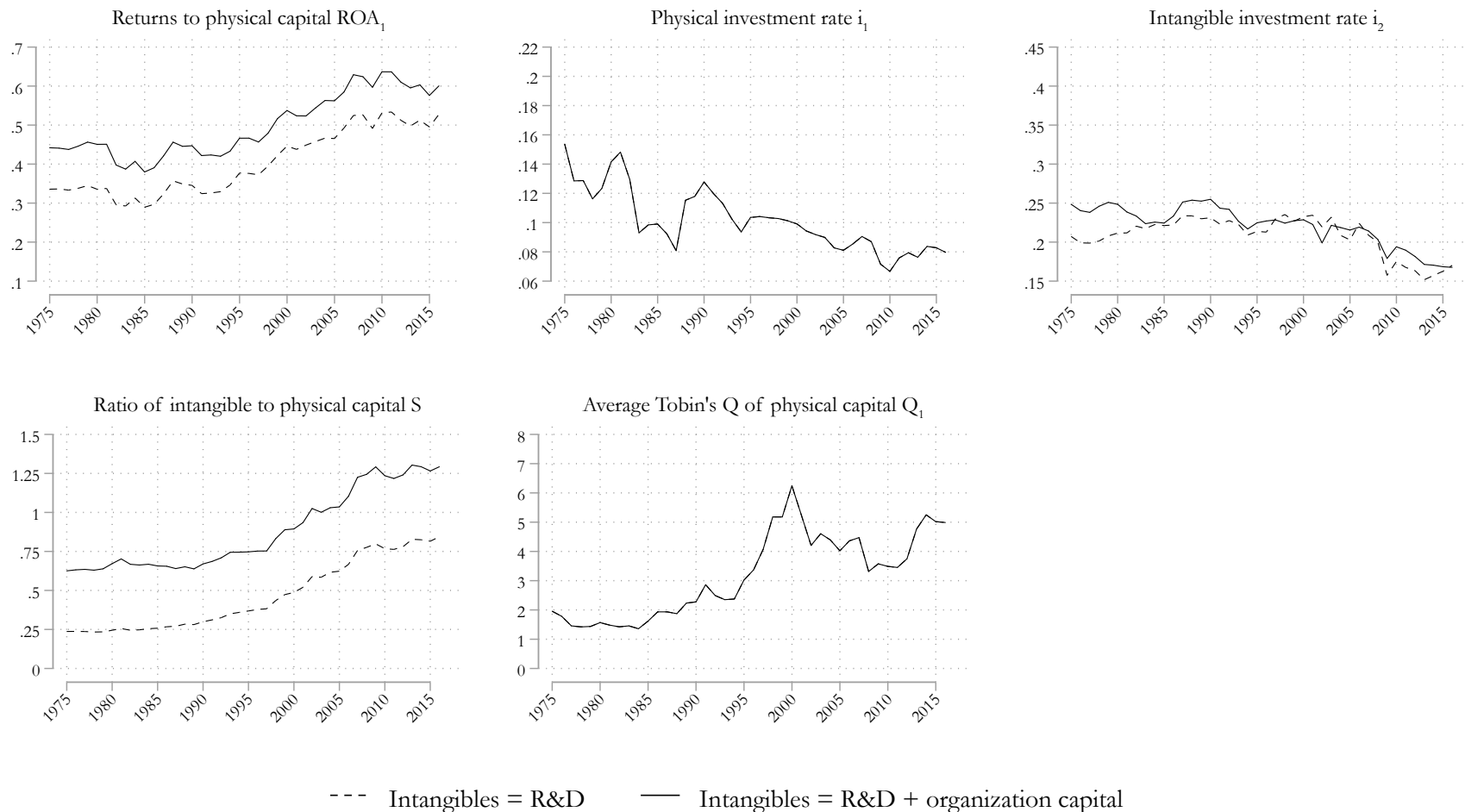


Figure C.13: Time series for the moments used in the construction of the physical investment gap decomposition (17), for the sample of publicly traded non-financial firms belonging to the Healthcare sector. The variable definitions are the same as those described in section 3 and in the caption to figure 4. The time series reported are the raw data; in particular, they are not averaged over seven-year windows. Data sources and variables are described in more detail the main text and in appendix B.

