A Macroeconomic Model with Financially Constrained Producers and Intermediaries *

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July 20, 2016

Abstract

We evaluate the quantitative effects of macroprudential policy. To do so, we solve a general equilibrium model with three types of agents and a government. Borrower-entrepreneurs produce output financed with long-term debt issued by financial intermediaries, subject to a leverage constraint. Intermediaries fund these loans combining deposits and their own equity, and are subject to a regulatory capital constraint. Savers provide funding to banks and to the government. Both entrepreneurs and banks make optimal default decisions. The government issues debt to finance budget deficits and to pay for bank rescue operations. We solve for macroeconomic quantities, the price of capital, the yield on safe bonds, and the credit spread. We study how financial and non-financial recessions differ, show that high credit spreads forecasts future declines in economic activity, and study macro-prudential policies. Policies that limit corporate leverage and financial leverage reduce welfare. Their benefits for financial and macro-economic stability are outweighed by the costs from a smaller-sized economy. The two types of macroprudential policies have different implications for the wealth distribution.

JEL: G12, G15, F31.

Keywords: financial intermediation, macroprudential policy, credit spread, intermediary-based asset pricing

*First draft: February 15, 2016. We thank Pierre Mabille for excellent research assistance.

1 Introduction

The financial crisis and Great Recession of 2007-09 underscored the importance of the financial system for the broader economy. Borrower default rates, bank insolvencies, government bailouts, and credit spreads all spiked while real interest rates were very low. The disruptions in financial intermediation fed back on the real economy. Consumption, investment, and output all fell substantially and persistently.

These events have caused economists to revisit the role of the financial sector in models of the macro economy. Building on early work that emphasized the importance of endogenous developments in credit markets in amplifying business cycle shocks,[1] a second generation of models has added nonlinear dynamics and a richer financial sector.[2] While a lot of progress has been made in understanding how financial intermediaries affect asset prices and macroeconomic performance, an important remaining challenge is to deliver a quantitatively successful model that can capture the dynamics of financial intermediary capital, asset prices, and the real economy during normal times and credit crises. Such a model requires a government, so that possible crisis responses can be studied, and explicit and implicit government guarantees to the financial sector can be incorporated. Indeed, Central Banks are in search of a model of the financial sector that can be integrated into their existing quantitative macro models. Our paper aims to make progress on this important agenda. It provides a calibrated model that matches key features of the U.S. macroeconomy and asset prices. In addition, we make three methodological contributions.

First, we separate out the role of producers and banks. The existing literature, as exemplified by the seminal Brunnermeier and Sannikov (2014) paper, combines the roles of financial intermediaries and producers ("experts"). This setup assumes frictionless interaction between banks and borrowers and focuses on the interaction between experts and saving households. It assumes that financial intermediaries hold equity claims in productive firms. In reality, banks make corporate loans and hold corporate bonds which are debt claims.[3] These debt contracts

[1] E.g., Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and Gertler and Karadi (2011).

[2] E.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), He and Krishnamurthy (2013), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), Moreira and Savov (2016).

[3] It is well understood that debt-like contracts arise in order to reduce the cost of gathering information and to mitigate principal-agent problems. See for example Dang, Gorton, and Holmstrom (2015).

are subject to default risk of the borrowers. Our model has three groups of agents, each with their own balance sheet: depositors who lend to intermediaries, entrepreneurs who own the production technology and borrow from intermediaries, and bankers who intermediate between the depositors and entrepreneurs. Banks perform the traditional role of maturity transformation and bear the credit risk in the economy. They help to optimally allocate risk across the various agents in the economy. Motivated by standard agency frictions, entrepreneurs/firms' borrowing capacity is limited by the equilibrium value of their capital stock. The maximum loan-to-value at loan origination is a first key macroprudential policy parameter. In order to discipline banks, we model a Basel II/III-style regulatory capital requirement that limits banks' liabilities at a fraction of their risk-weighted assets. The minimum regulatory capital that banks must hold is a second key macroprudential policy parameter.

Our second contribution is to introduce the possibility of default for financial intermediaries. The existing literature is usually cast in continuous time. As the financial sector approaches insolvency, intermediaries reduce risk and prices adjust so that they never go bankrupt. In discrete time, the language of quantitative macroeconomics, the possibility of default of intermediaries cannot be avoided. Far from a technical detail, bank insolvency is an important reality that keeps policy makers up at night. As Reinhart and Rogoff (2009) and Jorda, Schularick, and Taylor (2014a) make clear, banks frequently become insolvent. When they do, the banks' creditors (mostly the depositors) are bailed out by the government. In our model we assume that banks have limited liability and choose to default optimally. When the market value of their assets falls below that of liabilities, the government steps in, liquidates the assets and makes whole the depositors. The banking sector starts afresh the next period with zero wealth. The expectation of a bailout affects banks' risk taking incentives (e.g., Farhi and Tirole (2012)). By allowing for the possibility of bank insolvencies, our model can help explain how a corporate default wave can trigger financial fragility. Vice versa, weak financial balance sheets reduce firms' ability to borrow, invest, and grow.

The third methodological contribution is to endogenize the risk-free interest rate on safe debt. Most models in the intermediary-based macro and asset pricing literature keep the interest rate on safe assets (deposits or government debt) constant, sometimes by virtue of an assumption of risk neutrality of the savers. Once savers are risk averse, a natural assumption given that they invest in guaranteed deposits, the dynamics of the model change substantially. In a crisis, intermediaries contract the size of their balance sheet, thereby reducing the supply of safe debt in the economy. Simultaneously, risk averse depositors with strong precautionary savings motives increase their demand for safe assets. As a result, the equilibrium quantity of safe debt changes little but the price increases substantially. Real interest rates may turn negative. The low cost of debt allows the intermediaries to recapitalize quickly, dampening the effect of the crisis. Put differently, the endogenous price response of safe debt short-circuits the amplification mechanism that arises in a balance sheet recession in partial equilibrium models that hold the interest rate fixed. One might argue that there are other investors in the market for safe assets whose demand for safe assets may not rise as much because they are less risk averse (maybe institutional investors), but their demand for safe debt would have to be negatively correlated with that of the risk averse savers to offset the effect. Foreigners' demand for U.S. safe debt also increased dramatically in the global financial crisis, further amplifying domestic demand by savers rather than offsetting it. We argue that a partial solution lies in carefully modeling the government side of the model. With counter-cyclical spending and procyclical tax revenues, the government deficit is highly counter-cyclical. This expands the supply of safe debt in bad times, offsetting the contraction in the supply by the intermediation sector. While rates may still fall in crises, the decline is not as large as it would be without the government sector, and restores the amplification of the balance sheet recession models. Importantly, because the risk averse saver must absorb more debt in bad times, she must reduce spending in high marginal utility states. The ex-ante precautionary savings effect this triggers reduces the unconditional mean interest rate in the economy. While automatic stabilizers in fiscal policy may still be desirable for aggregate welfare, a new insight is that they slow down the recapitalization of banks in a crisis through their general equilibrium effect on the real interest rate.

What results is a rich and quantitatively relevant framework of the interaction between four balance sheets: those of borrower-entrepreneurs, financial intermediaries, saving households, and the government. Our model allows for occasionally binding borrowing constraints for both borrower-entrepreneurs and for intermediaries, and bankruptcy of both borrowers and intermediaries. The model generates amplification whereby aggregate shocks, for example to productivity, not only directly affect production and investment, but also affect the financial and non-financial sectors' leverage. Tighter financial constraints on banks reduce the availability of credit to firms which hurts investment and output, beyond the effects familiar from standard accelerator models.

Our model quantitatively matches the maturity, default risk, and loss-given default of corporate debt. It generates a large and volatile credit spread, again matching the data. The endogenous price of credit risk dynamics amplify the dynamics in the quantity of credit risk. We use the model to study the differences between regular non-financial recessions and financial recessions, which are recessions that coincide with credit crisis. We also show that credit spreads predict future economic activity, as has been emphasized in the recent literature.[4] Our second main exercise is to investigate the quantitative effects of macro-prudential policies for financial stability, economic growth, economic stability, fiscal stability, and economy-wide welfare. We compare and contrast bank capital regulation and loan-to-value limits on borrowers. Our model belongs to the class of models where incomplete markets and borrowing constraints create room for macro-prudential policy intervention.[5] We find that while macroprudential policies improve financial stability and reduce macroeconomic volatility, they also shrink the size of the economy. On net, they are welfare-reducing. Our model offers a quantitative answer to this important policy question.

Our paper provides a state-of-the-art solution technique. The model has two exogenous and persistent sources of aggregate risk. Standard TFP shocks hit the production function. In addition, shocks to the cross-sectional dispersion of capital depreciation govern credit risk. The model also has five endogenous aggregate state variables: the capital stock, corporate debt stock, intermediary net worth, household wealth, and the government debt stock. To solve this complex problem, we provide a nonlinear global solution method, called policy time iteration, which is a variant of the parameterized expectations approach. Policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of nonlinear equations including

[4]See for example, Friedman and Kuttner (1993), Gilchrist and Zakrajsek (2012), Greenwood and Hanson (2013), Jorda, Schularick, and Taylor (2014b), Lopez-Salido, Stein, and Zakrajsek (2015), Krishnamurthy and Muir (2016), and Mian, Sufi, and Verner (2016).

[5]Other models in this class are Lorenzoni (2008), Mendoza (2010), Korinek (2012), Bianchi and Mendoza (2013), Bianchi and Mendoza (2015), and Guerrieri and Lorenzoni (2015). Farhi and Werning (2014) study macroprudential policy in a model with demand externalities.

the Euler equations and the Kuhn-Tucker conditions expressed as equalities.[6] Our method improves on existing methods which compute two non-stochastic steady states: one steady state when the constraint never binds and one where it always binds, and then linearizes the solution around both of these states.[7].

The rest of the paper is organized as follows. Section 2 discusses the model setup. Section 3 presents the calibration. Section 4 contains the main results. Section 5 uses the model to study various macro-prudential policies. Section 6 concludes. All model derivations and some details on the calibration are relegated to the appendix.

2 The Model

2.1 Preferences, Technology, Timing

Preferences The model features a government and three groups of households: borrower-entrepreneurs (denoted by superscript B), intermediaries (denoted by superscript I) and savers (denoted by S). Savers are more patient than borrower-entrepreneurs and intermediaries, implying for the discount factors that $\beta_B = \beta_I < \beta_S$. For the coefficient of relative risk aversion we assume that $\sigma_I = \sigma_B \leq \sigma_S$. All agents have Epstein-Zin preferences over consumption streams $\{C_t^j\}_{t=0}^{\infty}$ with intertemporal elasticity of substitution ν.

$$U_t^j = \left\{ (1-\beta) \left(C_t^j\right)^{1-1/\nu} + \beta_j \left(\mathrm{E}_t \left[(U_{t+1}^j)^{1-\sigma_j} \right] \right)^{\frac{1-1/\nu}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu}}, \tag{1}$$

for $j = B, I, S$.

[6]One output of this research project will be a set of computer code which will be made publicly available. Discussions with the research department at three different Central Banks indicate that there is a demand for this type of output.

[7]In this approach, agents inside the model do not take into account the fact that borrowing constraints may become binding in the future due to future shock realizations. As a result, the approach ignores agents' precautionary savings motives related to future switches between "regimes" with and without binding constraints. While the piecewise-linear solution may prove sufficiently accurate in some contexts, it remains an open question whether it offers an appropriate solution to models with substantial risk and higher risk aversion, designed to match not only macroeconomic quantities but also asset prices (risk premia). See Guerrieri and Iacoviello (2015) for a nice discussion on these issues.

Technology Borrower-entrepreneurs own the productive capital stock of the economy and operate its production technology of the form

$$Y_t = (K_t)^{(1-\alpha)}(Z_t L_t)^{\alpha}, \tag{2}$$

where K_t is capital, L_t is labor, and Z_t is labor productivity. We assume that productivity grows at a stochastic rate g_t which follows an AR(1) process.

In addition to the technology for producing consumption goods, borrower-entrepreneurs also have access to a technology that can turn consumption into capital goods.

Borrower-entrepreneurs, intermediaries, and savers are endowed with $\bar{L}^B$, $\bar{L}^I$ and $\bar{L}^S$ units of labor, respectively. We assume that all types of households supply their complete labor endowment inelastically.

There are two more assets in the economy. One risky long-term bond that borrower-entrepreneurs can issue to intermediaries (corporate loans), and one short-term risk free bond that intermediaries can issue to savers (deposits).

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Aggregate TFP and stochastic depreciation shocks for borrower-entrepreneurs are realized. Production occurs.

2. Intermediaries decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown.[8] All agents know its probability distribution, and intermediaries maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[9]

3. Borrower households decide what fraction of debt to default on.

[8]Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. This randomization assumption is common in labor market models (Hansen (1985)).

[9]The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

4. Intermediaries' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of bankruptcy, the government picks up the shortfall in repayments to debt holders (depositors).

5. All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and intermediaries. We now describe each of the three types of household problems and the government problem in detail.

2.2 Borrower-Entrepreneurs' Problem

There is a representative family of borrower-entrepreneurs, consisting of a measure one of members. Each member owns the same quantity of capital k_t^B s.t. $\int_0^1 k_t^B di = K_t^B$, and the same quantity of outstanding bonds a_t^B s.t. $\int_0^1 a_t^B di = A_t^B$. The bond is a long-term contract, modeled as a perpetuity. Bond coupon payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the bond. We introduce a "face value" $F = \frac{\theta}{1-\delta}$, a fixed fraction θ of all repayments for each bond issued. Per definition, interest payments are the remainder $\frac{1-\theta}{1-\delta}$.

At the time of production, borrower-entrepreneurs hire their own labor and the labor of intermediaries and savers, denoted by L_t^j, with $j = B, I, S$. As payment each group receives a competitive wage w_t^j per unit of labor. During production, the labor inputs of the three types are combined into aggregate labor:

$$L_t = (L_t^B)^{1-\gamma_S-\gamma_I}(L_t^S)^{\gamma_S}(L_t^I)^{\gamma_I}.$$

Capital used for production are all capital units jointly held by borrower-entrepreneurs at the beginning of the period, such that output is $Y_t = (K_t^B)^{(1-\alpha)}(Z_t L_t)^{\alpha}$.

During the production process, the capital units owned by individual borrower-entrepreneurs experience depreciation shocks. Each borrower-member draws an idiosyncratic depreciation shock $\omega_{i,t} \sim F_\omega(\cdot)$ which proportionally lowers the amount of capital by $(1-\omega_{i,t})k_t^B$. The capital left after stochastic depreciation is $\omega_{i,t}k_t^B$. We denote the cross-sectional mean and standard deviation by $\mu_\omega = \mathrm{E}_i[\omega_{i,t}]$ and $\sigma_{t,\omega} = (\mathrm{Var}_i[\omega_{i,t}])^{0.5}$, where the latter varies over time. The mean implies an average depreciation rate of capital of $\delta_K = 1 - \mu_\omega$.

The family of borrower-entrepreneurs jointly decides which members should default on their outstanding debt. The capital units owned by the members who default are turned over to (seized by) the lender. Let the function $\iota(\omega) : [0, \infty) \to \{0, 1\}$ indicate the borrower-entrepreneurs' decision to default on a capital unit of quality ω. We conjecture and later verify that the optimal default decision is characterized by a threshold level ω_t^*, such that borrower-entrepreneurs default on all capital units with $\omega_{i,t} \le \omega_t^*$ and repay the debt for all other capital. Using the threshold level ω_t^*, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*)p_tK_t^B$ to be the value of the capital stock to the borrowers after default decisions have been made, where:

$$Z_A(\omega_t^*) = \int_0^\infty (1 - \iota(\omega))\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \ge \omega_t^*], \tag{3}$$

$$Z_K(\omega_t^*) = \int_0^\infty (1 - \iota(\omega))\omega\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \ge \omega_t^*]\ \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \ge \omega_t^*] \tag{4}$$

After making a coupon payment of 1 per unit of remaining outstanding debt, the amount of outstanding debt declines to $\delta Z_A\left(\omega_t^*\right) A_t^B$.

The profit of the borrower-entrepreneur's business is subject to a tax with rate τ_Π^B. The profit for tax purposes is defined as sales revenue net of labor expenses, and capital depreciation and interest payments of non-defaulting entrepreneurs

$$\Pi_t^B = Y_t - \sum_{j=B,I,S} w_t^j L_t^j - \delta_K Z_A(\omega_t^*)p_tK_t^B - (1-\theta)Z_A(\omega_t^*)A_t^B.$$

The fact that interest expenditure $(1-\theta)Z_A(\omega_t^*)A_t^B$ is deducted from taxable profit creates a "tax shield" and hence a preference for debt funding.

The borrower-entrepreneur family's problem is to choose consumption C_t^B, capital for next period K_{t+1}^B, default threshold ω_t^*, new debt A_{t+1}^B, investment X_t and labor inputs L_t^j to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$C_t^B + X_t + \Psi(X_t/K_t^B)K_t^B + Z_A(\omega_t^*)A_t^B(1+\delta q_t^m) + p_t K_{t+1}^B + \sum_{j=B,I,S} w_t^j L_t^j + \tau_\Pi^B \Pi_t^B$$
$$\leq Y_t + (1-\tau_t^B)w_t^B \bar{L}^B + p_t(X_t + Z_K(\omega_t^*)K_t^B) + q_t^m A_{t+1}^B + G_t^{T,B}, \tag{5}$$

and a leverage constraint:

$$FA_{t+1}^B \leq \phi p_t Z_K(\omega_t^*)K_t^B. \tag{6}$$

Outstanding debt at the end of the period A_{t+1}^B is the sum of the remaining debt after default and new borrowing. The borrower household uses after-tax labor income, net transfer income from the government ($G_t^{T,B}$), sales of old (K_t^B) and newly produced (X_t) capital units, and new debt raised, to pay for consumption, debt service, new capital purchases, investment including adjustment costs, wage and profit tax payments. New debt raised is $q_t^m A_{t+1}^B$, where q_t^m is the price of one bond in terms of the consumption good.

The borrowing constraint in (6) caps the face value of debt at the end of the period, FA_{t+1}^B, to a fraction of the market value of the available capital units after depreciation and default, $p_t Z_K(\omega_t^*)K_t^B$, where ϕ is the maximum leverage ratio. With such a constraint, declines in capital prices (in bad times) tighten borrowing constraints.

2.3 Savers

Savers can invest in one-period risk free bonds (deposits and government debt). They inelastically supply their unit of labor $\bar{L}^S$. Entering with wealth W_t^S, the saver's problem is to choose consumption C_t^S and short-term bonds B_t^S to maximize life-time utility U_t^S in (1), subject to the budget constraint:

$$C_t^S + q_t^f B_t^S \leq (1-\tau_t^S)w_t^S \bar{L}^S + G_t^{T,S} + W_t^S \tag{7}$$

and a short-sale constraints on bond holdings:

$$B_t^S \quad \geq 0. \tag{8}$$

The budget constraint (7) shows that saver uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, and purchases of short-term bonds

2.4 Intermediaries

After TFP and depreciation shocks have been realized, financial intermediaries choose whether or not to declare bankruptcy. Intermediaries who declare bankruptcy have all their assets and liabilities liquidated. They also incur a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, intermediaries do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \to \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Intermediaries choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that intermediaries default for all realizations for which the utility cost is below the threshold.

After the realization of the penalty, intermediaries execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem to be described below. First, while intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^I = \frac{C_t^I}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Intermediaries' portfolio choice consists of loans to borrower-entrepreneurs (A_t^I) and short-term bonds (B_t^I). Loans are modeled as bonds aggregating the debt of the borrowers. The coupon payment on performing loans in the current period is $A_t^I Z_A(\omega_t^*)$. For borrower-entrepreneurs that enter into foreclosure, the intermediaries repossess their capital units as collateral. These capital units are worth $(1-\zeta)\left(\mu_\omega - Z_K(\omega_t^*)\right) p_t K_t^B$, where ζ is the fraction of capital value destroyed in bankruptcy, a deadweight loss. Thus, the total payoff per bond is:

$$M_t = Z_A(\omega_t^*) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_t^B}{A_t^I}.$$

The price of the bond is q_t^m. In addition, intermediaries can trade in short-term bonds with savers and the government. They are allowed to take a short position in these bonds, using their loans to borrower-entrepreneurs as collateral. They are subject to a leverage constraint:

$$-B_t^I \leq q_t^m \xi A_{t+1}^I. \tag{9}$$

A negative position in the short-term bond is akin to intermediaries issuing deposits. The negative position in the short-term bond must be collateralized by the market value of intermediaries' holdings of long-term loan bonds. The parameter ξ determines how useful loans are as collateral.

Denote the wealth of an intermediary that did not go into bankruptcy by:

$$W_t^I = (M_t + \delta Z_A(\omega_t^*) q_t^m) A_t^I + B_{t-1}^I \tag{10}$$

Intermediaries are subject to corporate profit taxes at rate τ_Π^I. Their profit for tax purposes is defined as the net interest income on their loan business[10]

$$\Pi_t^I = (1-\theta) Z_A(\omega_t^*) A_t^I + r_t^f B_{t-1}^I.$$

Intermediaries' also receive income for inelastically supplying their labor to borrower-entrepreneurs[11]. They further need to pay a deposit insurance fee (κ) to the government that is proportional to the amount of short-term bonds they issue. Their budget constraint is

$$(1 - D(\rho_t)) W_t^I + (1-\tau^I) w_t^I \bar{L}^I + G_t^{T,I} \geq C_t^I + q_t^m A_{t+1}^I + (q_t^f + \mathrm{I}_{\{B_t^I < 0\}} \kappa) B_t^I + \tau_\Pi^I \Pi_t^I. \tag{11}$$

Note that intermediaries only receive wealth W_t^I if they do not declare bankruptcy at the beginning of the period; in case of bankruptcy their wealth is zero.

[10]We define the risk free interest rate as the yield on risk free bonds, $r_t^f = 1/q_t^f - 1$.

[11]Stand-in for managerial labor or similar.

2.5 Government

The actions of the government are determined via fiscal rules: taxation, spending, bailout, and debt issuance policies. Government tax revenues, T_t, are labor income tax receipts plus deposit insurance fee receipts and corporate profit tax receipts:

$$T_t = \sum_{j=B,I,S} \tau_t^j w_t^j L_t^j + \tau_\Pi^B \Pi_t^B + \tau_\Pi^I \Pi_t^I - \mathrm{I}_{\{B_t^I<0\}} \kappa B_t^I$$

Government expenditures, G_t are the sum of financial sector bailouts, other exogenous government spending, G_t^o, and transfer spending G_t^T:

$$G_t = G_t^o + G_t^T - D(\rho_t) W_t^I$$

The bailout to the financial sector equals the negative of the financial wealth of intermediaries, W_t^I, in the event of a bankruptcy.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{12}$$

We impose a transversality condition on government debt:

$$\lim_{u\to\infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^S B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^S$ is the SDF of the saver.[12]

Because of its unique ability to tax and repay its debt, the government can spread out the cost of default waves and financial sector rescue operations over time.

Government policy parameters are $\Theta_t = (\tau_t^i, \tau_\Pi^i, \kappa, G_t^o, \phi, \xi)$. The parameters ϕ in equation (6) and ξ in equation (9) can be thought of as macro-prudential policy tools. One could add

[12] We show below that the risk averse saver is the marginal agent for short-term risk-free debt. In the numerical work below, we keep the ratio of government debt to GDP contained between $\underline{b^G}$ and $\overline{b^G}$ by decreasing taxes linearly when the debt-to-GDP threatens to fall below $\underline{b^G}$ and raising taxes linearly when debt-to-GDP threatens to exceed $\overline{b^G}$.

the parameters that govern the utility cost of bankruptcy of intermediaries to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy.

2.6 Equilibrium

Given a sequence of income shocks $\{Y_t\}$, depreciation shocks $\{\omega_{t,i}\}_{i \in B}$, and utility costs of default shocks ρ_t, and given a government policy Θ_t, a competitive equilibrium is an allocation $\{C_t^B, K_{t+1}^B, X_t, A_{t+1}^B, L_t^j\}$ for borrower-entrepreneurs, $\{C_t^S, B_t^S\}$ for savers, $\{C_t^I, A_{t+1}^I, B_t^I\}$ for intermediaries, default policies $\iota(\omega_{it})$ and $D(\rho_t)$, and a price vector $\{p_t, q_t^m, q_t^f\}$, such that given the prices, borrower-entrepreneurs, savers, and intermediaries maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

1. Risk-free bonds: $B_t^G = B_t^S + B_t^I$
2. Loans: $A_{t+1}^B = A_{t+1}^I$
3. Capital: $K_{t+1}^B = \mu_\omega K_t^B + X_t$
4. Labor: $L_t^j = \bar{L}^j$ for all $j = B, I, S$
5. Consumption: $Y_t = C_t^B + C_t^I + C_t^S + G_t^o + X_t + K_t^B \Psi(X_t/K_t^B) + \zeta(\mu_\omega - Z_K(\omega_t^*))p_t K_t^B$

The last equation states that total output equal the sum of consumption expenditures, (wasteful) spending by the government, consumption goods used for capital-goods production, and intermediary expenditure incurred when liquidating capital goods of bankrupt firms[13].

2.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function

[13] This assumption implies that intermediaries need to spend a fraction ζ of the fair value of the capital to execute the distressed sale

summing value functions of the agents according to their population weights ℓ:

$$\mathcal{W}_t(\cdot;\Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^I V_t^I,$$

where the $V^j(\cdot)$ functions are the value functions defined in the appendix.

3 Calibration

The model is calibrated at annual frequency. The parameters of the model and their targets are summarized in Table 1.

Aggregate Productivity We assume that aggregate productivity grows with a stochastic rate g_t that follows an AR(1) process:

$$Z_{t+1} = \exp(g_{t+1})Z_t \tag{13}$$

$$g_{t+1} = (1-\rho_g)\bar{g} + \rho_g g_t + \epsilon_{t+1} \quad \epsilon_{t+1} \sim iid\, \mathcal{N}(0,\sigma_g) \tag{14}$$

Given the persistence of TFP growth, g_t becomes a state variable. We discretize the g_t process into a 5-state Markov chain using the method of Rouwenhorst (1995). The procedure matches the mean, volatility, and persistence of GDP growth, which is endogenous, by choosing both the productivity grid points and the transition probabilities between them. Consistent with our model, our concept of GDP excludes net exports, housing investment, changes in inventories, and government investment. We define the GDP deflator correspondingly. The observed real per capita GDP growth between 1953 and 2014 has a mean of 2.00%, a volatility of 1.94% and a persistence of 0.34. The model matches these moments exactly.

Depreciation In each period, firms face idiosyncratic stochastic capital depreciation shocks $\omega_{i,t}$, which are drawn from a Gamma distribution characterized by shape and a scale parameters $(\chi_{t,0}, \chi_{t,1})$. $F_\omega(\cdot;\chi_{t,0},\chi_{t,1})$ is the corresponding CDF. We choose $\{\chi_{t,0},\chi_{t,1}\}$ to keep the mean μ_ω constant at 0.925, implying annual depreciation of capital of 7.5%. This is the observed depreciation rate in the 1953-2014 BEA fixed asset data, calculated as the average ratio of depreciation

Table 1: Calibration

This table reports parameter values.

Par	Description	Value	Target
		Exogenous Shocks	
$\bar{g}$	mean TFP growth	2.0%	Mean rpc GDP gr 53-14 of 2.00%
σ_g	vol. TFP growth	2.85%	Vol rpc GDP gr 53-14 of 1.95%
ρ_g	persistence TFP growth	0.22	AC(1) rpc GDP gr 53-14 of 0.34
μ_ω	mean idio. depr. shock	7.5%	Capital depreciation BEA 53-14
σ_ω	vol. idio. depr. shock	{0.25,0.40}	Corporate default rates
p^ω_{LL}, p^ω_{HH}	transition prob	0.2,0.99	Frequency and duration of credit crises
		Production, Population, Labor Income Shares	
ℓ^i	pop. shares $i \in \{S, B, I\}$	{69,28.3,2.7}%	Population shares SCF 95-13, QCEW 01-15
γ^i	inc. shares $i \in \{S, B, I\}$	{60,37.4,2.6}%	Labor inc. shares SCF 95-13, QCEW 01-15
α	labor share in production	0.66	Standard value (Kydland-Prescott)
ψ	marginal adjustment cost	1	Vol. investment-to-GDP ratio 53-14 of 1.23%
		Corporate loans	
δ	average life loan pool	0.937	Duration Fcn. (Appendix B)
θ	principal fraction	0.582	Duration Fcn. (Appendix B)
ϕ	maximum LTV ratio	0.50	FoF nonfinancial sectors 85-14
ζ	DWL of bankruptcy	{0.2,0.5}	Corporate loan and bond severities 81-15
		Preferences	
$\beta^B = \beta^I$	time discount factor B, I	0.86	Mean investment-to-GDP ratio 53-14 of 13.3%
$\sigma^B = \sigma^I$	risk aversion B, I	2	Financial leverage 85-14 of 95.6%
β^S	time discount factor S	0.99	Mean risk-free rate 85-14
σ^S	risk aversion S	20	Vol. risk-free rate 85-14
ν	intertemp. elasticity of subst.	1	
		Government Policy	
τ	personal income tax rate	26.21%	BEA govt pers tax rev to GDP 53-14 of 17.30%
τ_Π	corporate income tax rate	15.64%	BEA govt corp. tax rev to GDP 53-14 of 3.41%
G^o	exogenous govt spending	17.57%	BEA govt. discr. spending to GDP 53-14
G^T	govt transfers to agents	3.14%	BEA govt. net transfers to GDP 53-14
κ	deposit insurance fee	0	Deposit insurance fee
ξ	margin	0.95	Basel II reg. capital charge (C&I loans)
σ_ρ	bank bankruptcy	5%	Technical assumption

of total private nonresidential fixed assets to the net stock of total private nonresidential fixed assets, both measured at current cost.

Credit crises We let the cross-sectional standard deviation $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ govern aggregate corporate credit risk and represent the second source of exogenous aggregate risk. We refer to states with the high value for $\sigma_{t,\omega}$ as *credit crises*. We set the two values $(\sigma_{H,\omega}, \sigma_{L,\omega}) = (0.25, 0.40)$. We also allow the deadweight losses of default to vary across the two aggregate credit risk states $(\zeta_H, \zeta_L) = (0.2, 0.5)$. Together, these four parameters are important drivers of the default rate and the severity rate (loss given default rate) in normal times and in credit crises. Our baseline model generates an average default rate of 1.46%, an average severity (loss given default) of 36.1%, and thus an average loss rate of 0.69%. We look at two sources of data: corporate loans and corporate bonds. From the Flow of Funds, we obtain delinquency and charge-off rates on Commercial and Industrial loans and Commercial Real Estate loans by U.S. Commercial Banks for the period 1991-2015. The average delinquency rate is 3.1% and the average loss rate is 0.7%. Default rates and severity rates are much higher in the recessions[14] of 1991, 2001, and 2007-09. The second source of data is Standard & Poors' default rates on publicly-rated corporate bonds for 1981-2014. The average default rate is 1.5%; 0.1% on investment-grade bonds and 4.1% on high-yield bonds. The average severity rate on S&P and Moody's rated defaults between 1985 and 2004 is 44%[15]. Our average default, loss, and severity rates are all close to the data. Thus our model generates the right amount of corporate credit risk.

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high (g is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth (g is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), p^{ω}_{LL} and p^{ω}_{HH}, to match the frequency and length of credit crises. Thus, the model implies that not all recessions are credit crises, but all credit crises are recessions.

[14] In the 1991 recession, the delinquency rate spiked at 8.2% and the charge-off rate at 2.2%. For the 2007-09 crisis, the respective numbers are 6.8% and 2.7%.

[15] Again, default and severity rates are higher during recessions. During the 2001 recession, for example, the default rate on high-yield bonds was 9.9%, with an average loss-given-default of 55%.

Based on the historical frequency of financial crises in Reinhart and Rogoff (2009), we target a 10% probability of a credit crisis. Conditional on a crisis, we set the expected length to 2 years, based again on evidence in Reinhart and Rogoff.

Production We set the marginal adjustment cost parameter $\psi = 1$ in order to match the observed volatility of the ratio of investment to GDP of 1.23%. The model generates a value of 1.17%, which is close. We set the parameter α which governs the overall labor income share in the Cobb-Douglas production function equal to its standard value of 0.66.

Population and labor income shares To pin down the population shares of our three different types of households we turn to the Survey of Consumer Finance (SCF).[16] We define savers as those households who hold a low share of their wealth in the form of risky assets. In particular, we compute for each household in the survey the share of assets, net of all real estate, held in stocks or private business equity, considering both direct and indirect holdings of stock. Using this definition of the risky asset share, we then calculate the fraction of households whose share is less than one percent. This amounts to 69% of SCF households. The remaining 31% of households have a large risky asset share. We split them into 28.3% borrowers-entrepreneurs and 2.7% financial intermediaries based on the share of employees that work in the financial sector, defined as "Securities, Investments" and "Credit Intermediation" from the Quarterly Census of Employment and Wages, averaged over the longest available sample 2001-2015. The population shares are used for the welfare calculations.

From the same QCEW data, we obtain the wage share for the intermediaries of 2.6%. The labor income share of savers in the SCF is 60%. The income share of the borrower-entrepreneurs must then be the remaining 37.4%. The income shares determine the Cobb-Douglas parameters γ_I, γ_B, and γ_S. By virtue of the calibration, the model matches basic aspects of the observed income distribution.[17]

[16] We use all survey waves from 1995 until 2013 and average across them.

[17] Intermediaries' labor income is 2.6% of total labor income and 1.72% of GDP pre-tax and 1.27% post-tax. After-tax profits are an additional 1.28% of GDP. Total post-tax intermediary income is 2.55% of GDP in the model. The market value of intermediated assets is 67% of GDP. Thus, intermediary income is 3.8% of intermediated assets. Intermediary profits are 1.9% of intermediated assets. Philippon (2015) reports that the cost of financial intermediation has historically been about 2% of intermediated assets.

Corporate Loans In our model, a corporate loan is a geometric bond. The issuer of one bond (firm) at time t promises to pay the holder (intermediary) 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Given that the present value of all payments $(1/(1-\delta))$ can be thought of as the sum of a principal (share θ) and an interest component (share $1-\theta$), we define the book value of the debt as $\theta/(1-\delta)$. This book value of debt is used in the firm's collateral constraint. We set $\delta = 0.937$ and $\theta = 0.582$ to match the observed duration of corporate bonds. Appendix B.1 contains the details. The model's corporate loans have a duration of 7 years on average.