Compustat NF, Manufacturing sector

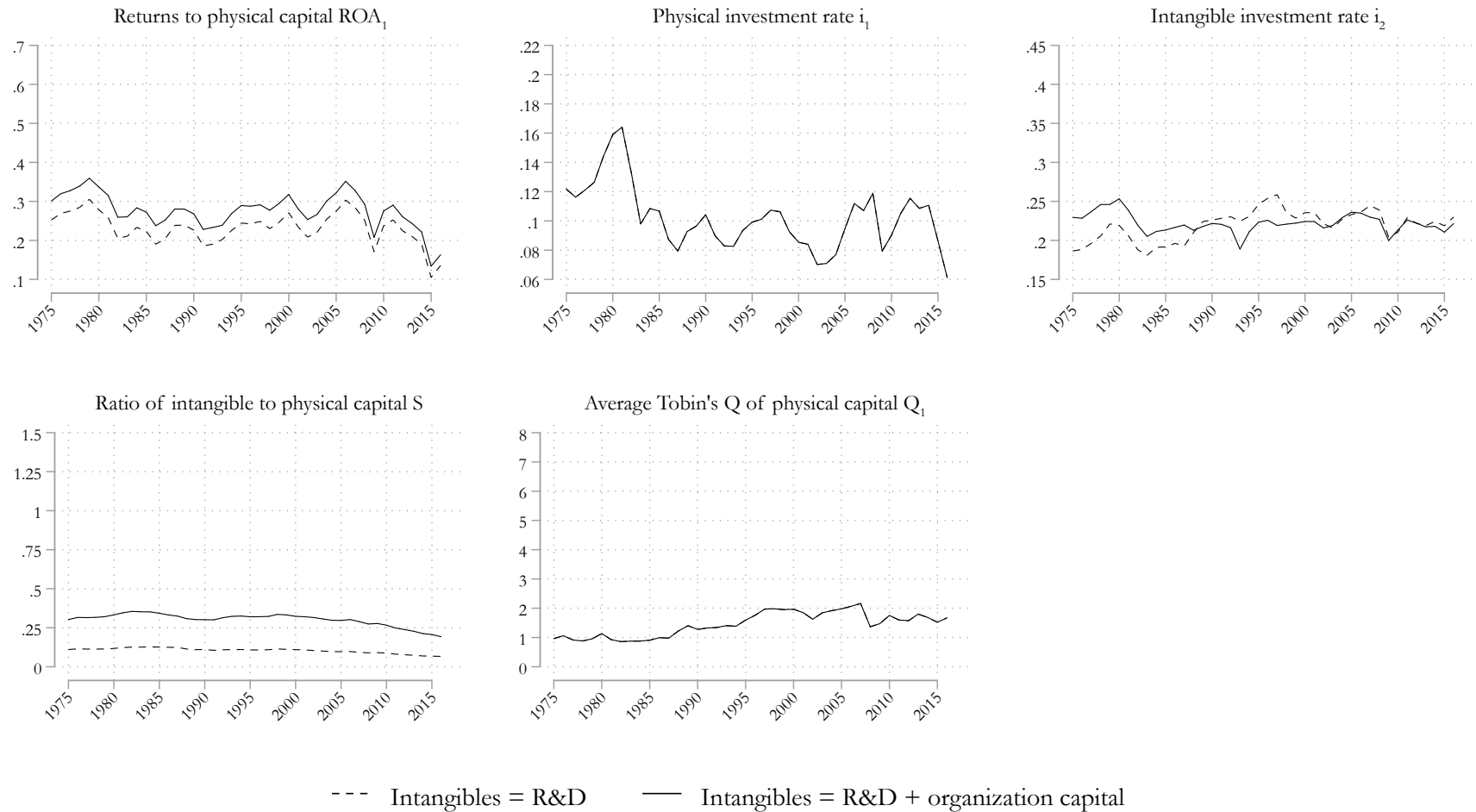


Figure C.14: Time series for the moments used in the construction of the physical investment gap decomposition (17), for the sample of publicly traded non-financial firms belonging to the Manufacturing sector. The variable definitions are the same as those described in section 3 and in the caption to figure 4. The time series reported are the raw data; in particular, they are not averaged over seven-year windows. Data sources and variables are described in more detail the main text and in appendix B.