Borrowers can obtain a loan with principal value up to a fraction ϕ of the market value of their house. We set the LTV ratio parameter $\phi = 0.5$ to target non-financial sector leverage. In the Flow of Funds data, the average ratio of loans and debt securities of the nonfinancial corporate and nonfinancial noncorporate businesses to their non-financial assets is 37%.[18] The model generates average corporate leverage of 44.5%.

Preference parameters Preference parameters are harder to pin down directly by data since they affect many equilibrium quantities and prices simultaneously. For simplicity, we assume all three agents have unit elasticity of intertemporal substitution ν, a common value in the asset pricing literature. The subjective time discount factor and risk aversion coefficients are agent-specific. In order to highlight the separate roles of intermediaries' and firms' balance sheets, we purposely set the time discount factor and the risk aversion coefficient of borrowers and intermediaries equal. We set $\beta_B = 0.86$ to instill a strong borrowing motive in the borrower-entrepreneur. This parameter is important for matching the economy's investment-to-output ratio. The model generates a ratio of 13.9% of total private nonresidential fixed investment to GDP, close to the observed 13.3% in the data.

Given $\beta_I = 0.86$, intermediaries' risk aversion ($\sigma_B = \sigma_I = 2$) helps to pin down the financial leverage ratio. The average ratio of total debt to total assets for 1985-2014 is 95.6%.[19] The

[18]For the Flow of Funds leverage data, we use the post-1987 sample. Only in this sample is nonfinancial leverage stationary.

[19]Specifically, we include U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations, Life and Property-Casualty Insurance Companies, GSEs, Agency- and GSE-backed Mortgage pools, Issuers of ABS, and REITs. Krishnamurthy and Vissing-Jorgensen (2015) identify a group of financial institutions as net suppliers of safe, liquid assets. This group is the same as ours

model generates average intermediary (book) leverage of 94.9%. The model generates a volatility of changes in corporate debt to GDP of 3.5%, compared to 4.4% for the observed changes in the Flow of Funds corporate debt to GDP ratio.

The time discount factor and risk aversion of the saver disproportionately affect the mean short-term interest rate and its volatility. We set $\beta^S = 0.99$ to generate a low mean real rate of interest of 1.85%. Risk aversion of the saver σ_S controls the volatility of the real interest rate which is 2.35% in the model. In the data, the mean real interest rate is 1.2% with a volatility of 2.0% over the period 1985-2014.[20]

Government parameters Our goal is to capture average government spending and tax revenues as well as their cyclical properties. The model has two sources of government spending and two sources of tax revenue.

Discretionary and transfer spending as a fraction of GDP are modeled as follows: $G_t^i/Y_t = G^i \exp\{b_i(g_t - \bar{g})\}, i = o, T$. The scalars G^o and G^T are set to match the observed average discretionary spending to GDP of 17.58% in the 1953-2014 NIPA data, and transfer spending to GDP of 3.19%, respectively.[21] We set $b_o = -0.5$ and $b_T = -5.5$ in order to match the slope in a regression of log spending to GDP on GDP growth and a constant. We closely match these slopes: -7.86 and -0.71 in model versus -7.26 and -0.74 in the 1953-2014 data.

Similarly, we model the labor income tax rate as $\tau_t = \tau \exp\{b_\tau(g_t - \bar{g})\}$. We set the tax rate $\tau = 26.21\%$ in order to match observed average income tax revenue to GDP of 17.30%.[22]

except that we add insurance companies and take out money market mutual funds, since we are interested in leveraged financial firms. For comparison, leverage for the Krisnamurthy and Vissing-Jorgensen institutions is 90.7% for the 1985-2014 sample. The group of excluded, non-levered financial institutions are Money Market Mutual Funds, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. Total financial sector leverage, including these non-levered institutions, is 60.6%.

[20] To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months from the Survey of Professional Forecasters.

[21] We divide by $\exp\left\{b_i/2\sigma_g^2/(1-\rho_g^2)(b_i-1)\right\}$, a Jensen correction, ensure that average spending means match the targets.

[22] We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line 9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures to the rest of the world rather than being consumed domestically. Since the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption

We set the sensitivity of the tax rate to aggregate productivity growth $b_\tau = 0.5$ to match the observed sensitivity of log income tax revenue to GDP to GDP growth.

Fourth, we set the corporate tax rate that both financial and non-financial corporations pay to a constant $\tau_\Pi = 15.64\%$. This allows us to match observed corporate tax revenues of 3.41% of GDP. The tax shield of debt that firms and banks enjoy in the model reduces the tax they pay. The model endogenously generates cyclicality in corporate tax revenues.

The final source of government spending is interest service on the debt, which is endogenous since both quantity and price of government debt are determined in equilibrium. In the data, net interest payments on government debt average to 2.98% of GDP.[23] This number is close to the observed average budget deficit of 3.04% of GDP. We do not aim to match this number since the government cannot run a 3% deficit in perpetuity lest the debt explode. In our calibration, the personal and corporate tax revenue is very close to the discretionary and transfer spending; the primary deficit averages close to 0% of GDP. Government debt to GDP averages around 50% of GDP in a long simulation of the benchmark model. While it fluctuates meaningfully over prolonged periods of time (standard deviation of 37%), the government debt to GDP ratio remains stationary.[24]

We can interpret the risk-taker borrowing constraint parameters, ξ, as a regulatory capital constraint set by the government. Under Basel II and III, corporate loans and bonds have a risk weight that depends on their credit quality. For a 40% loss given default, the risk weight on C&I loans with 2.5 year maturity ranges from 13% for AAA, 54% for BBB-, 125% for B+, to 325% for CCC. A blended regulatory capital requirement of 5% (8% times a blended risk weight of 62.5%) seems appropriate. This implies that $\xi = 0.95$.

We set the deposit insurance fee parameter $\kappa = 0$ to reflect the fact that banks were not

back to domestic agents.

[23]Net interest expenses are interest payments to persons and businesses (line 28) minus income receipts on asses (line 10).

[24]In our numerical work, we guarantee the stationarity of the ratio of government debt to GDP by gradually decreasing personal tax rates τ_t when debt-to-GDP falls below $\underline{b^G} = 0.1$ and by gradually increasing personal tax rates when debt-to-GDP exceed $\overline{b^G} = 1.2$. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -10%. Tax rates are gradually and convexly increased until they hit 60% at a debt-to-GDP ratio of 160%. Our simulations never reach the -30% and +160% debt/GDP states. The simulation spends 9% of the time in the profligacy and 9% of the time in the austerity region. Profligacy and austerity tax policies do not affect the amount of resources that are available for private consumption in the economy.

required to pay any deposit insurance fees between 1997 and 2006.[25]

Utility cost of risk-taker bankruptcy The model features a random utility penalty that intermediaries suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty at least some of the time. We assume ρ_t is normally distributed with a mean of μ_ρ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.05$. The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower μ_ρ, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of intermediaries) depends on the frequency of credit crises, and the endogenous choices (asset and liability choice) of the intermediaries.

4 Results

We present results from a long simulation of the model (10,000 years). For all variables of interest, we report averages and standard deviations over time, as well as averages conditional on being in a good state (positive TFP growth and no credit crisis, i.e. $\sigma_{\omega,L}$), non-financial recession (negative TFP growth, $\sigma_{\omega,L}$), and financial recession (negative TFP growth and $\sigma_{\omega,H}$). We start by presenting standard macroeconomic moments, before turning to the financial sector.

4.1 Macro Quantities

The model matches several aggregate quantity moments by virtue of the calibration. These include the mean investment-to-output ratio (14.1% vs. 13.3% in data), its volatility (1.12% vs. 1.23% in data), the mean capital-to-output ratio (1.48 vs. 1.27 in the data), mean government consumption-to-output (17.58% vs. 17.58% in data), mean transfer spending-to-output (3.19% vs. 3.19% in data), personal income tax revenue-to-output (17.34% vs. 17.30% in data). The model also matches the mean, standard deviation, and autocorrelation of changes in log real

[25]FDIC premia were raised after the crisis. Well capitalized banks currently pay 2.5 cents per $100 insured.

Table 2: Unconditional Macroeconomic Quantity Moments

	mean	stdev	output corr.	AC
Data				
Δy	2.00%	1.94%	1.000	0.339
Δx	3.25%	6.14%	0.784	0.242
Δc	2.14%	1.78%	0.873	0.321
Δg	0.56%	2.53%	0.292	0.460
X / K	10.51%	0.89%	0.442	0.822
X / Y	13.29%	1.23%	0.187	0.867
Model				
Δy	2.00%	2.17%	1.000	0.244
Δx	2.00%	10.89%	0.566	-0.457
Δc	2.00%	2.28%	0.763	-0.024
Δg	2.00%	1.85%	0.623	0.522
X / K	9.52%	1.12%	0.660	0.655
X / Y	14.08%	1.12%	0.621	0.410

per capita GDP. Finally, the model generates amplification in the sense that GDP growth has higher persistence (34% autocorrelation vs. 34% in data) than TFP growth (22% in model).

Table 2 reports the unconditional moments of log changes in (real per capita) GDP and its four main components for the data (Panel A) and the model (Panel B). The model is able to generate investment growth which is substantially more volatile than GDP growth. The model actually overstates the volatility of business investment growth (10.9% vs. 6.1% in the data). It also overstates the volatility of aggregate consumption growth (2.3% vs. 1.8% in data). The reason that both are overstated is that investment-to-output and consumption-to-output are even more negatively correlated in the model (-83%) than in data (-46%). Both investment and consumption growth lack the persistence observed in the data. The model somewhat understates the volatility in government discretionary spending growth Δg (1.8% vs. 2.5%), but generates pro-cyclicality and about the right persistence. The investment to capital (X/K) and the investment to GDP (X/Y) ratio moments are reasonably close in model and data.

To understand the workings of the model, it is useful to separate out three types of periods: expansions, non-financial recessions, and financial recessions. Financial recessions are states where both TFP growth is below average and there is a credit crisis, i.e., σ_ω is high. Non-financial recessions are below-average TFP growth and low σ_ω states. GDP growth[26] between period $t-1$ and t is 3.1% when period t is an expansion, -0.24% when t is a non-financial recession, and -1.5% when it is a financial recession. As in the data, financial recessions are worse than non-financial recessions in terms of economic growth. The differences in investment growth are larger still. It is 5.3% in expansions, -2.7% in non-financial recessions, and -11.4% in financial recessions. Aggregate consumption growth is only slightly worse in financial (-0.17%) than in non-financial (-0.14%) recessions, suggesting that the agents in the model do a decent job sharing risk across dates and states. We come back to this point below.

4.2 Financial Variables

Next, we turn to the various balance sheet variables, reported in Table 3.

Firms The first two rows of the borrower panel show the market value of assets ($p_t K_t^B$) and the market value of liabilities of the non-financial corporate sector ($q_t^m A_t^B$), both scaled by GDP and measured at the end of the period. The difference between these two is the market value of firm equity scaled by GDP. The ratio of the two is the mark-to-market leverage ratio, i.e., the ratio of the market value of debt to the market value of assets (row 4). Entrepreneurs own slightly more than half of their firms in the form of corporate equity, on average. Corporate equity amounts to 84% of GDP in the model.

The market leverage ratio is about the same in expansions as in non-financial recessions. Non-financial recessions see an increase in both the market value of assets and liabilities relative to expansions, keeping the market value of equity unchanged. The increase in the market value of capital is entirely accounted for by a rise in the capital-to-output ratio K_t^B/Y_t; there actually is a modest decline in the price of capital p (row 15). Naturally, capital does not fall as much as output in (non-financial) recessions. For the same reason, the market value of debt rises because the quantity of debt increases relative to GDP, FA_t^B/Y_t (row 3), while the price of corporate debt q_t^m does not change much between expansions and non-financial recessions (row 17).

[26]We define GDP growth as growth in producer output Y_t net of deadweight losses caused by corporate bankruptcies.

Financial recessions affect corporations quite differently. First, the drop in the price of capital is much larger. But the capital to output ratio increases a lot more as well so that the market value of capital relative to GDP still ends up higher in financial recessions than in expansions. The main difference is that the market price of debt is much lower in financial recessions, or equivalently the interest rate on corporate debt is much higher. Book debt actually increases (row 3), at least relative to a depressed GDP.

The borrower-entrepreneurs' book leverage is 45% on average, measured as book value of debt to market value of assets (row 5). Both the mean and volatility of the corporate debt-to-GDP ratio are close to the data. In the model, borrowers are always up against their leverage constraint. The reasons are that they are sufficiently impatient and the tax shield adds further value to taking out debt. In financial recessions, the market value of firm collateral falls and borrowing constraints tighten. This reduces the firm's debt capacity and its actual leverage, given that the constraint binds.

Borrowers default when the book value of their debt falls below the market value of their (collateral) assets. The model generates average corporate default and loss rates of 1.4% (row 6) and 0.7% points (row 8), respectively, implying an average loss-given-default rate of 36% (row 7). All these numbers are in line with the data. Loss rates are a lot higher in financial recessions (4.52%) than in non-financial recessions (0.31%) and expansions (0.28%). This is the result of both higher default rates and higher losses-given-default in financial recessions, both of which are important features of the data. Thus the model generates the right amount of corporate credit risk and generates the strong cyclicality in that quantity of risk observed in the data.

Intermediaries Intermediary leverage is 91.5% in market value and 93.1% in book value terms on average in the model. It comes close to matching the 95.6% financial leverage target. Intermediaries choose to be so highly levered for a number of reasons. Like the corporate firms, they are impatient and enjoy a tax shield. As the only agent with access to deposits, they alone can earn a large spread between the short-term deposit rate (1.88%) and the rate on corporate loans (4.60%). They bear the interest rate risk associated with the maturity transformation they perform, as well as the credit risk on the loans. Given the low (but realistically calibrated)

Table 3: Balance Sheet Variables and Prices

	Unconditional		**Expansions**	**Non-fin Rec.**	**Fin Rec.**
	mean	stdev	mean	mean	mean
			Borrower		
1. Mkt Val of Capital / Y	1.511	0.041	1.503	1.539	1.511
2. Mkt Val of Corp Debt / Y	0.671	0.035	0.670	0.690	0.635
3. Book val corp debt / Y	0.682	0.023	0.677	0.698	0.679
4. Market corp leverage	44.39%	1.56%	44.58%	44.80%	42.03%
5. Book corp leverage	45.11%	0.36%	45.06%	45.33%	44.97%
6. Default rate	1.44%	1.85%	0.83%	0.94%	7.15%
7. Loss-given-default rate	36.14%	8.71%	33.47%	33.02%	63.11%
8. Loss Rate	0.68%	1.24%	0.28%	0.31%	4.52%
			Intermediary		
9. Mkt fin leverage	91.46%	2.46%	90.88%	90.97%	96.84%
10. Book fin leverage	93.14%	2.14%	92.85%	93.61%	94.21%
11. Fraction leverage constr binds	30.35%	45.98%	17.09%	50.45%	80.02%
12. Bankruptcies	0.49%	6.98%	0.00%	0.00%	5.24%
			Saver		
13. Deposits / Y	0.625	0.030	0.622	0.645	0.598
14. Government Debt / Y	0.455	0.372	0.443	0.492	0.455
			Prices		
15. Tobin's q	1.000	0.011	1.004	0.996	0.983
16. Risk-free rate	1.88%	2.27%	2.17%	2.90%	-2.69%
17. Corporate bond rate	4.60%	0.32%	4.54%	4.56%	5.16%
18. Credit spread	2.73%	2.30%	2.37%	1.66%	7.85%
19. Excess return on corp. bonds	2.10%	3.14%	2.90%	1.91%	-3.27%

average loss rate, they choose to take up substantial leverage to reach their desired risk-return trade-off.

Intermediary leverage is lower in non-financial recessions and higher in financial recessions than in expansions. In other words, the behavior of financial leverage over the cycle is opposite that of non-financial leverage. The main reason for the rise in intermediary leverage in financial recessions is that the market value of banks' assets, corporate loans, falls substantially. This occurs because default risk and default risk premia rise, both of which increase corporate bond yields. A lower value of bank assets tightens their regulatory capital constraint. The intermediary leverage constraint binds in 80% of the financial crises compared to 50% of non-financial recessions and 17% of expansions. In those periods, intermediaries must reduce deposits to meet capital requirements in the wake of their credit losses. Given the low cost of deposit funding in financial crises (-2.7%), intermediaries would like to raise more deposits and increase corporate lending but they cannot. In contrast, intermediaries are unconstrained in half of the non-financial recessions. The expected return per unit of risk must not be conducive to expand their lending activities during these periods.

Intermediary net worth, or bank equity, is an important state variable in all intermediary-based models. Intermediary net worth is the difference between the market value of bank assets (row 2) and the book value of deposits (row 13). Intermediary equity to GDP is 4.6% on average. From a high of 4.8% of GDP in expansions, it shrinks to 3.7% of GDP in financial recessions, while it only falls to 4.5% in non-financial recessions. The reduction in intermediary net worth levels is larger still since GDP is lowest in financial recessions. Intuitively, the reduction in net worth makes intermediaries effectively more risk averse, leading them to charge larger risk premia on new lending.