Physical investment gap
Compustat NF, High-tech sector

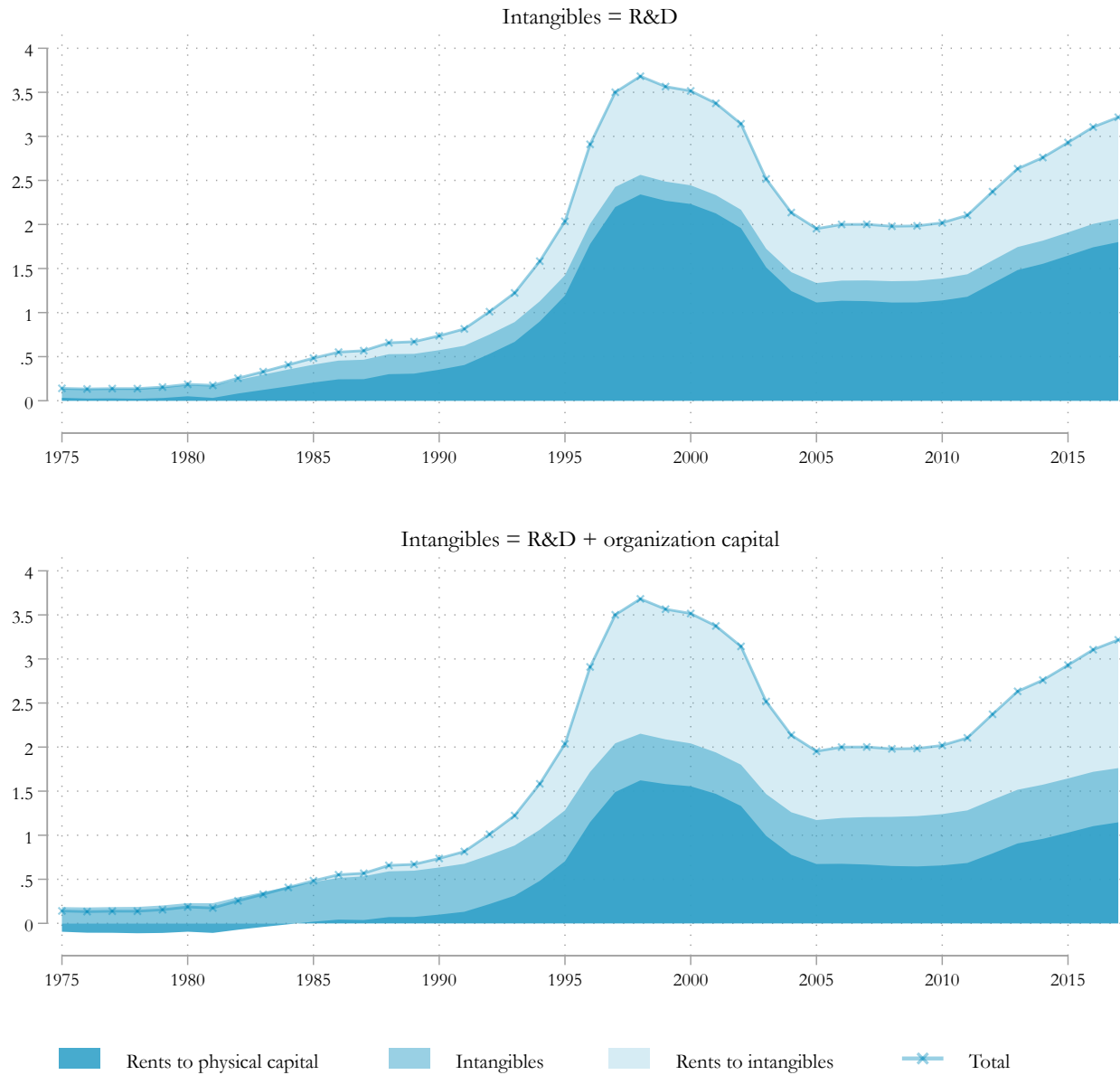


Figure C.15: The physical investment gap among publicly traded non-financial firms belonging to the High-tech sector, with two different measures of the intangible capital. The top panel uses only R&D capital as the measure of intangibles, while the bottom panel uses the sum of R&D and organization capital. The time series for the moments used to construct the decomposition are reported in figure C.11.