In the equilibrium of our model, the intermediation sector is insolvent in 0.5% of the periods. Bank bankruptcy only occurs in financial recessions (10% of the years), when it happens with 5% probability (row 12). In those periods, the government makes whole the depositors and takes over the assets of the banks at their market value. The banking sector restarts with zero wealth the next period. Deposit insurance lowers the cost of funding and provides banks with a risk shifting motive vis-a-vis the government. But, as risk averse agents, bankers are reluctant to hit low net worth states since they imply low consumption and high marginal utility. The trade-off between these two factors generates rare financial disasters.

Savers Risk averse savers only hold safe debt: they hold the safe debt supplied by the government and the intermediaries (rows 13 and 14). In the model, these two sources are 46% and 63% of GDP, respectively (rows 13 and 14). Because of a counter-cyclical primary deficit to GDP ratio, the government must raise more resources in recessions. The marginal agent absorbing fluctuations in safe assets is the risk averse saver. Both during financial and non-financial recessions, the total supply of safe debt from deposits and government debt (relative to GDP) is lower than during expansion. However, when compared to non-financial recession, the riskfree rate is much lower during financial recessions, implying that the price of safe debt is much higher. What causes this much higher price in financial recessions given the roughly same reduction in the quantity as in non-financial recessions? The reason is increased demand for safe debt. The conditional expectation of consumption growth of savers is lower in financial recessions, boosting their consumption smoothing motive and demand for safe assets. Because intermediaries are much more likely to be constrained in financial recessions, they cannot increase their supply of deposits, and prices of safe assets must rise.

Prices Real interest rates on safe debt are 1.88% on average and have a volatility of 2.27% (row 16). Both are very reasonable numbers matching historical averages. Non-financial recessions have higher risk-free rates since banks must pay up to attract deposits from savers with weaker consumption smoothing demands. Faced with strong corporate collateral values (row 15) and low default rates, banks view the corporate lending environment as favorable. Corporate loan rates are the same as in expansions. In contrast, financial recessions see large declines in collateral values, negative (excess) returns on bank assets (row 19), high corporate credit spreads (row 18), and very low real interest rates (row 16). All of these are important features of real-life financial crises.

The model is able to generate a high unconditional credit spread while matching the observed amount of default risk.[27] The credit spread is also highly volatile and counter-cyclical. The

[27] We define the credit spread as the difference between the yield on a blended portfolio of investment grade and high yield bonds and the yield on a one-year constant maturity Treasury yield. We use the longest available sample from Barclays U.S. corporate IG and HY bond indices from February 1987 to December 2015. To determine the portfolio weights on the high yield versus investment grade bonds, we use market values of the

reason is that the model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical SDF of the intermediary, who is the marginal agent in the corporate loan market.

4.3 Consumption and Welfare

Table 4 reports the moments of consumption for each agents, as well as each agent's value function, and aggregate welfare. By virtue of being the largest groups of agents, borrowers and savers have the highest consumption shares (relative to GDP). More interesting is consumption growth in the second panel. It reveals that the intermediary has by far the most volatile consumption growth, followed by the saver, and the borrower. All agents have the same elasticity of intertemporal substitution, but the saver has the highest risk aversion. Thus, the borrower has a relatively low consumption growth volatility and the saver a relatively high one, given their risk aversions. The reason that the borrower is able to smooth consumption so well is that she owns a large share of the firm and receives a large share of aggregate labor income. Thus her net worth buffer is high and gets replenished with a large and regular labor income stream.[28] Financial recessions have a positive element for borrowers: they can shed some of their debt and buy back the bank-repossessed capital assets at a much lower price. Borrower consumption growth is not that low in financial crises (1.38% versus 1.94% unconditionally). The saver is stuck absorbing fluctuating amounts of safe assets, expanding saving and cutting consumption in the worst states of the world. Her consumption growth is only 0.9% in financial crises. Intermediaries' role is to help the saver and borrower smooth consumption by absorbing most of the aggregate risk in the economy. The intermediary absorbs all credit losses and suf-

amounts outstanding, also from Barclays. The average weight on HY is 19.4%. The resulting credit spread has a mean of 3.36% and a volatility of 1.46%. We compare this with a difference measure of the credit spread which takes a 19.4%-80.6% weighted average of the Moody's Aaa and Baa yields and subtracts the one-year CMT rate. Over the same February 1987-December 2015 period, the mean credit spread is 3.05% with a volatility of 1.39%. The second measure of the credit spread has a correlation of 86% with the first one. The advantage of this second measure is that we can compute it back to 1953. The mean spread over the 1953-2015 period is 2.08% with a volatility of 1.55%. While the second credit spread is downward biased, compared to the better first measure, considering a longer sample period would lead us to consider a lower mean credit spread target than 3.36%. For example, we could add the 31 basis point difference between our favorite Barclays measure and the Moody's measure over the post-1987 sample to the full-sample Moody's mean of 2.08% to get to a target mean credit spread for the full 1953-2015 sample of 2.39%. Our model is close with a mean credit spread of 2.73%.

[28] Bernanke, Gertler, and Gilchrist's (1999) model gives the entrepreneurs a much smaller labor income share.

Table 4: Consumption and Welfare

	Unconditional		**Expansions**	**Non-fin Rec.**	**Fin Rec.**
	mean	stdev	mean	mean	mean
			Consumption to Output		
Consumption, B	0.344	0.005	0.342	0.344	0.354
Consumption, I	0.025	0.006	0.027	0.026	0.011
Consumption, S	0.312	0.007	0.312	0.311	0.319
			Consumption growth		
Consumption, B	2.00%	1.72%	2.67%	0.17%	1.41%
Consumption, I	2.00%	38.96%	10.79%	-0.29%	-57.01%
Consumption, S	2.00%	3.26%	2.92%	-0.45%	0.96%
			Welfare		
DWL / Y	0.003	0.005	0.001	0.001	0.018
Value function, B	0.378	0.006	0.377	0.379	0.382
Value function, I	0.026	0.002	0.026	0.027	0.023
Value function, S	0.528	0.004	0.530	0.524	0.524
Aggregate welfare	0.472	0.003	0.473	0.470	0.470
			Marginal utility ratios		
log(MU B / MU I)	0.009	0.300	0.096	0.065	-0.753
log(MU B / MU S)	2.208	0.026	2.206	2.211	2.218
log(MU S / MU I)	-2.199	0.301	-2.110	-2.146	-2.971

fers a 57% consumption drop in a financial recession. Financial recessions transfer wealth from intermediaries to borrowers, and to a lesser extent to savers.

The third panel reports moments related to aggregate welfare. Deadweight losses from corporate bankruptcies are small relative to GDP (0.3%), but larger in financial recessions when the loss rate spikes (1.8%). This takes away resources from the economy. Still, fluctuations in DWL only account for a small share of overall fluctuations in GDP. Aggregate welfare is computed as the population-weighted average of the value functions of the three types of agents. It is lower in recessions than in expansions, largely because of the saver's reduction in welfare in recessions. Borrowers fare well in financial recessions, welfare is slightly higher in financial than non-financial recessions. Intermediaries suffer the most.

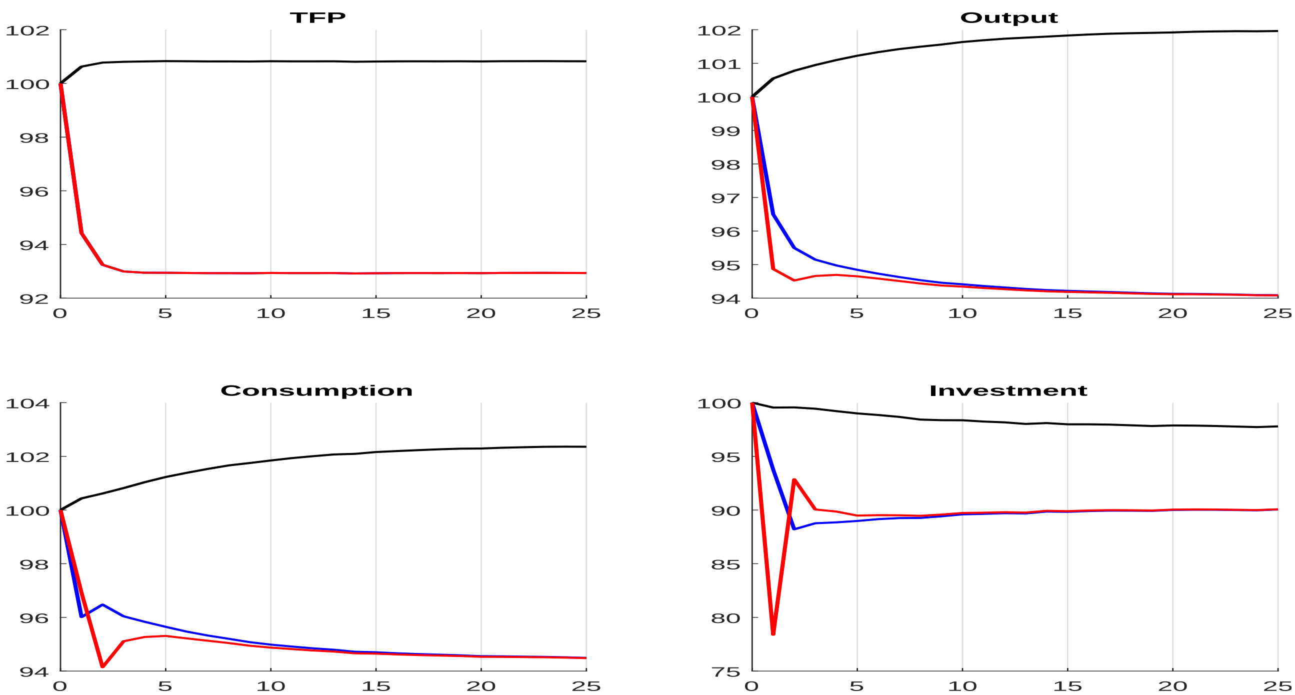
The last panel reports ratios of marginal utilities between pairs of agents. In a complete markets model with agents whose preference parameters differ, these ratios would differ across pairs of agents but be constant over time. Our model is an incomplete markets model featuring imperfect risk sharing; the marginal utility ratios display non-trivial volatility. For example, the borrower has higher marginal utility than the intermediary in expansions, but the reverse is true in financial crises. This occurs despite the fact that the borrower and intermediary have identical preferences.

4.4 Model Dynamics in Financial Crises

To further understand the dynamics of a credit crisis in the model, we compute impulse-responses. We start off the model in a high-growth state, the second highest of five points on the TFP growth grid, and the low σ_ω state. All other endogenous variables take their average values for that exogenous state. In period 1 the model undergoes a change to the lowest TFP growth grid point. In one case, the recession is accompanied by a switch to the high σ_ω state, a financial recession or credit crisis. In the second case, the economy remains in the low σ_ω state, a non-financial recession. From period 2 onwards, the two exogenous states follow their stochastic laws of motion in each case. For comparison, we compute an additional series where we let the exogenous states stochastically revert to the mean starting from period 0. Specifically, we simulate 10,000 sample paths of additional years and average across paths. In periods 1 through 25, the endogenous state variables take on their average value corresponding to the particular exogenous states that are realized on that particular sample path. The exercise allows us to study the transition from a strong economy with a credit boom to a bust with a financial crisis, simulating the experience of the years 2006–2015. Comparison of the two cases helps us to understand the differences between financial and non-financial recessions. Figures 1-4 plot the response functions for the key variables. All quantity variables are normalized to 100 in year 0. All growing quantity variables are detrended by the constant long-run productivity growth rate (of 2.00% per year).

Figure 1 shows that GDP, consumption, and investment all fall precipitously in the first period of the recession relative to their boom-era values. Government consumption increases by our assumption of countercyclical government spending. What is interesting is that output and consumption continue to fall sharply in the year following a financial recession, while they stabilize in the year following a non-financial recession. This is not a mechanical effect because the probability of a recession in year 2 is independent of whether or not the economy was in a high credit risk state in year 1. The model generates endogenous amplification. Most of the additional period-2 decline in consumption is due to borrowers who cut spending to rebuild the capital stock in those period 2 states where TFP recovers. A second interesting finding is that investment takes a much bigger hit in financial than in non-financial recessions in period 1. After a partial recovery in period 2, investment drops again in period 3. Even ten years later, investment remains 10% below the previous peak (and 8% below the mean reverting path). The same is trough to a lesser extent for the other macroeconomic quantities.

Figure 1: Financial vs. Non-financial Recessions: Macro Quantities



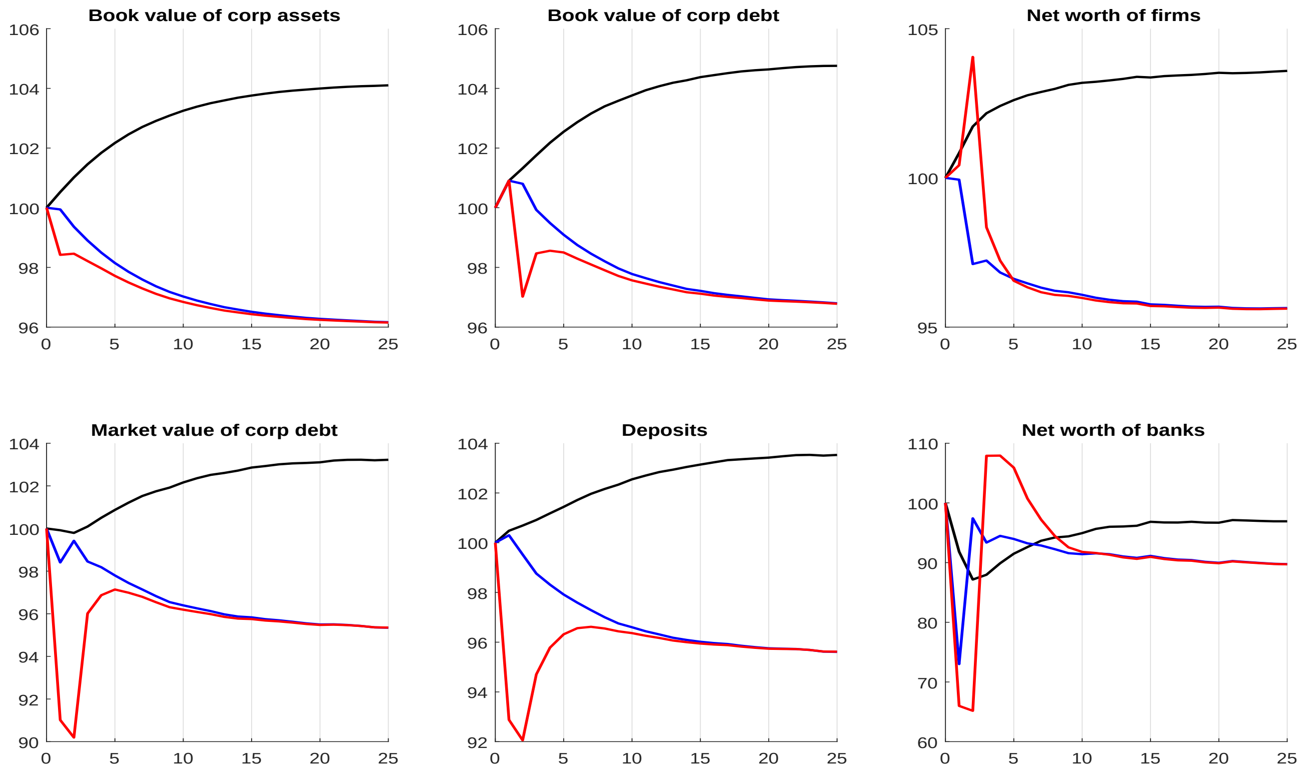
The graphs show the average path of the economy through a recession episode which starts at time 1. In the previous period, the economy was in a high growth state. The recession is either accompanied by a high dispersion of depreciation (high σ_ω, financial recession), or low dispersion (low σ_ω, non-financial recession). From period 2 onwards, the economy evolves according to its regular probability laws. We obtain these via a Monte Carlo simulation of 10,000 paths of periods 2-10, and averaging across these paths. **Blue line:** non-financial recession **Red line:** financial recession.

Figure 2 focuses on the assets and liabilities of borrowers and intermediaries. All balance sheet variables are measured at the end of the period so that the shock at time 1 affects them in period 1 of the transition graph. Variables that contain market prices, like the market value of firm debt or both net worth series, are affected by movements in prices and quantities. The first two panels show that firms shrink their book value of assets and liabilities much more in a financial recession than a non-financial recession. Firms' debt falls more in market value (bottom left panel) than in book value terms due to the decline in the price of debt. The market value of assets also falls more sharply than the book value because of a decline in the price of capital assets. The decline in the market value of assets is smaller than that in the market value of liabilities in financial recessions so that the net worth of the corporate sector increases sharply in financial recessions (top right panel). Most of the increase in net worth occurs in period 2 and it is the result of a rebound in the price of capital and a further decline in the price of corporate loans. It occurs despite an increase in the book value of corporate debt in period 2. Entrepreneurs cut consumption thereby increasing the resources available inside the firm. One interpretation is that firms are building reserves in anticipation of future investment needs rather than paying out dividends to their owners. As we saw in Figure 1, firms indeed resume investment in period 2.