Physical investment gap
Compustat NF, Healthcare sector

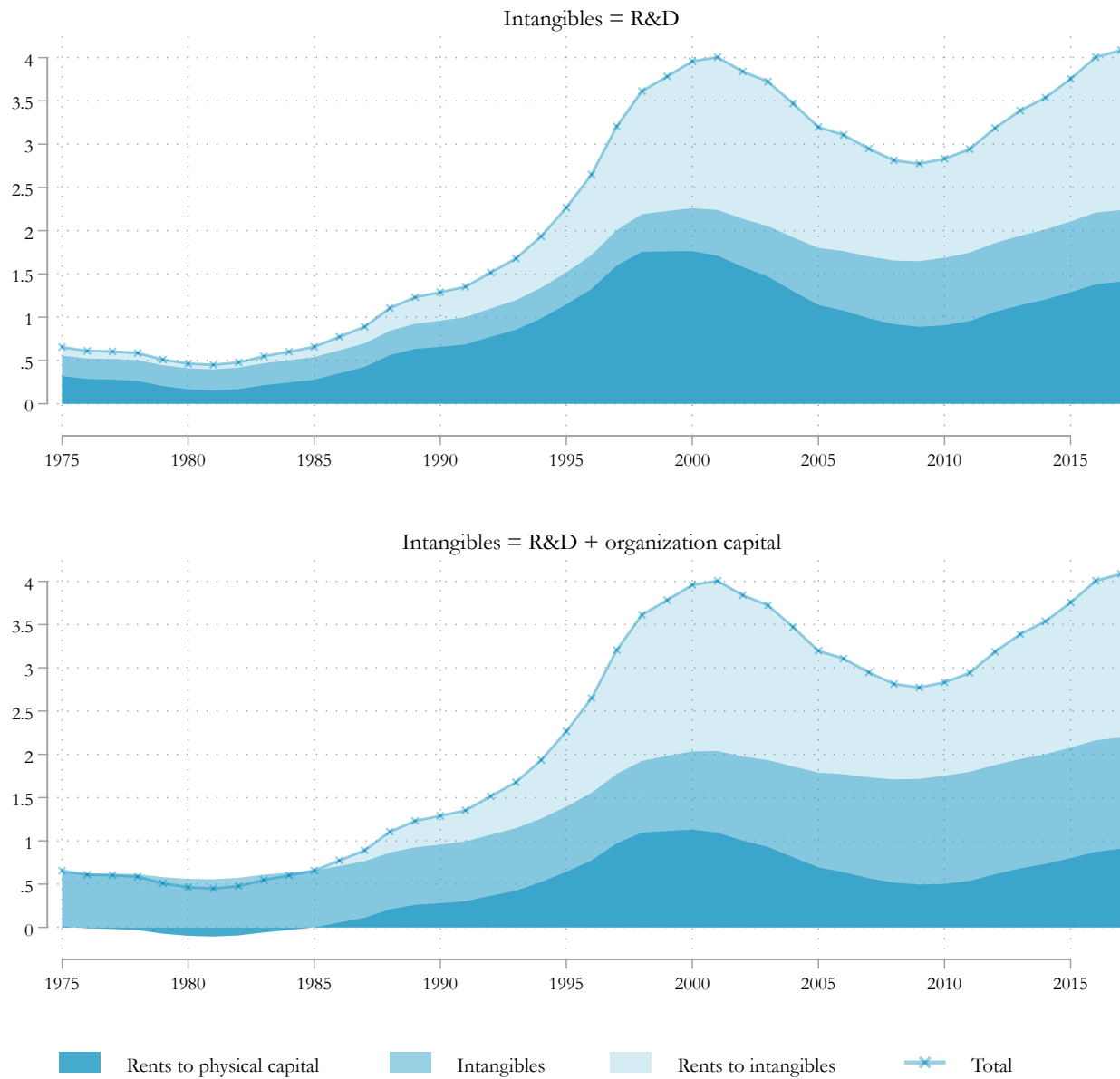


Figure C.16: The physical investment gap among publicly traded non-financial firms belonging to the Healthcare sector, with two different measures of the intangible capital. The top panel uses only R&D capital as the measure of intangibles, while the bottom panel uses the sum of R&D and organization capital. The time series for the moments used to construct the decomposition are reported in figure C.11.

Physical investment gap
Compustat NF, Manufacturing sector

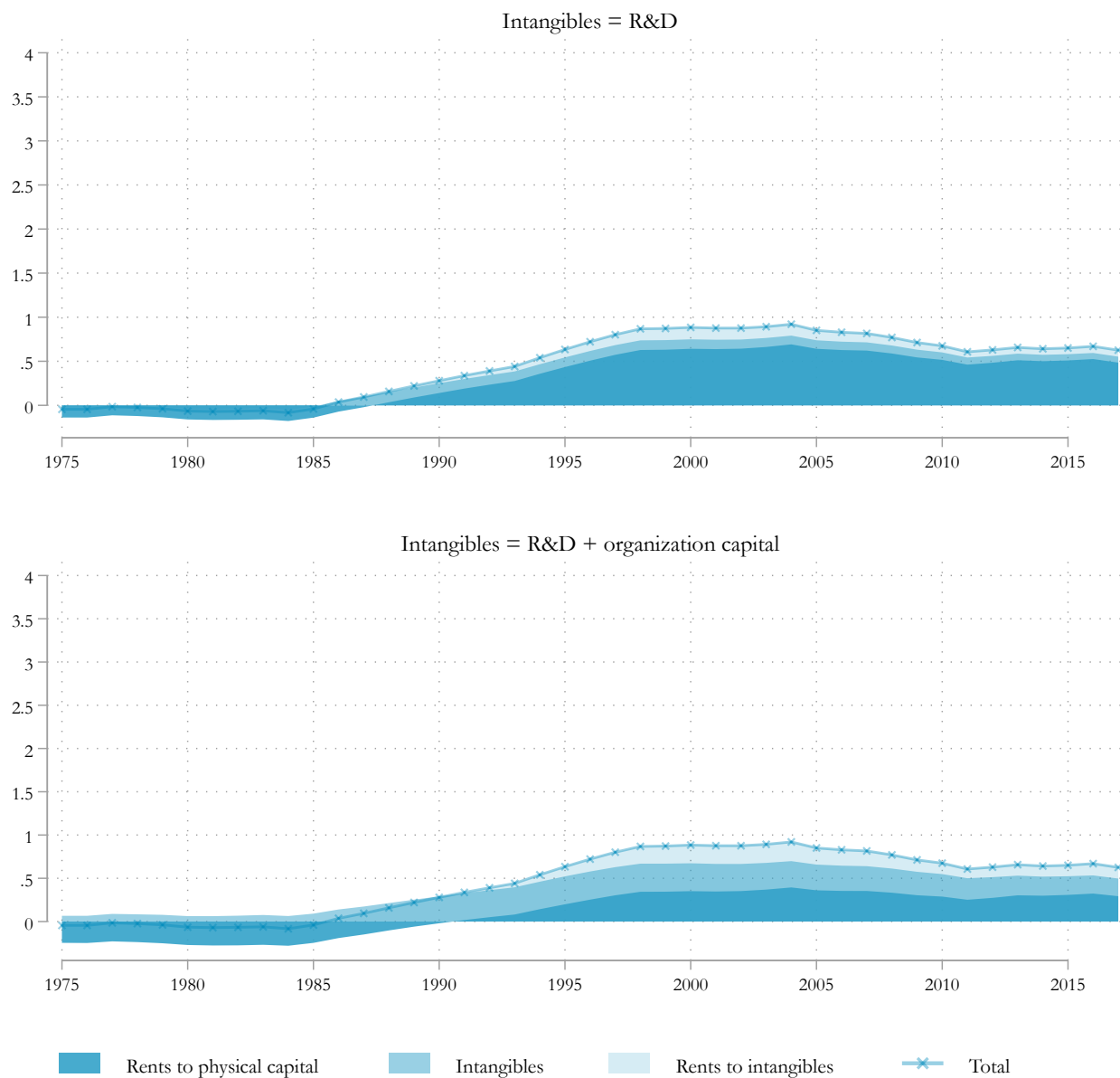


Figure C.17: The physical investment gap among publicly traded non-financial firms belonging to the Manufacturing sector, with two different measures of the intangible capital. The top panel uses only R&D capital as the measure of intangibles, while the bottom panel uses the sum of R&D and organization capital. The time series for the moments used to construct the decomposition are reported in figure C.11.