The bottom panels of Figure 2 show banks' balance sheet items. The market value of banks' assets is in the bottom left, while the deposits in the middle panel are the banks' liabilities (both in market and book value terms). The difference between the two is the banks' net worth, plotted in the right panel. A financial crisis coincides with a sharp drop in bank assets and liabilities. Bank leverage increases sharply in market and book value terms. There is no recovery in period 2 as bank assets, bank liabilities, and bank equity continue to fall in the year after the shock. Bank equity falls by more than one-third in two years. There is a rebound in the market value of banks' assets in year 3, which is to a large extent driven by an increase in the price of corporate loan assets. The recovery in loan values relaxes banks' borrowing constraint and allows them to increase deposits. The quantity of deposits rebound more modestly and gradually. Even though the deposit rate turns negative and banks could make large profits be expanding deposits, they are constrained and thus unable to take advantage of the low rates immediately after the shock. Financial crises redistribute wealth from financial to non-financial firms in the model. By defaulting on their debt and buying back the seized collateral from the banks at fire sale prices, firms are able to recapitalize. Banks absorb the credit losses, resulting in a massive decline in equity.

Figure 3 shows the interest rates, the credit spread, and the price of capital. In the first period of a financial recession following a boom, the real risk-free rate turns negative and the credit spread blows out reaching 800 basis points. Financial recessions are periods of high credit risk

Figure 2: Financial vs. Non-financial Recessions: Balance Sheet Variables

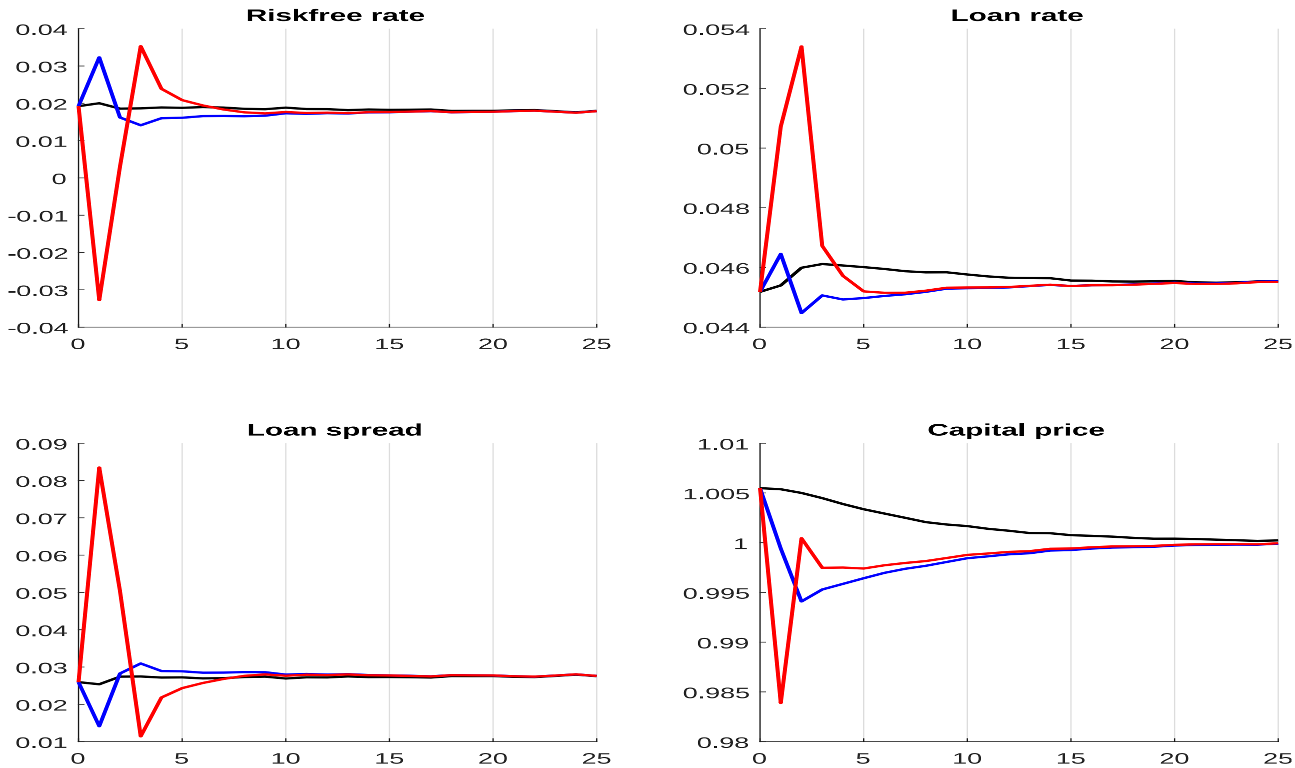


Blue line: non-financial recession **Red line:** financial recession.

and credit risk premia, both of which enter in the credit spread. Strong precautionary savings motives depress the real rate. In sharp contrast, non-financial recessions see an initial increase in the risk-free rate and a decline in the credit spread. The price of capital falls most in financial recessions. In the year after a financial crisis, the risk-free rate and credit spreads reverse. This occurs because the recession and the credit crisis end with substantial probability in period 2. The loan rate, which is their sum, continues to increase putting further pressure on the value of bank assets. The period-2 reversal can also be seen in the price of capital.

Figure 4 shows the value functions of the three types of agents, and aggregate welfare, their population-weighted average. The main distinction between non-financial (left panel) and financial recessions is that intermediaries suffer much greater welfare losses in a financial recession. They also rebound much more strongly in period 2. The welfare gains that borrowers experience and the losses that savers experience are similar in magnitude. Borrowers can recapitalize at the expense of the banks in financial recessions, shedding debt in bankruptcy and buying capital at distressed prices. The net result is that aggregate welfare losses from a

Figure 3: Financial vs. Non-financial Recessions: Prices



Blue line: non-financial recession **Red line:** financial recession.

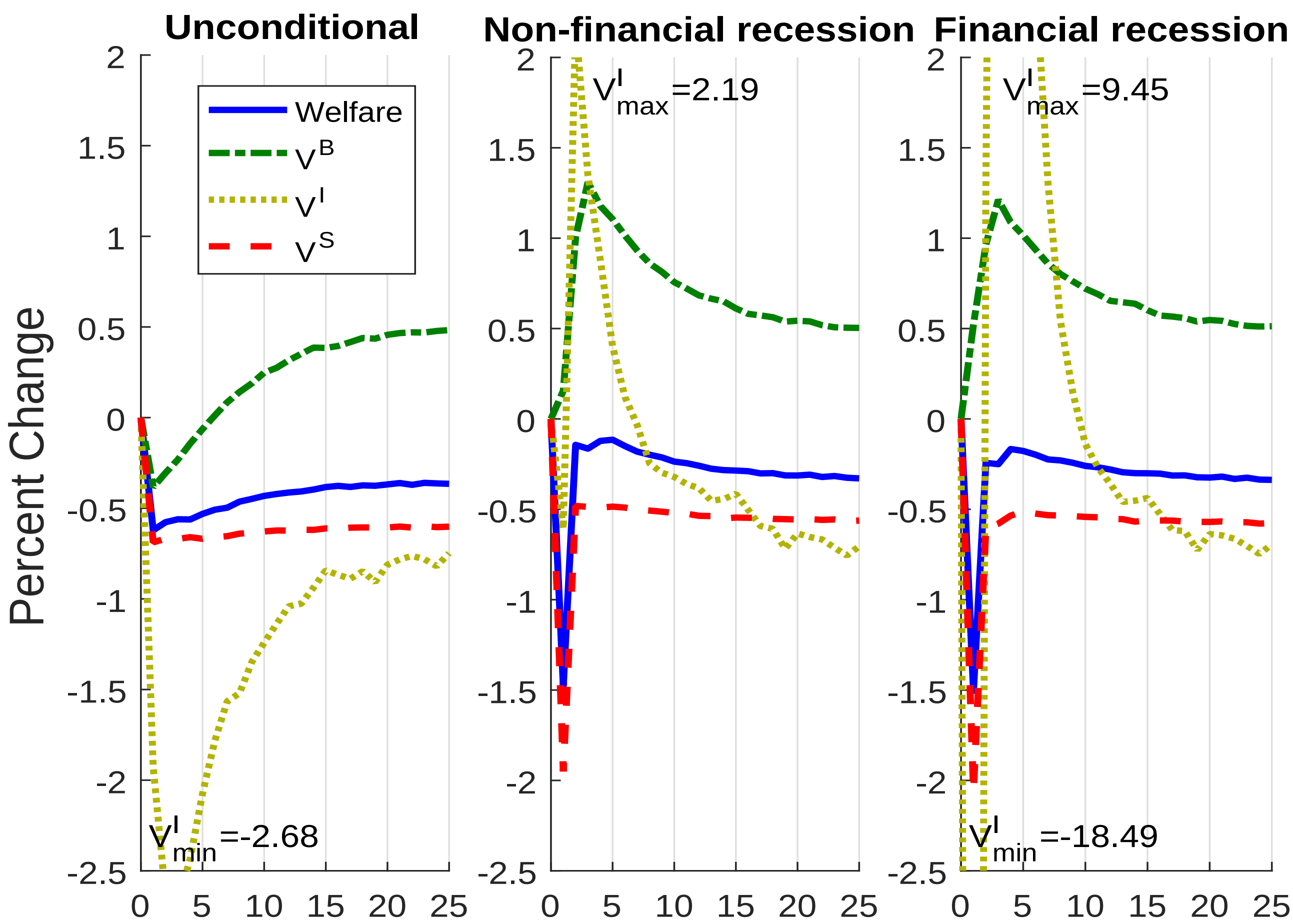
recession are about the same for financial and non-financial recessions.

4.5 Predictive Regressions and the Credit Risk Premium

A literature cited in the introduction documents that credit spreads are good predictors of future economic growth. We add to this literature by revisiting this predictive relationship inside a rich structural model that generates endogenous fluctuations in both credit spreads and macroeconomic aggregates.

Table 5 summarizes our predictability results. In the first three columns we focus on predicting real per capita GDP growth between period t-1 and t, Δy_t. The first column uses the credit spread at time t-1, s_{t-1}, as the predictor. As before, the credit spread is the difference between the yield on long-term defaultable corporate loans and the yield on the short-term safe asset. Since the model is cast at annual frequency, the regression are run based on 10,000 periods of annual simulated data. As is common in the empirical literature, the regression controls for

Figure 4: Financial vs. Non-financial Recessions: Welfare



one lag of the dependent variable. T-statistics are reported below the point estimates.

The first column shows that the credit spread forecasts next year's GDP growth with a slope of -0.052. The R^2 is 11.9%. The coefficient implies that a one standard deviation increase in the credit spread changes GDP growth by -0.05 standard deviations. In the second column we replace the credit spread at time t-1 by the change in the credit spread between t-2 and t-1, Δs_{t-1}. If the credit spread is persistent, this may be the better specification. The results are very close in terms of economic magnitudes and R^2. Using annual data from 1929-2013 and the Baa-minus-Treasury spread, Lopez-Salido, Stein, and Zakrajsek (2015) report a standardized response of -0.37. While our model generates the correct sign as well as significance, the magnitude of the effect is about 1/6th of that found in the data.

The credit spread fluctuates because of changes in the expected default rate, the quantity of risk, and because of changes in the price of credit risk. In addition, since both agents trading the

Table 5: Predicting Economic Activity with the Credit Spread

	(1) Δy_t	(2) Δy_t	(3) Δy_t	(4) CRP_t	(5) CRP_t	(6) Δy_t
s_{t-1}	-0.0516		-0.0320			
	(-5.28)		(-2.90)			
Δs_{t-1}		-0.0447				
		(-6.25)				
CRP_{t-1}			-0.195			
			(-3.85)			
G_t				-0.0483	-0.0515	
				(-36.10)	(-50.92)	
crisis_t				0.0070	0.0093	
				(52.44)	(85.87)	
ILev_t					0.0843	
					(41.32)	
FLev_t					0.248	
					(83.38)	
$1[\text{ILev}_{t-1} > 0.97]$						-0.0027
						(-2.62)
ILev_{t-1}						-0.0056
						(-0.39)
FLev_{t-1}						-0.123
						(-5.79)
Δy_{t-1}	0.333	0.339	0.315			0.342
	(34.99)	(36.08)	(29.69)			(35.45)
Constant	0.0144	0.0128	0.0158	0.0574	-0.129	0.0727
	(35.98)	(49.66)	(28.50)	(41.83)	(-42.31)	(3.48)
Adjusted R^2	0.119	0.120	0.120	0.391	0.668	0.122

corporate bonds are potentially constrained, their respective shadow values of marginal wealth also affect the spread. The expected return on corporate bonds in excess of the one-period safe bond rate is hence both a measure of the risk premium and the tightness of the intermediaries' lending constraint. The theoretical literature on intermediary-based asset pricing emphasizes the role of high risk premia as indicators of financial crises. To isolate the risk premium, we directly compute it in the model as the (conditional) negative covariance of the intermediary's stochastic discount factor and the corporate bond's return. Column (3) uses this definition of the credit risk premium (CRP) and shows that it, too, forecasts future GDP growth with a negative sign. The regression still includes lagged GDP growth and the lagged credit spread. The R^2 is the same 12% as for the regression only including the credit spread and lagged GDP. Greenwood and Hanson (2013) and Lopez-Salido, Stein, and Zakrajsek (2015) provide empirical proxies for the expected excess returns associated with bearing credit risk, interpret them as investor sentiment measures, and show that they predict future economic activity. Our model generates fluctuating "investor sentiment," in that risk premia are time-varying, which predicts economic activity. The fact that both the CRP and the credit spread enter significantly shows that the other components of the spread, namely risk-neutral compensation for expected defaults and constraint tightness, have additional predictive power for GDP growth.

In columns (4) and (5), we investigate which of the state variables in the model are the key drivers of the credit risk premium. In intermediary-based asset pricing models, the net worth share of the intermediary is usually a key state variable. For example, in He and Krishnamurty (2013), when intermediaries represent a small share of total wealth, they require high risk premia to be willing to hold risky assets. Our model features much more complex risk premium dynamics since there are three agents whose wealth shares matter (plus government debt, plus the capital stock). The effects of all state variables on the risk premium may be nonlinear.

As a first step, we regress the credit risk premium in the model on the two exogenous state variables, measured contemporaneously, in column (4).[29] They explain 39.1% of the risk premium variation. The credit shock is naturally a stronger driver than TFP growth: being in a credit crisis leads to a 70 basis points higher premium (relative to an average premium of 90 bp). When we add financial intermediary and corporate leverage as explanatory variables

[29]The variable crisis_t is an indicator for states with high $\sigma_{\omega,t}$.

for the credit risk premium in column (5), to capture the nonlinear effect of corporate and intermediary wealth, the R^2 increases to 66.8%. Both kinds of leverage have large and positive explanatory power for the premium.

In column (6), we directly use these state variables of the model as predictors of future GDP growth, instead of controlling for their effect indirectly through prices as in column (3). To capture the nonlinear effect of intermediary leverage, we include an indicator variable that is one in periods with intermediary leverage greater than 97%, which are the 5% of all periods with the highest leverage[30]. Both high corporate and intermediary leverage predict low future growth. The R^2 is slightly higher than in columns (1)-(3) at 12.2%. The significant indicator term shows that periods of very high intermediary leverage correspond to financial crises that lead to low future growth: being in such a crisis today causes 27 bp lower growth tomorrow on average.

Figure 5 shows the histogram of the intermediary wealth share plotted against the credit spread. Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary's wealth share is low. In a financial crisis, our model generates quantitatively large credit spreads and risk premia.[31]

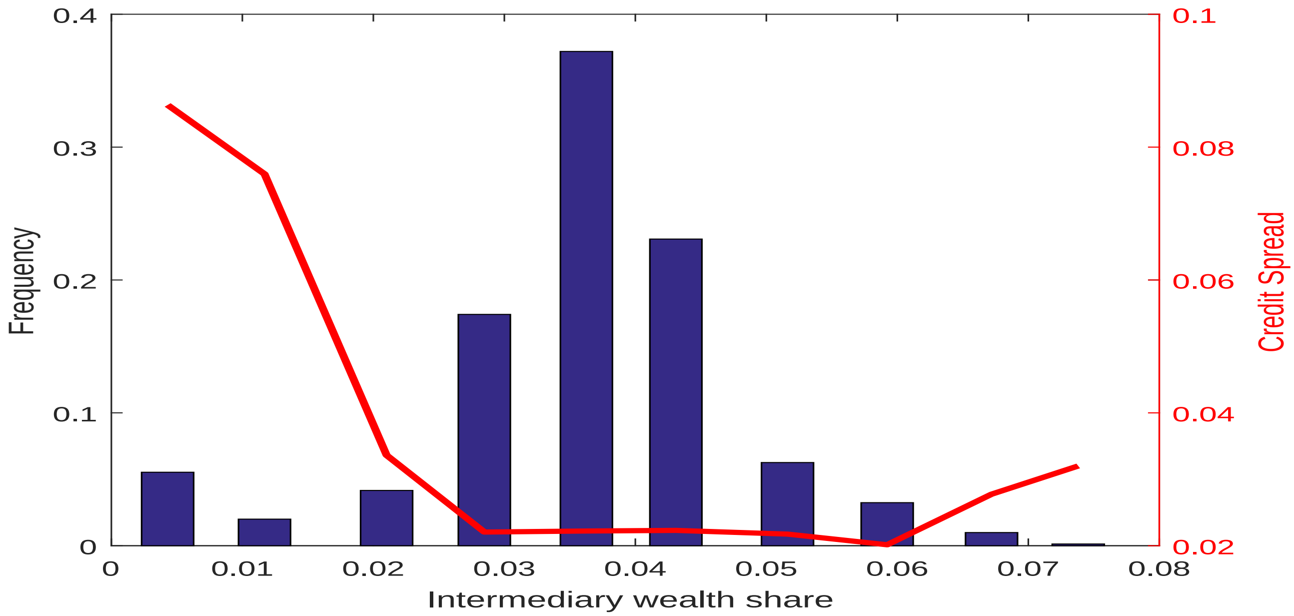
5 Macro-prudential Policy

We use our calibrated model to investigate the effects of macro-prudential policy choices. The first exercise we consider is tightening the maximum loan-to-value ratio that applies to corporate borrowers (ϕ). It directly constrains the maximum leverage of the non-financial firms. The second experiment increases the regulatory capital weight that enters in banks' Basel constraint (ξ). It directly constrains the maximum leverage of the financial intermediaries. Table 6 presents the financial results, while Table 7 presents macro and welfare results.