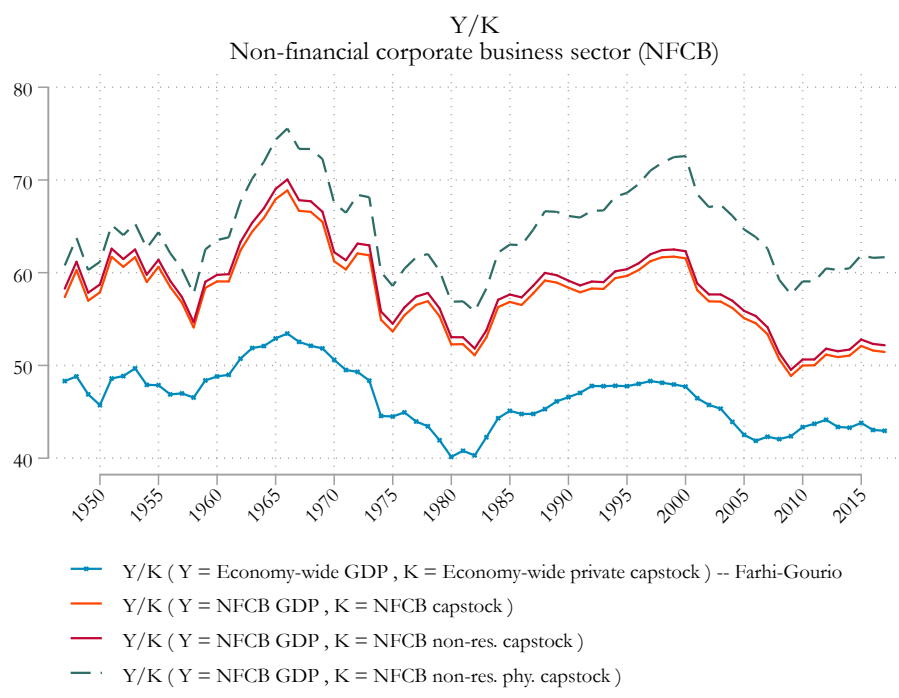
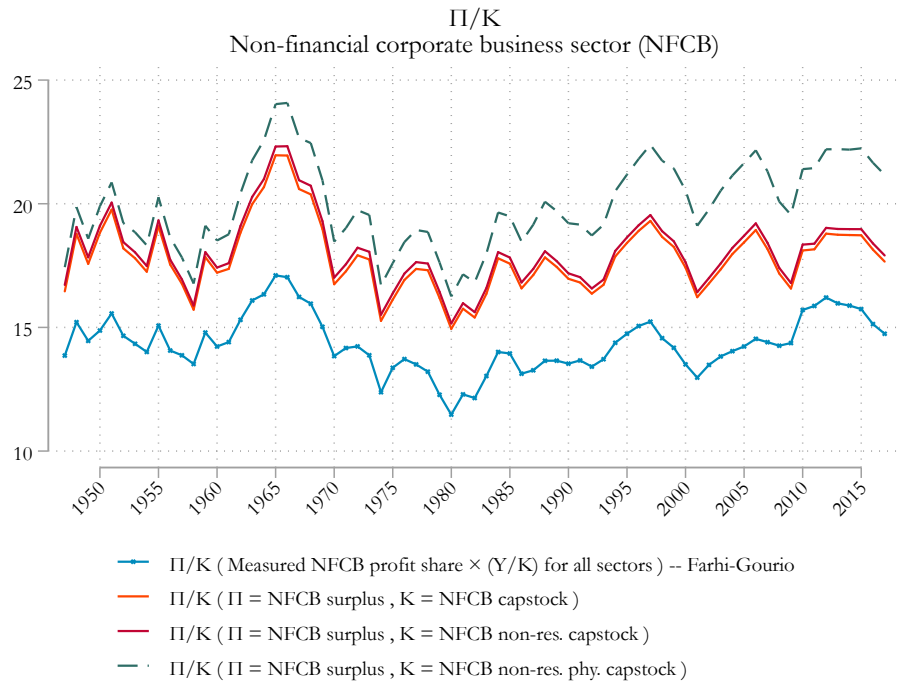
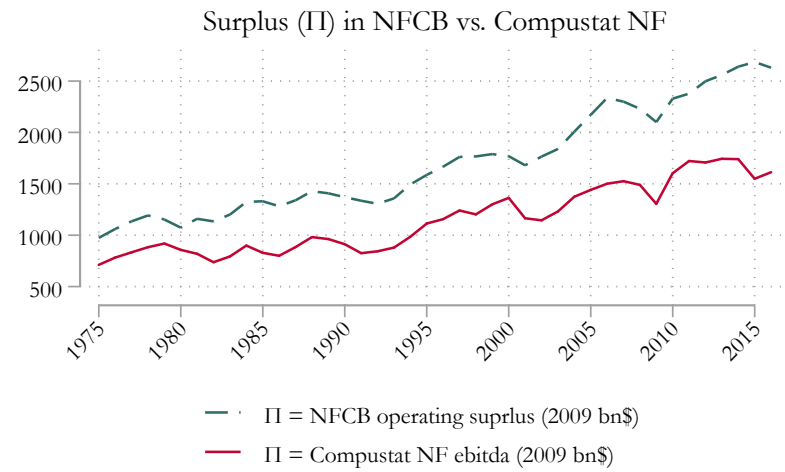
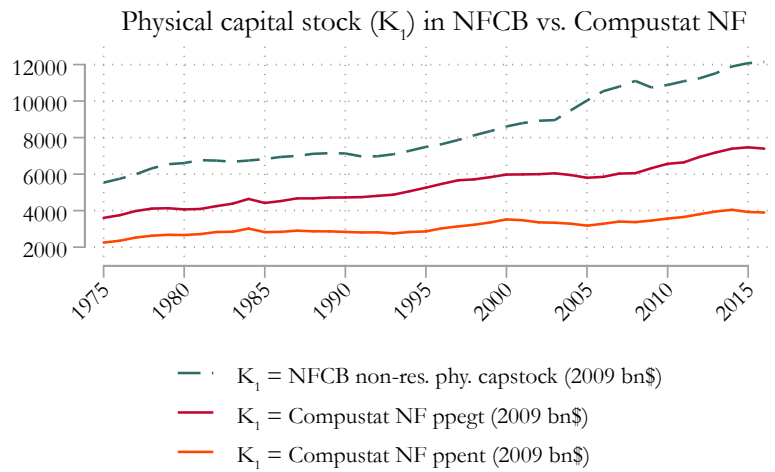