[30] ILev_t is intermediary leverage at the beginning of the period. It may hence be higher than the maximum leverage at the end of a period, given by ξ.

[31] Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and the credit risk premium.

Figure 5: The Credit Spread and the Financial Intermediary Wealth Share



Tightening Firms' LTV Constraint Tightening LTV limits in column 2, we find that firm book leverage falls by 9 percentage points when ϕ is reduced from 0.50 to 0.40. With lower leverage, default rates drop substantially from 1.44% in the benchmark to 0.64%. Average loss given default also falls, resulting in a loss rate that drops from 4.5% to 2.8% in financial recessions, making the latter substantially less severe events. The concomitant reduction in deadweight losses from foreclosure (-57%) is a first source of welfare gain from this macro-prudential policy.

The second result is that tighter LTV constraints and the improved safety of corporate loans increase the willingness of banks to lend. Intermediary leverage constraints bind in 59% of the periods compared to 30% in the benchmark model. Because these financial recessions are less severe, banks regularly want to increase lending to the point where their constraint binds. In expansions intermediary constraints bind in 52% of periods compared to 17% in the benchmark. In financial recessions, banks' regulatory capital constraint binds in 98% of the periods versus 80% in the benchmark model. On average, banks' leverage itself is almost unchanged.

Banks are smaller. Deposits represent only 52% of GDP compared to 63% in the benchmark model. The reduction in the size of the financial intermediation sector follows directly from the macro-prudential policy. Bank bankruptcies are entirely eliminated. The increased financial stability reduces macro-economic volatility: GDP growth (-0.9%), consumption growth (-8.5%), and investment growth (-49.7%) are all less volatile.

The improvement in economic and financial stability comes at the cost of a smaller economy. The reduction in corporate credit reduces the size of the entire economy: the capital stock and output are smaller than in the benchmark economy by 5.4 and 1.88 percent, respectively.

On net, tightening firms' maximum LTV ratio lowers welfare by 0.49% in consumption equivalent units. The reduction in credit risk in the system benefits the stability of the economy, helping all three agents smooth consumption better. The volatility of all pairwise marginal utility ratios falls. But the smaller size of the economy hurts the mean consumption of all agents. There are interesting distributional implications. Maybe surprisingly, borrowers benefit from having tighter LTV constraints imposed on them. Their value function increases by 1.42%. They consume a larger share of total output compared to the benchmark (+3.6%). An important reason for the higher average consumption share is that borrowers pay a lower interest rate on their debt, especially during financial crises.

Savers, who are by far the largest group in the economy, lose from the policy: their value function falls by 1.0% and their consumption share by 1.2%. They earn lower interest rates on their savings. The scarcity of safe debt allows banks and the government to pay a lower safe interest rate. Lower rates in turn reduce the interest expense and the safe debt the government must issue.

Finally, the banks are the biggest losers from this policy because (i) their operations shrink in size, and (ii) the difference between the yield on their assets and liabilities, the credit spread, shrinks by 42 basis points. Intermediaries earn a risk premium as compensation for absorbing large fluctuations in output resulting from occasional crises. Tightening the corporate leverage constraint reduces the size of these crises, and lowers the compensation intermediaries receive from insuring the other types of agents from these crises.

Tightening Banks' Leverage Constraint The next experiment we investigate is a increase in bank equity requirements. Rather than being able to borrow 95 cents against every dollar in assets ($\xi = 0.95$), we only allow banks to raise 90 cents in deposits ($\xi = 0.90$). This change lowers equilibrium bank leverage from 93.1% to 88.9% in book value terms and by a similar amount in market value terms. The regulatory capital constraint naturally binds more

Table 6: Macroprudential Policy

	Benchmark mean	**Lower ϕ** mean	**Lower ξ** mean	**Lower ϕ and ξ** mean
		Borrowers		
1. Mkt Val of Capital / Output	1.511	1.457	1.481	1.425
2. Mkt Val of Corp Debt / Output	0.671	0.558	0.634	0.517
3. Book val corp debt / Y	0.682	0.530	0.670	0.519
4. Market corp leverage	44.39%	38.27%	42.78%	36.26%
5. Book corp leverage	45.11%	36.39%	45.22%	36.43%
6. Default rate	1.44%	0.64%	1.22%	0.51%
7. Loss-given-default rate	36.14%	34.87%	35.75%	34.45%
8. Loss Rate	0.68%	0.33%	0.58%	0.27%
		Intermediaries		
9. Mkt fin leverage	91.46%	91.08%	87.36%	87.24%
10. Book fin leverage	93.14%	92.54%	88.95%	88.72%
11. Fraction leverage constr binds	30.35%	58.93%	56.60%	87.93%
12. Bankruptcies	0.49%	0.00%	0.00%	0.00%
		Savers		
13. Deposits / Y	0.625	0.516	0.564	0.459
14. Government Debt / Y	0.455	0.150	0.306	0.133
		Prices		
15. Tobin's q	1.000	1.000	1.000	1.000
16. Risk-free rate	1.88%	1.62%	1.87%	1.71%
17. Corporate bond rate	4.60%	3.92%	5.02%	4.48%
18. Credit spread	2.73%	2.31%	3.16%	2.77%
19. Excess return on corp. bonds	2.10%	2.02%	2.63%	2.55%

frequently: 56% of the periods compared to 30% in the benchmark. All bank bankruptcies are eliminated by this policy.

Bank deposits shrink by 6% of GDP, which is a smaller decline than in the previous policy. Because the intermediary is better capitalized, with bank equity being 7.0% rather than 4.7% of GDP, she is better able to absorb credit losses. Credit losses are also smaller than in the benchmark. Nevertheless, the intermediary earns a high credit spread (43 bps higher than in the benchmark), reflecting the fact that the intermediary leverage constraint is binding more often. Because of the higher profit margin, the intermediary's consumption share grows by 8.6%. Her consumption growth volatility also shrinks because of the higher equity buffer, further adding to the welfare gain. Like in the previous experiment, it is the agent whose constraint is tightened that benefits the most from the policy change. In this case it is the intermediary whose welfare increases by +11.60%.

Tighter bank capital requirements have a modest adverse effect on non-financial corporations; their welfare falls by -0.7%. They face a higher corporate bond rate (+42bps). The equilibrium amount of corporate debt and the economy's capital stock (-3.0%) shrink somewhat. Effects on production are more modest (-1% decline in output) than in the first experiment. Nevertheless, the smaller size of the economy is again the culprit for the overall welfare loss.

Savers are also worse off (-0.46%) as their consumption share decreases and their consumption growth volatility increases. There is much less safe debt for them to hold (22% of GDP less) in equilibrium, largely due to the tighter bank capital requirements. This hampers savers' wealth accumulation. The equilibrium interest rate is unchanged indicating that the changes in supply of safe debt are offset by changes in the saver's demand.

Overall, the welfare effect of tighter bank capital requirements is a welfare loss of -0.50% which is very similar to that from the LTV policy. The gains from lower volatility and lower deadweight losses from foreclosure are not as big as in the first experiment but the costs from a smaller economy are not as large either.

Combining Both The combined welfare effects from both macro-prudential policy changes are nearly additive in terms of the aggregate welfare loss: -1.04%. The main cost from the policy is a much smaller economy (2.98% decline in output). The main benefit is lower DWL

Table 7: Macroprudential Policy: Macro and Welfare

	Benchmark	**Lower ϕ**	**Lower ξ**	**Lower ϕ and ξ**
	mean	mean	mean	mean
			Welfare	
Aggregate welfare	0.472	-0.49%	-0.50%	-1.04%
Value function, B	0.378	+1.42%	-0.70%	+0.39%
Value function, I	0.026	-7.73%	+11.60%	+0.57%
Value function, S	0.528	-1.03%	-0.46%	-1.47%
DWL / Y	0.003	-56.85%	-17.36%	-66.46%
			Size of Economy	
Output	0.975	-1.88%	-1.03%	-2.98%
Capital stock	1.447	-5.42%	-3.00%	-8.51%
			Volatility	
Output growth	1.97%	-0.90%	-0.28%	-0.89%
Consumption growth	2.28%	-8.52%	-3.85%	-10.26%
Investment growth	10.89%	-49.65%	-12.07%	-54.25%

from defaults (-66%), greater financial stability, and lower macroeconomic volatility. The effects on inequality from both policies are different. Tighter LTV requirements mostly redistribute wealth from intermediaries to borrower-entrepreneurs, while the opposite is true for restrictions on bank leverage. Savers lose from both policies, whereas banks and entrepreneurs each gain modestly. The reason that savers are worse off is the reduced supply of risk free debt, which in equilibrium equals savers' wealth, and the reduction in the risk free interest rate, which is the return savers earn on their wealth.

6 Conclusion

We provide the first calibrated macro-economic model which features banks who extend long-term defaultable loans to non-financial firms producing output and raise deposits from risk averse savers, and in which both banks and firms can default. The model incorporates a rich set of fiscal policy rules, including deposit insurance, and endogenizes the risk-free interest rate.

The model features a double financial accelerator. Like in the standard accelerator model, shocks to the macro-economy affect entrepreneurial net worth. Since firm borrowing is constrained by net worth, macro-economic shocks are amplified by tighter borrowing constraints.

Unlike the original models, ours features impatient but risk averse and infinitely-lived entrepreneurs. The second financial accelerator arises from explicitly modeling the financial intermediaries' balance sheet as separate from that of the entrepreneur-borrowers and saving households. Intermediaries are subject to regulatory capital constraints. Macro-economic shocks that lead to binding intermediary borrowing constraints amplify the shocks through their direct effect on intermediaries' net worth and the indirect effect on borrowers to whom the intermediaries lend. However, when intermediaries are well enough capitalized to absorb the fundamental shock without constraining the firms, they can dampen the first accelerator mechanism.

We provide a global nonlinear solution and a realistic calibration to the U.S. economy. We explore the dynamics of quantities and prices in this setting and compare them to U.S. data, with a focus on understanding differences between financial and non-financial recessions. Our main application studies macro-prudential policy and contrasts restrictions on firm leverage to those on bank leverage. While such policies reduce the credit risk and promote macro-economic stability and better risk sharing among the agents, they shrink the size of the economy and are ultimately welfare-reducing.

Extensions to this model could introduce New Keyenesian elements such as nominal rigidities, monopolistic competition, and monetary policy. Our setting is an interesting one to evaluate the effect of the zero lower bound (ZLB) on nominal short rates. A binding ZLB during a financial crisis would keep the real rate elevated. The ZLB economy would prevent the intermediaries from recapitalizing in a crisis through a negative real deposit rate. Negative real rates mitigate the severity of financial recessions in the current model. The upshot is that a binding ZLB may lead to a severe crisis exactly because it prevents a recapitalization of financial intermediaries.

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A Model Appendix

We reformulate the problem to ensure stationarity of the problem. We do so by scaling all variables by productivity.

A.1 Borrower-entrepreneur problem

A.1.1 Preliminaries

We start by defining some preliminaries.

$$\begin{aligned} Z_A(\omega_t^*) &= [1 - F_\omega(\omega_t^*; \chi)] \\ Z_K(\omega_t^*) &= [1 - F_\omega(\omega_t^*; \chi)]\mathrm{E}\left[w_{i,t} \mid \omega_{i,t} \geq \omega_t^*; \chi\right] \end{aligned}$$

and $F_\omega(\cdot; \chi)$ is the CDF of $\omega_{i,t}$ with parameters χ. Assume $\omega_{i,t}$ are drawn from a Gamma distribution with shape and scale parameters $\chi = (\chi_0, \chi_1)$ such that

$$\begin{aligned} \mu_\omega &= \mathrm{E}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1 \\ \sigma^2_{t,\omega} &= \mathrm{Var}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1^2 \end{aligned}$$

From Landsman and Valdez (2004, equation 22), we know that

$$\mathrm{E}[\omega | \omega \geq \bar{\omega}] = \mu_\omega \frac{1 - F_\omega(\bar{\omega}; \chi_0 + 1, \chi_1)}{1 - F_\omega(\bar{\omega}; \chi_0, \chi_1)}$$

so the closed form expression for Z_K is

$$Z_K(\omega_t^*) = \mu_\omega \left[1 - F_\omega(\omega_t^*; \chi_0 + 1, \chi_1)\right]$$

It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$\begin{aligned} \frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_\omega(\omega) d\omega = -\omega_t^* f_\omega(\omega_t^*), \\ \frac{\partial Z_A(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_\omega(\omega) d\omega = -f_\omega(\omega_t^*), \end{aligned}$$

where $f_\omega(\cdot)$ is the p.d.f. of a Gamma distribution with parameters (χ_0, χ_1).

Capital Adjustment Cost Let

$$\Psi(X_t, K_t^B) = \frac{\psi}{2} \left(\frac{X_t}{K_t^B} - (e^{\bar{g}} - \mu_\omega) \right)^2 K_t^B.$$

Then partial derivatives are

$$\Psi_X(X_t, K_t^B) = \psi \left(\frac{X_t}{K_t^B} - (e^{\bar{g}} - \mu_\omega) \right) \tag{15}$$

$$\Psi_K(X_t, K_t^B) = -\frac{\psi}{2} \left(\left(\frac{X_t}{K_t^B} \right)^2 - (e^{\bar{g}} - \mu_\omega)^2 \right) \tag{16}$$

A.1.2 Statement of stationary problem

We consider the borrower's problem in the current period after productivity and depreciation shocks have been realized, after the intermediary has chosen a default policy, and after the intermediary's random utility penalty is realized. To ensure stationarity of the borrower-entrepreneur's problem we define the following transformed variables,

$$\left\{ \hat{C}_t^B, \hat{X}_t, \hat{A}_t^B, \hat{K}_t^B, \hat{w}_t^j, \hat{G}_t^{T,B} \right\},$$

where for any variable $v\hat{a}r_t$ denotes division by the current realization of productivity Z_t:

$$v\hat{a}r_t = \frac{var_t}{Z_t}.$$

It follows that transformed output is $\hat{Y}_t = (\hat{K}_t^B)^{1-\alpha} L_t^\alpha$. For the choices of capital and debt for the next period we further define

$$\hat{\hat{K}}_{t+1}^B = \frac{K_{t+1}^B}{Z_t}$$

and

$$\hat{\hat{A}}_{t+1}^B = \frac{A_{t+1}^B}{Z_t}.$$

Let $\mathcal{S}_t^B = \left(g_t, \sigma_{\omega,t}, \hat{W}_t^I, \hat{W}_t^S, \hat{B}_{t-1}^G \right)$ represent state variables exogenous to the borrower-entrepreneur's decision. Then the borrower's value function, transformed to ensure stationarity, is:

$$\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) = \max_{\{\hat{C}_t^B, \hat{\hat{K}}_{t+1}^B, \omega_t^*, \hat{X}_t, \hat{\hat{A}}_{t+1}^B, L_t^j\}} \left\{ (1-\beta_B) \left(\hat{C}_t^B \right)^{1-1/\nu} + \right.$$
$$\left. + \beta_B \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^B(e^{-g_{t+1}} \hat{\hat{K}}_{t+1}^B, e^{-g_{t+1}} \hat{\hat{A}}_{t+1}^B, \mathcal{S}_{t+1}^B) \right)^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$\begin{aligned}\hat{C}_t^B = {} & (1-\tau_\Pi^I)\hat{Y}_t^B + (1-\tau_t^B)\hat{w}_t^B \bar{L}^B + \hat{G}_t^{T,B} + p_t[\hat{X}_t + (Z_K(\omega_t^*) + \tau_\Pi^I \delta_K Z_A(\omega_t^*))\hat{K}_t^B] \\ & + q_t^m \hat{\hat{A}}_{t+1}^B - Z_A(\omega_t^*)\hat{A}_t^B(1-(1-\theta)\tau_\Pi^I + \delta q_t^m) \\ & - p_t \hat{\hat{K}}_{t+1}^B - \hat{X}_t - \Psi(\hat{X}_t, \hat{K}_t^B) - (1-\tau_\Pi^I)\sum_{j=B,I,S} \hat{w}_t^j L_t^j \end{aligned} \tag{17}$$

$$\phi p_t Z_K(\omega_t^*)\hat{K}_t^B \geq F\hat{\hat{A}}_{t+1}^B \tag{18}$$

$$\mathcal{S}_{t+1}^B = h(\mathcal{S}_t^B) \tag{19}$$

where the functions Z_K and Z_A are defined in the preliminaries above.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that that default decision takes the form of a cutoff rule:

$$\begin{aligned}\tilde{\tilde{V}}^B\left(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B\right) = {} & F_\rho(\rho_t^*)\mathrm{E}_\rho\left[\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) \mid \rho < \rho_t^*\right] + (1-F_\rho(\rho_t^*))\,\mathrm{E}_\rho\left[\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B) \mid \rho > \rho_t^*\right] \\ = {} & F_\rho(\rho_t^*)\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^S(\rho_t < \rho_t^*)) + (1-F_\rho(\rho_t^*))\,\hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^S(\rho_t > \rho_t^*)),\end{aligned} \tag{20}$$

where (20) obtains because the expectation terms conditional on realizations of ρ_t and ρ_t^* only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned}\hat{V}_t^B &\equiv \hat{V}(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B), \\ \hat{V}_{A,t}^B &\equiv \frac{\partial \hat{V}(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B)}{\partial \hat{A}_t^B}, \\ \hat{V}_{K,t}^B &\equiv \frac{\partial \hat{V}(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B)}{\partial \hat{K}_t^B}.\end{aligned}$$