Figure C.18: Measures of Π/K (surplus per unit of capital) and Y/K (value added per unit of capital) in BEA and Compustat data. The construction of each time series is described in appendix B. The blue line reproduces the measures of Π/K and Y/K used in Farhi and Gourio (2018), which differ from our measures primarily because we focus only on the NFCB sector; more discussion is in appendix B.



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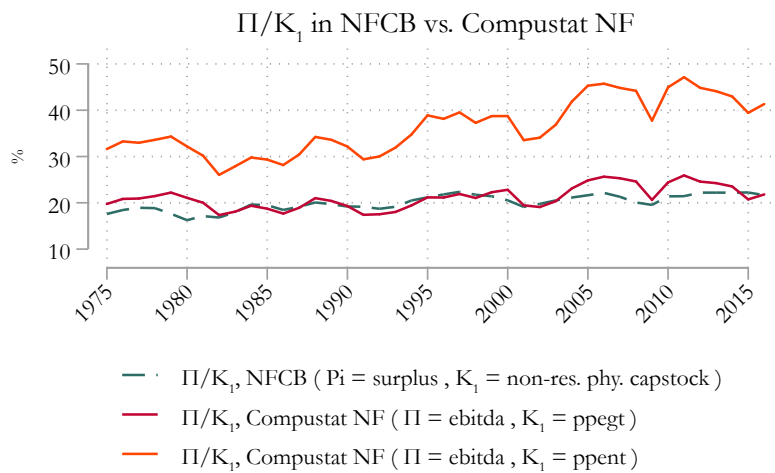


Figure C.19: Measures of the total physical capital stock at current cost (K_1), of surplus (Π), and of the ratio of surplus to capital (Π/K_1) in BEA and Compustat data. All nominal data are deflated using the CPI with base 2009. Differences between the series are discussed in appendix B.

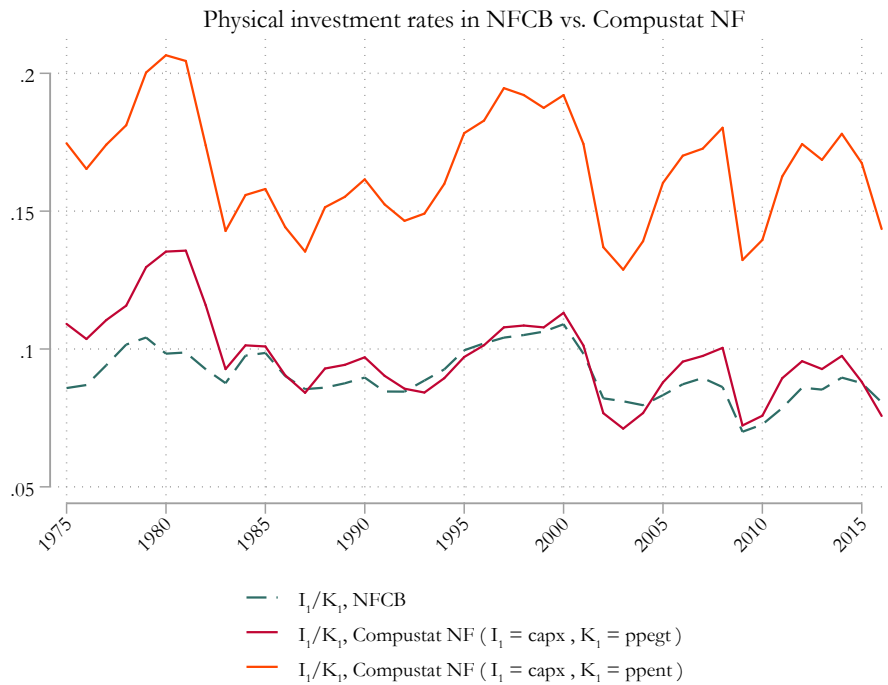
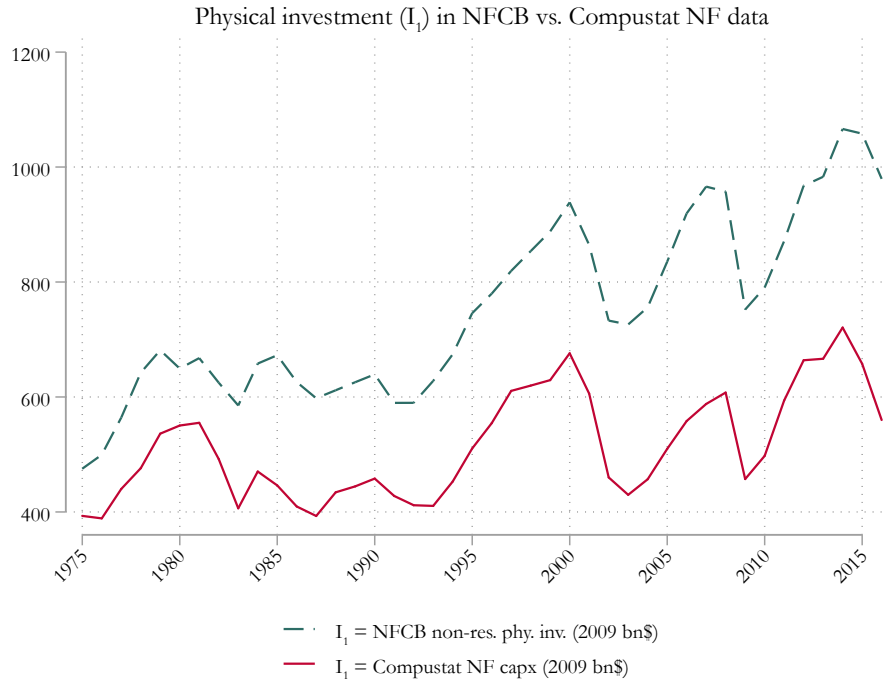


Figure C.20: Measures of the dollar value of investment in the NFCB sector (top panel), and of the investment rate (bottom panel). Dashed lines are measures obtained using BEA data, while solide lines are measures obtained using Compustat data. Differences between the series are discussed in appendix B.