Therefore the marginal values of borrowing and of capital of $\tilde{V}^B(\cdot)$ are:

$$\begin{aligned}\tilde{\hat{V}}_{A,t}^B &= F_\rho(\rho_t^*)\frac{\partial \hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial \hat{A}_t^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial \hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*))}{\partial \hat{A}_t^B} \\ \tilde{\hat{V}}_{K,t}^B &= F_\rho(\rho_t^*)\frac{\partial \hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho_t < \rho_t^*))}{\partial \hat{K}_t^B} + (1-F_\rho(\rho_t^*))\,\frac{\partial \hat{V}^B(\hat{K}_t^B, \hat{A}_t^B, \mathcal{S}_t^B(\rho_t > \rho_t^*))}{\partial \hat{K}_t^B}\end{aligned}$$

Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^B(\hat{K}_{t+1}^B, \hat{A}_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}.$$

A.1.3 First-order conditions

Loans The FOC for loans $\hat{\hat{A}}^B_{t+1}$ is:

$$
\begin{aligned}
0 = \frac{1}{1-1/\nu} \Bigg\{ & (1-\beta_B) \left[\hat{C}^B_t\right]^{1-1/\nu} + \\
& + \beta_B \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^B(\hat{K}^B_{t+1}, \hat{A}^B_{t+1}, \mathcal{S}^B_{t+1}) \right)^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}} \Bigg\}^{\frac{1}{1-1/\nu}-1} \times \\
\times \Bigg\{ & (1-1/\nu)(1-\beta_B) \left[\hat{C}^B_t\right]^{-1/\nu} q^m_t + \\
& + \beta_B \frac{1-1/\nu}{1-\sigma_B} \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^B(\hat{K}^B_{t+1}, \hat{A}^B_{t+1}, \mathcal{S}^B_{t+1}) \right)^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}-1} \times \\
& \times E_t \left[(1-\sigma_B) \left(e^{g_{t+1}} \tilde{\hat{V}}^B(\hat{K}^B_{t+1}, \hat{A}^B_{t+1}, \mathcal{S}^B_{t+1}) \right)^{-\sigma_B} e^{g_{t+1}} \tilde{\hat{V}}^B_{A,t+1} e^{-g_{t+1}} \right] \Bigg\} - \lambda^B_t F
\end{aligned}
$$

where λ^B_t is the Lagrange multiplier on the borrowing constraint.

Simplifying, we get:

$$
\begin{aligned}
& q^m_t (\hat{C}^B_t)^{-1/\nu} (1-\beta_B)(\hat{V}^B_t)^{1/\nu} = \\
& \qquad \lambda^B_t F - \beta_B \mathrm{E}_t[(e^{g_{t+1}} \tilde{\hat{V}}^B_{t+1})^{-\sigma_B} \tilde{\hat{V}}^B_{A,t+1}](CE^B_t)^{\sigma_B - 1/\nu} (V^B_t)^{1/\nu}
\end{aligned} \tag{21}
$$

Observe that we can rewrite equation (21) as:

$$
q^m_t = \frac{(\hat{C}^B_t)^{1/\nu}}{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}} \left\{ \lambda^B_t F - \beta_B \mathrm{E}_t[(e^{g_{t+1}} \tilde{\hat{V}}^B_{t+1})^{-\sigma_B} \tilde{\hat{V}}^B_{A,t+1}](CE^B_t)^{\sigma_B - 1/\nu} (\hat{V}^B_t)^{1/\nu} \right\}.
$$

We define the rescaled Lagrange multiplier of the borrower as the original multiplier divided by marginal utility of current consumption:

$$
\tilde{\lambda}^B_t = \lambda^B_t \frac{(\hat{C}^B_t)^{1/\nu}}{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}}.
$$

Then we can solve for the mortgage price as:

$$
q^m_t = \tilde{\lambda}^B_t F - \beta_B \frac{(\hat{C}^B_t)^{1/\nu}}{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}} \mathrm{E}_t[(e^{g_{t+1}} \tilde{\hat{V}}^B_{t+1})^{-\sigma_B} \tilde{\hat{V}}^B_{A,t+1}](CE^B_t)^{\sigma_B - 1/\nu} (\hat{V}^B_t)^{1/\nu}. \tag{22}
$$

Capital The FOC for new purchases of capital $\hat{\hat{K}}^B_{t+1}$ is:

$$0 = \frac{1}{1-1/\nu}(\hat{V}^B_t)^{1/\nu} \times \Big\{ -(1-1/\nu)(1-\beta_B)(\hat{C}^B_t)^{-1/\nu} p_t + \\ + \frac{1-1/\nu}{1-\sigma_B}\beta_B (CE^B_t)^{\sigma_B - 1/\nu} \mathrm{E}_t[(1-\sigma_B)(e^{g_{t+1}}\tilde{\hat{V}}^B_{t+1})^{-\sigma_B} e^{g_{t+1}} \tilde{\hat{V}}^B_{K,t+1} e^{-g_{t+1}}] \Big\}.$$

Simplifying, we get:

$$p_t \frac{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}}{(\hat{C}^B_t)^{1/\nu}} = \\ \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}^B_{t+1})^{-\sigma_B}\tilde{\hat{V}}^B_{K,t+1}](CE^B_t)^{\sigma_B-1/\nu}(\hat{V}^B_t)^{1/\nu} \tag{23}$$

Default Threshold Taking the first-order condition with respect to $\omega^\star_t$ and using the expressions for the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$ in the preliminaries above yields:

$$f_\omega(\omega^*_t)\left[\omega^*_t p_t \hat{K}^B_t (1+\phi\tilde{\lambda}^B_t) - (1-(1-\theta)\tau^I_\Pi + \delta q^m_t)\hat{A}^B_t\right] \frac{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}}{(\hat{C}^B_t)^{1/\nu}} = 0. \tag{24}$$

This can be simplified to give:

$$\omega^*_t = \frac{\hat{A}^B_t(1-(1-\theta)\tau^I_\Pi + \delta q^m_t)}{p_t \hat{K}^B_t(1+\phi\tilde{\lambda}^B_t)}. \tag{25}$$

Investment The FOC for investment $\hat{X}_t$ is:

$$[1+\Psi_X(\hat{X}^B_t, \hat{K}^B_t) - p_t]\frac{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}}{(\hat{C}^B_t)^{1/\nu}} = 0,$$

which simplifies to

$$1+\Psi_X(\hat{X}^B_t, \hat{K}^B_t) = p_t.$$

Labor Inputs Defining $\gamma_B = 1-\gamma_I-\gamma_S$, aggregate labor input is

$$L_t = \prod_{j=B,I,S}(L^j_t)^{\gamma_j}.$$

The FOC for labor input L^j_t is then

$$(1-\tau^\Pi)\left[\alpha\gamma_j \frac{L_t}{L^j_t}\left(\frac{\hat{K}^B_t}{L_t}\right)^{1-\alpha} - \hat{w}^j_t\right]\frac{(1-\beta_B)(\hat{V}^B_t)^{1/\nu}}{(\hat{C}^B_t)^{1/\nu}} = 0,$$

which yields

$$\alpha\gamma_j \frac{L_t}{L_t^j}\left(\frac{\hat{K}_t^B}{L_t}\right)^{1-\alpha} = \hat{w}_t^j. \tag{26}$$

A.1.4 Marginal Values of State Variables and SDF

Loans Taking the derivative of the value function with respect to $\hat{A}_t^B$ gives:

$$\hat{V}_{A,t}^B = -\left(1-(1-\theta)\tau_\Pi^I + \delta q_t^m\right) Z_A(\omega_t^*)\frac{(1-\beta_B)(\hat{V}_t^B)^{1/\nu}}{(\hat{C}_t^B)^{1/\nu}}. \tag{27}$$

Capital Taking the derivative of the value function with respect to $\hat{K}_t^B$ gives:

$$\begin{aligned}\hat{V}_{K,t}^B = &\left[p_t\left(Z_K(\omega_t^*)(1+\phi\tilde{\lambda}_t^B) + \tau_\Pi^I \delta_K Z_A(\omega_t^*)\right) + (1-\alpha)(1-\tau_I^\Pi)\left(\frac{\hat{K}_t^B}{L_t}\right)^{-\alpha} - \Psi_K(\hat{X}_t^B, \hat{K}_t^B)\right] \\ &\times \frac{(1-\beta_B)(\hat{V}_t^B)^{1/\nu}}{(\hat{C}_t^B)^{1/\nu}}. \end{aligned}\tag{28}$$

SDF Define the borrower-entrepreneur's intertemporal marginal rate of substitution between t and $t+1$, conditional on a particular realization of ρ_{t+1} as:

$$\begin{aligned}\mathcal{M}_{t,t+1}^B(\rho_{t+1}) &= \frac{\partial V_t^B/\partial C_{t+1}^B}{\partial \hat{V}_t^B/\partial \hat{C}_t^B} = \frac{\partial \hat{V}_t^B}{\partial \hat{V}_{t+1}^B(\rho_{t+1})} e^{-g_{t+1}} \frac{\partial \hat{V}_{t+1}^B/\partial \hat{C}_{t+1}^B}{\partial \hat{V}_t^B/\partial \hat{C}_t^B} \\ &= (\hat{V}_t^B)^{1/\nu}\beta_B (CE_t^B)^{\sigma_B - 1/\nu}(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^B(\rho_{t+1}))^{-\sigma_B}\frac{(\hat{C}_{t+1}^B)^{-1/\nu}(1-\beta_B)(\hat{V}_{t+1}^B(\rho_{t+1}))^{1/\nu}}{(\hat{C}_t^B)^{-1/\nu}(1-\beta_B)(\hat{V}_t^B)^{1/\nu}} \\ &= \beta_B e^{-\sigma_B g_{t+1}}\left(\frac{\hat{C}_{t+1}^B}{\hat{C}_t^B}\right)^{-1/\nu}\left(\frac{\hat{V}_{t+1}^B(\rho_{t+1})}{CE_t^B}\right)^{-(\sigma_B - 1/\nu)}\end{aligned}$$

We can then define the stochastic discount factor (SDF) of borrowers as:

$$\tilde{\mathcal{M}}_{t,t+1}^B = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*),$$

where $\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*)$ and $\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*)$ are the IMRSs, conditional on the two possible realizations of state variables.

A.1.5 Euler Equations

Loans Recall that $\tilde{\hat{V}}_{A,t+1}^B$ is a linear combination of $\hat{V}_{A,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $\hat{V}_{A,t+1}^B$ given by equation (27). Substituting in for $\tilde{\hat{V}}_{A,t+1}^B$ in

(21) and using the SDF expression, we get the recursion:

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*) \left(1 - (1-\theta)\tau_\Pi^I + \delta q_{t+1}^m\right)\right]. \tag{29}$$

Capital Likewise, observe that we can write (23) as:

$$p_t = \frac{\beta_B \mathrm{E}_t[e^{g_{t+1}} (\tilde{\hat{V}}_{t+1}^B)^{-\sigma_B} \tilde{\hat{V}}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu} (\hat{V}_t^B)^{1/\nu}}{(\hat{C}_t^B)^{-1/\nu}(1-\beta_B)(\hat{V}_t^B)^{1/\nu}}$$

Recall that $\tilde{\hat{V}}_{K,t+1}^B$ is a linear combination of $\hat{V}_{K,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $\hat{V}_{K,t+1}^B$ given by equation (28). Substituting in for $\tilde{\hat{V}}_{K,t+1}^B$ and using the SDF expression, we get the recursion:

$$\begin{aligned} p_t = \mathrm{E}_t \Bigg[\tilde{\mathcal{M}}_{t,t+1}^B \Bigg\{ & p_{t+1} \left(Z_K(\omega_{t+1}^*)(1 + \phi \tilde{\lambda}_{t+1}^B) + \tau_\Pi^I \delta_K Z_A(\omega_{t+1}^*) \right) \\ & + (1-\alpha)(1-\tau^\Pi) \left(\frac{\hat{K}_{t+1}^B}{L_{t+1}} \right)^{-\alpha} - \Psi_K(\hat{X}_{t+1}, \hat{K}_{t+1}^B) \Bigg\} \Bigg]. \end{aligned} \tag{30}$$

A.2 Intermediaries

A.2.1 Statement of stationary problem

As for borrower-entrepreneurs, we define the following transformed variables for intermediaries:

$$\{\hat{W}_t^I, \hat{C}_t^I, \hat{A}_{t+1}^I, \hat{G}_t^{T,I}, \hat{B}_t^I\}.$$

Denote by $\hat{W}_t^I$ risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{\hat{W}}_t^I = (1 - D(\rho_t))\hat{W}_t^I,$$

and the effective utility penalty is:

$$\tilde{\rho}_t = D(\rho_t)\rho_t.$$

Let $\mathcal{S}_t^I = \left(g_t, \sigma_{\omega,t}, \hat{K}_t^B, \hat{A}_t^B, \hat{B}_{t-1}^G, \hat{W}_t^S\right)$ denote all other aggregate state variables exogenous to intermediaries.

After the default decision, intermediaries face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\hat{V}^I(\tilde{\hat{W}}_t^I, \tilde{\rho}_t, \mathcal{S}_t^I) = \max_{\hat{C}_t^I, \hat{A}_{t+1}^I, \hat{B}_t^I} \left\{ (1-\beta_I) \left[\frac{\hat{C}_t^I}{e^{\tilde{\rho}_t}} \right]^{1-1/\nu} + \beta_I \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^I \left(\hat{W}_{t+1}^I, \mathcal{S}_{t+1}^I \right) \right)^{1-\sigma_R} \right]^{\frac{1-1/\nu}{1-\sigma_R}} \right\}^{\frac{1}{1-1/\nu}} \tag{31}$$

subject to:

$$(1-\tau^I)\hat{w}_t^I\bar{L}^I + \tilde{\hat{W}}_t^I + \hat{G}_t^{T,I} = \hat{C}_t^I + q_t^m \hat{A}_{t+1}^I + (q_t^f + \tau_I^\Pi r_t^f - \kappa I_{\{\hat{B}_t^I<0\}})\hat{B}_t^I, \tag{32}$$

$$\hat{W}_{t+1}^I = e^{-g_{t+1}}\left[\left(\tilde{M}_{t+1} + Z_A(\omega_{t+1}^*)\delta q_{t+1}^m\right)\hat{A}_{t+1}^I + \hat{B}_t^I\right], \tag{33}$$

$$\hat{B}_t^I \geq -\xi q_t^m \hat{A}_{t+1}^I, \tag{34}$$

$$\hat{A}_{t+1}^R \geq 0, \tag{35}$$

$$\mathcal{S}_{t+1}^I = h(\mathcal{S}_t^I). \tag{36}$$

For the statement of the problem above, we have defined the after-tax payoff per bond as

$$\tilde{M}_t = Z_A(\omega_t^*)(1-(1-\theta)\tau_\Pi^I) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_t^B}{A_t^B}.$$

The continuation value $\tilde{\hat{V}}^I\left(\hat{W}_{t+1}^I, \mathcal{S}_{t+1}^I\right)$ is the outcome of the optimization problem intermediaries face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^I) = \max_{D(\rho)} \mathrm{E}_\rho\left[D(\rho)\hat{V}^I(0,\rho,\mathcal{S}_t^I) + (1-D(\rho))\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)\right] \tag{37}$$

Define the certainty equivalent of future utility as:

$$CE_t^I = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^I\left(\hat{W}_{t+1}^I, \mathcal{S}_{t+1}^I\right)\right)^{1-\sigma_I}\right]^{\frac{1}{1-\sigma_I}}. \tag{38}$$

A.2.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0,1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $\hat{V}^I(W, \rho, \mathcal{S}_t^R)$ defined in (31) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given $\hat{W}_t^I$, intermediaries optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (37) as

$$\begin{aligned}\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho\left[\mathbb{1}[\rho<\rho^*]\ \hat{V}^I(0,\rho,\mathcal{S}_t^I) + (1-\mathbb{1}[\rho<\rho^*])\,\hat{V}^I(\hat{W}_t^I,0,\mathcal{S}_t^I)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\ \mathrm{E}_\rho\left[\hat{V}^I(0,\rho,\mathcal{S}_t^I)\,|\,\rho<\rho^*\right] + (1-F_\rho(\rho^*))\,\hat{V}^I(\hat{W}_t^I,0,\mathcal{S}_t^I).\end{aligned}$$

The solution ρ_t^* is characterized by the first-order condition:

$$\hat{V}^I(0,\rho_t^*,\mathcal{S}_t^I) = \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $\hat{V}^I(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = \hat{V}^I(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)), \tag{39}$$

and by substituting the solution into (37), we obtain

$$\tilde{\hat{V}}^I(\hat{W}_t^I, \mathcal{S}_t^I) = F_\rho(\rho_t^*)\mathrm{E}_\rho\left[\hat{V}^I(0, \rho, \mathcal{S}_t^I) \mid \rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I). \tag{40}$$

Equations (31), (39), and (40) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (39) is given by:

$$\mathcal{F}(x) = \begin{cases} \log((1-\beta_I)\hat{C}_t^I) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_I(CE_t^I)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1-\beta_I)\log(\hat{C}_t^I) + \beta_I\log(CE_t^I) - \log(x) - (1-\beta_I) & \text{if } \nu = 1. \end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned} &(q_t^f + \tau^\Pi r_t^f - \kappa I_{\{\hat{B}_t^I < 0\}})\frac{(1-\beta_I)(\hat{V}_t^I)^{1/\nu}}{(\hat{C}_t^I)^{1/\nu}} = \\ &\lambda_t^I + \beta_I \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^I)^{-\sigma_I}\tilde{\hat{V}}_{W,t+1}^I](CE_t^I)^{\sigma_I - 1/\nu}(\hat{V}_t^I)^{1/\nu} \end{aligned} \tag{41}$$

where λ_t^I is the Lagrange multiplier on the borrowing constraint (34).

The first order condition for loans is:

$$\begin{aligned} &(q_t^m + \tau^\Pi r_t^m F)\frac{(1-\beta_I)(\hat{V}_t^I)^{1/\nu}}{(\hat{C}_t^I)^{1/\nu}} = \lambda_t^I \xi q_t^m + \mu_t^I \\ &+ \beta_I \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^I)^{-\sigma_I}\tilde{\hat{V}}_{W,t+1}^I\left(M_{t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m\right)](CE_t^I)^{\sigma_I - 1/\nu}(\hat{V}_t^I)^{1/\nu}, \end{aligned} \tag{42}$$

where μ_t^I is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (35).

A.2.3 Marginal value of wealth and SDF

Differentiating (40) gives the marginal value of wealth

$$\tilde{\hat{V}}_{W,t}^I = (1 - F_\rho(\rho_t^*))\,\frac{\partial \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)}{\partial \hat{W}_t^I},$$

where

$$\frac{\partial \hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I)}{\partial \hat{W}_t^I} = (\hat{C}_t^I)^{-1/\nu}(1-\beta_I)(\hat{V}^I(\hat{W}_t^I, 0, \mathcal{S}_t^I))^{1/\nu},$$

The stochastic discount factor of intermediaries is therefore

$$\mathcal{M}^I_{t,t+1} = \beta_I e^{-\sigma_I g_{t+1}} \left(\frac{\hat{V}^I(\hat{W}^I_{t+1}, 0, \mathcal{S}^I_{t+1})}{CE^I_t} \right)^{-(\sigma_I - 1/\nu)} \left(\frac{\hat{C}^I_{t+1}}{\hat{C}^I_t} \right)^{-1/\nu},$$

and

$$\tilde{\mathcal{M}}^I_{t,t+1} = (1 - F_\rho(\rho^*_{t+1}))\mathcal{M}^I_{t,t+1}$$

A.2.4 Euler Equations

It is then possible to show that the FOC with respect to $\hat{B}^I_t$ and $\hat{A}^I_{t+1}$, respectively, are:

$$q^f_t = \tilde{\lambda}^I_t + \mathrm{E}_t \left[\tilde{\mathcal{M}}^I_{t,t+1} \right] + \kappa I_{\{\hat{B}^I_t < 0\}} - \tau^\Pi r^f_t, \tag{43}$$

$$q^m_t (1 - \xi \tilde{\lambda}^I_t) = \tilde{\mu}^I_t + \mathrm{E}_t \left[\tilde{\mathcal{M}}^I_{t,t+1} \left(\tilde{M}_{t+1} + \delta Z_A(\omega^*_{t+1}) q^m_{t+1} \right) \right]. \tag{44}$$

A.3 Savers

A.3.1 Statement of stationary problem

For savers, we define the following transformed variables:

$$\{\hat{W}^S_t, \hat{C}^S_t, \hat{B}^S_t, \hat{G}^{T,S}_t\}.$$

Let $\mathcal{S}^S_t = \left(g_t, \sigma_{\omega,t}, \hat{K}^B_t, \hat{A}^B_t, \hat{W}^I_t, \hat{B}^G_{t-1}\right)$ be the saver's state vector capturing all exogenous state variables. Scaling by productivity, the stationary problem of the saver – after the intermediary has made default her decision and the utility cost of default is realized – is:

$$\hat{V}^S(\hat{W}^S_t, \mathcal{S}^S_t) = \max_{\{\hat{C}^S_t, \hat{B}^S_t\}} \left\{ (1 - \beta_S) \left[\hat{C}^S_t \right]^{1-1/\nu} + \beta_S \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{\hat{V}}^S(\hat{W}^S_{t+1}, \mathcal{S}^S_{t+1}) \right)^{1-\sigma_S} \right]^{\frac{1-1/\nu}{1-\sigma_S}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$\hat{C}^S_t = (1 - \tau^S_t)\hat{w}^S_t \bar{L}^S + \hat{G}^{T,S}_t + \hat{W}^S_t - q^f_t \hat{B}^S_t \tag{45}$$

$$\hat{W}^S_{t+1} = e^{-g_{t+1}} \hat{B}^S_t \tag{46}$$

$$\hat{B}^S_t \geq 0 \tag{47}$$

$$\mathcal{S}^S_{t+1} = h(\mathcal{S}^S_t) \tag{48}$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$\hat{V}_t^S \equiv \hat{V}_t^S(\hat{W}_t^S, \mathcal{S}_t^S),$$
$$\hat{V}_{W,t}^S \equiv \frac{\partial \hat{V}_t^S(\hat{W}_t^S, \mathcal{S}_t^S)}{\partial \hat{W}_t^S},$$

Denote the certainty equivalent of future utility as:

$$CE_t^S = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{\hat{V}}^S(\hat{W}_t^S, \mathcal{S}_t^S)\right)^{1-\sigma_S}\right].$$

Like borrower-entrepreneurs, savers must take into account the intermediary's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$\tilde{\hat{V}}_{W,t}^S = F_\rho(\rho_t^*)\frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t < \rho_t^*))}{\partial \hat{W}_t^S} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t > \rho_t^*)}{\partial \hat{W}_t^S}.$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f(\hat{C}_t^S)^{-1/\nu}(1-\beta_S)(\hat{V}_t^S)^{1/\nu} = \lambda_t^S + \beta_S \mathrm{E}_t[(e^{g_{t+1}}\tilde{\hat{V}}_{t+1}^S)^{-\sigma_S}\tilde{\hat{V}}_{W,t+1}^S](CE_t^S)^{\sigma_S - 1/\nu}(\hat{V}_t^S)^{1/\nu} \tag{49}$$

where λ_t^S is the Lagrange multiplier on the no-borrowing constraint (47).

A.3.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$\hat{V}_{W,t}^S = (\hat{C}_t^S)^{-1/\nu}(1-\beta_S)(\hat{V}_t^S)^{1/\nu}, \tag{50}$$

and for the continuation value function:

$$\tilde{\hat{V}}_{W,t}^S = F_\rho(\rho_t^*)\frac{\partial V^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t < \rho_t^*))}{\partial \hat{W}_t^S} + (1 - F_\rho(\rho_t^*))\,\frac{\partial \hat{V}^S(\hat{W}_t^S, \mathcal{S}_t^S(\rho_t > \rho_t^*)}{\partial \hat{W}_t^S}.$$

Defining the SDF in the same fashion as we did for the borrower, we get:

$$\mathcal{M}_{t,t+1}^S(\rho_t) = \beta_S e^{-\sigma_S g_{t+1}}\left(\frac{\hat{V}_{t+1}^S}{CE_t^S}\right)^{-(\sigma_S - 1/\nu)}\left(\frac{\hat{C}_{t+1}^S}{\hat{C}_t^S}\right)^{-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^S = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^S(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^S(\rho_{t+1} > \rho_{t+1}^*).$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (49) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^S + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^S\right] \tag{51}$$

where $\tilde{\lambda}_t^S$ is the original multiplier λ_t^S divided by the marginal value of wealth.

A.4 Steady State

This subsection outlines the calculation of the steady state of the deterministic version of the model (without aggregate shocks). The calculations directly operate on the stationary variables and omit the "hats".

Production Function The stationary version of the model has the production function

$$Y = K^{1-\alpha} L^{\alpha},$$

where $L = (L^B)^{\gamma_B}\,(L^I)^{\gamma_I}\,(L^S)^{\gamma_S}$.

From the first-order conditions for labor demand (26) we get for steady state wages

$$w^j = \alpha\gamma_j \frac{L}{L^j}\left(\frac{K^B}{L}\right)^{1-\alpha}.$$

By the usual derivations for CRS production functions this implies

$$w^j L^j = \alpha\gamma_j Y,$$

and

$$Y - \sum_j w^j L^j = (1-\alpha)Y.$$

Borrower Leverage Assuming a binding leverage constraint for borrower-entrepreneurs, the steady state of the model can be reduced to two nonlinear equations in the bond price q^m and the default threshold ω^*. The two equations that constitute the system for characterizing equilibrium are marked by (S1) and (S2) below.

If the constraint is binding we get

$$\phi p Z_K(\omega^*) K^B = F A^B,$$

which we can substitute into the first-order condition for the default threshold (25) to yield

$$\omega^* = \frac{\phi Z_K(\omega^*)}{e^g(1+\phi\lambda^B)} \frac{1-(1-\theta)\tau_\Pi^B + \delta q^m}{F}. \tag{S1}$$

The equation above is a nonlinear equation in ω^*.

Capital Price and Investment We can use the first-order condition for investment X and the market clearing condition for capital to express investment and the capital price p as functions of parameters and ω_*. First, rewriting the FOC for investment, we get

$$X = \left[(e^g - \mu_\omega) + \frac{p-1}{\psi}\right] K^B.$$

Substituting this into the capital market clearing condition and eliminating K^B from both sides gives

$$1 = e^{-g}\left[Z_K + (1-\zeta)(\mu_\omega - Z_K) + e^g - \mu_\omega - \frac{1-p}{\psi}\right],$$

which can be solved for the price

$$p = 1 + \psi\zeta(\mu_\omega - Z_K).$$

Reinserting this into the expression for X gives

$$X = \left[e^g + \zeta(\mu_\omega - Z_K) - \mu_\omega\right] K^B.$$

Household SDFs The (stochastic) discount factors of households reduce to

$$\tilde{\beta}_j = \beta_j e^{-g/\nu}.$$

Capital Stock and Borrower Consumption We can solve the borrower-entrepreneurs first-order condition for next period's debt (29) to get an expression for the Lagrange multiplier on the leverage constraint

$$\lambda^B = 1 - \tilde{\beta}_B Z_A \left(\frac{1-(1-\theta)\tau_\Pi^B}{q^m} + \delta\right).$$

We can further rewrite Ψ_K as

$$\Psi_K = -(p-1)\left(e^g - \mu_\omega + \frac{p-1}{2\psi}\right)$$

using that $X/K^B = e^g - \mu_\omega + (p-1)/\psi$.

Plugging this into the FOC for capital (30) gives

$$p = \tilde{\beta}_B\left[p\left(Z_K(1+\phi\lambda^B) + \tau_\Pi^B Z_A \delta_K\right) + (1-\alpha)(1-\tau_\Pi^B)\left(\frac{K^B}{L}\right)^{-\alpha} + (p-1)\left(e^g - \mu_\omega + \frac{p-1}{2\psi}\right)\right].$$

This equation can be solved for K^B as a function of labor input L

$$K^B = \left(\frac{(1-\alpha)(1-\tau^{\Pi})\tilde{\beta}_B}{p\left[1 - \tilde{\beta}_B\left(Z_K(1+\phi\lambda^B) + \tau_{\Pi}^B Z_A \delta_K\right)\right] - \tilde{\beta}_B(p-1)\left(e^g - \mu_\omega + \frac{p-1}{2\psi}\right)} \right)^{1/\alpha} L.$$

Since L only depends on parameters, we can therefore now also compute output

$$Y = \left(\frac{(1-\alpha)(1-\tau^{\Pi})\tilde{\beta}_B}{p\left[1 - \tilde{\beta}_B\left(Z_K(1+\phi\lambda^B) + \tau_{\Pi}^B Z_A \delta_K\right)\right] - \tilde{\beta}_B(p-1)\left(e^g - \mu_\omega + \frac{p-1}{2\psi}\right)} \right)^{(1-\alpha)/\alpha} L.$$

We can then use the solution for investment and the binding borrowing constraint to compute borrower consumption from the budget constraint.

Savers and Riskfree Bond Price Savers' FOC for holdings of risk free debt determines the risk free bond price

$$q = \tilde{\beta}_S.$$

Intermediary The intermediary's FOC for risk free debt implies

$$\lambda^I = 1 - \frac{\tilde{\beta}_I - \tau_{\Pi}^I r^f}{q}.$$

With $\tilde{\beta}_I = \tilde{\beta}_S$, and using $r^f = 1/q - 1 = 1/\tilde{\beta}_S - 1$, we get

$$\lambda^I = \frac{\tau_{\Pi}^I}{\tilde{\beta}_I}\left(\frac{1}{\tilde{\beta}_I} - 1\right).$$

By market clearing in the market for loans to borrower-entrepreneurs we have $A^I = A^B = \phi p K^B / q^m$, and then using the intermediary's binding leverage constraint

$$qB^I = -\xi q^m A^I = -\xi \phi p K^B.$$

Intermediary wealth is therefore

$$W^I = e^{-g} A^I \left[\tilde{M} + q^m\left(\delta Z_A - \frac{\xi}{q}\right)\right],$$

and intermediary consumption is

$$C^I = \alpha\gamma_I(1-\tau^I)Y + G^I + W^I - A^I\left[q^m\left(1 - \xi(q-\kappa)\right) - \tau_{\Pi}^I r^f \xi q^m / q\right].$$

The bond price q^m is determined by the intermediary's FOC for loans to borrowers

$$q^m = \frac{\tilde{\beta}_I \tilde{M}}{1 - \tilde{\beta}_I \delta Z_A - \xi\lambda^I}, \tag{S2}$$

with $\tilde{M}$ given by

$$\tilde{M} = Z_A(1 - (1-\theta)\tau_\Pi^I) + \frac{(1-\zeta)(\mu_\omega - Z_K)pK^B}{A^B}.$$

Government Government revenues are

$$T = \left[\alpha\left(\sum_j \gamma_j \tau^j\right) + (1-\alpha)\tau_\Pi^B\right] Y - A^B\left[(1-\theta)Z_A(\tau_\Pi^B - \tau_\Pi^I) + \xi\frac{q^m}{q}(\tau_\Pi^I - \kappa)\right] - \tau_\Pi^B Z_A \delta_K pK^B.$$

Assuming zero transfers to households ($G^B = G^I = G^S = 0$), the only difference between revenues and wasteful spending G^O are interest payments on the government debt to savers:

$$G^O = T - (e^{-g} - q)B^G,$$

for some fixed B^G that is indeterminate in the non-stochastic version of the model.

Saver Consumption From market clearing for risk free debt we know that $B^S = B^G - B^I$, and saver consumption is

$$C^S = \alpha\gamma_S(1-\tau^S)Y + G^S + (e^{-g} - q)B^S.$$

Resource Constraint The steady state allocation needs to satisfy

$$C^B + C^I + C^S = Y - G^O - e^{-g}(X + \Psi).$$

B Calibration Appendix

B.1 Long-term corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by $F = \frac{\theta}{1-\delta}$, a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for δ and θ.

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices[32] we obtain a time series of monthly market values, durations (the sensitivity

[32] Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively

of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices[33] we obtain a time series of option-adjusted spreads over the Treasury yield curve.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC c of 5.5% and WAM T of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate corresponding to that period's WAM to get a time series of bond yields r_t. Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^c(r_t) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_t)^{i/2}} + \frac{1}{(1+r_t)^T}$$

We can write the steady-state price of a geometric bond with parameter δ as

$$P^G(r_t) = \frac{1}{1+r_t}\left[1 + \delta P^G(r_t)\right]$$

Solving for $P^G(y_t)$, we get

$$P^G(r_t) = \frac{1}{1+r_t-\delta}$$

The calibration determines how many units X of the geometric bond with parameter δ one needs to sell to hedge one unit of plain vanilla bond P^c against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta,X} \sum_{t=1997.1}^{2015.12} \left[P^c(r_t) - XP^G(r_t;\delta)\right]^2$$

We estimate $\delta = 0.937$ and $X = 12.9$, yielding an average pricing error of only 0.41%. This value for δ implies a time series of durations $D_t = -\frac{1}{P_t^G}\frac{dP_t^G}{dr_t}$ with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it D_t years from now. The principal of this loan is just the price of the corresponding D_t maturity zero-coupon bond $\frac{1}{(1+r_t)^{D_t}}$

We set the "principal" F of one unit of the geometric bond to be some fraction θ of the undiscounted sum of all its cash flows $\frac{\theta}{1-\delta}$, where

$$\theta = \frac{1}{N}\sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get $\theta = 0.582$ and $F = 9.18$.

[33] They are named C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively.