

Technological Innovation: Winners and Losers*

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Abstract

We analyze the effect of innovation on asset prices in a tractable, general equilibrium framework with heterogeneous households and firms. Innovation has a heterogeneous impact on households and firms. Technological improvements embodied in new capital benefit workers, while displacing existing firms and their shareholders. This displacement process is uneven: newer generations of shareholders benefit at the expense of existing cohorts; and firms well positioned to take advantage of these opportunities benefit at the expense of firms unable to do so. Under standard preference parameters, the risk premium associated with innovation is negative. Our model delivers several stylized facts about asset returns, consumption and labor income. We derive and test new predictions of our framework using a direct measure of innovation. The model's predictions are supported by the data.

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Introduction

The history of technological innovation is a story of displacement. New technologies emerge that render old capital and processes obsolete. Further, these new technologies are typically embodied in new vintages of capital, so the process of adoption is not costless. For instance, the invention of the automobile by Karl Benz in 1885 required investment in new types of capital, such as paved highways and an infrastructure for fuel distribution. Resources therefore needed to be diverted into investment in the short run in order for the economy to benefit in the long run. Not all economic agents benefitted from the automobile. Railroad firms, which in the late 19th century accounted for 50% of the market capitalization of all NYSE-listed firms, were displaced as the primary mode of transport.¹

We analyze the effect of innovation on the stock market using a general equilibrium model. We model innovation as technological change embodied in new vintages of capital goods.² A key feature of innovation is that it leads to benefits and losses that are unevenly distributed. Hence we consider an economy where both households and firms vary in their exposure to innovation shocks. This heterogeneous impact differentiates innovation from disembodied technical change – in our case a labor augmenting productivity shock – that affects equally all vintages of capital goods.

Innovation leads to displacement of existing owners of capital and therefore to an increase in the marginal utility of consumption of stock market participants. This process of displacement occurs through two channels. First, innovation leads to wealth reallocation between shareholders and workers. Innovation reduces the value of older vintages of capital. In contrast, in our model, labor benefits from innovation since their skill is not tied to a particular technology. As long as shareholders and workers do not fully share risks – for instance, due to limited stock market participation by workers – aggregate innovation shocks lead to wealth reallocation between the owners of capital and workers.

¹Flink (1990, p. 360) writes: “The triumph of the private passenger car over rail transportation in the United States was meteoric. Passenger miles traveled by automobile were only 25 percent of rail passenger miles in 1922 but were twice as great as rail passenger miles by 1925, four times as great by 1929.”

²We use the terms innovation and capital-embodied change interchangeably in this paper. More precisely, we study a particular form of technological innovation, specifically innovation that is embodied in new vintages of intermediate goods. Accordingly, our empirical measure of embodied shocks relies on patent data, since innovation that is embodied in new products is more easily patentable (see, for example, Comin, 2008, for a discussion on patentable innovation). The type of innovation that we study could be related to the concept of skill-biased technical change, but the two concepts are in general distinct. For instance, the first industrial revolution, a technological change embodied in new forms of capital – the factory system – led to the displacement of skilled artisans by unskilled workers, who specialized in a limited number of tasks (see e.g. Sokoloff, 1984, 1986; Atack, 1987; Goldin and Katz, 1998). Further, skill-biased technical change need not be related to firms’ growth opportunities in the same manner as the embodied technical change we consider in this paper.

Second, innovation results in reallocation of wealth across generations. Intergenerational risk sharing is limited in our model. Households have finite lives; each new cohort of households brings with it embodied technological advances in the form of blueprints. Only part of the rents from innovation are appropriated by existing shareholders. Since households cannot share risks with future generations, periods of significant innovation result in wealth transfer from the existing set of households to the newer generations.

Firms have heterogeneous exposure to innovation shocks, leading to cross-sectional differences in risk premia. Firms that are able to capture a larger share of rent from the new inventions benefit more from improvements in the frontier level of technology relative to firms that are heavily invested in older vintages of capital. Since firms with high growth opportunities are less susceptible to the displacive effect of innovation, they are valued highly by financial market participants, earning relatively low average returns in equilibrium. This result is consistent with extensive empirical evidence on stock returns of growth firms. Further, due to their similar exposure to the aggregate innovation shock, stock returns of firms with similar access to growth opportunities comove with each other, above and beyond of what is implied by their exposures to the market returns.

We calibrate our model to match several moments of real economic variables and asset returns, including the mean and volatility of the aggregate consumption growth rate, the equity premium, and the risk-free rate. Our main focus is on the model's cross-sectional predictions. Observable firm characteristics, such as valuation ratios or past investment rates, are correlated with firms' growth opportunities. This endogenous relation allows the model to replicate several stylized facts about the cross-section of asset returns. Both in the data, and in the model, firms with high market-to-book ratios or investment rates (growth) have lower average returns than firms with low market-to-book ratios or investment rates (value). Most importantly, our model captures the tendency of growth (or value) firms to comove with each other, over and above their exposure to the market portfolio. Further, our model replicates the failure of the CAPM and the consumption CAPM in pricing the cross-section of stock returns, since neither the market portfolio nor aggregate consumption is a sufficient statistic for the marginal utility of market participants.

We test the direct implications of our mechanism using a novel measure of embodied technology shocks constructed in Kogan, Papanikolaou, Seru, and Stoffman (2012), which infers the value of innovation from stock market reactions to news about patent grants. The measure of Kogan et al. (2012) has a natural interpretation in the context of our model; we construct this measure in simulated data and show that it is a close match to the current real investment opportunity set in the economy. Armed with a proxy for the key unobservable variable in our model, we concentrate our empirical analysis on the properties of

the model directly linked to its main economic mechanism – displacement in the cross-section of households and firms generated by embodied innovation shocks.

Our empirical tests support the model’s predictions regarding household consumption and innovation. First, innovation shocks generate displacement in the cross-section of households. The level of technological innovation during the year when household heads enter the economy is associated with higher lifetime consumption; by contrast, innovation shocks following the cohort’s entry tend to lower its consumption level relative to the rest of the economy. These patterns exist only for households that own stocks. Moreover, and consistent with our model, higher innovation predicts lower consumption growth of stockholders relative to non-stockholders.

Next, we relate the measure of innovation to firm outcomes. We find that firms with high growth opportunities are displaced less than firms with low growth opportunities when their competitors innovate. Similarly, growth firms have higher return exposure to embodied shocks than value firms. We find that this difference in innovation risk exposures is quantitatively sufficient to account for the observed differences in average returns among value and growth firms in the data. We approximate the stochastic discount factor of our model using our innovation series and data on total factor productivity or consumption. The point estimates of the market price of innovation risk are negative and statistically significant, and most importantly they are close in magnitude to the estimates implied by the calibrated general equilibrium model.

Our work is related to asset pricing models with explicit production.³ Papers in this literature construct structural models with heterogeneous firms and analyze the economic sources of cross-sectional differences in firms’ systematic risk, with a particular focus on understanding the origins of average return differences among value and growth firms. Most of these models are in partial equilibrium (e.g., Berk, Green, and Naik, 1999; Carlson, Fisher, and Giammarino, 2004; Zhang, 2005; Kogan and Papanikolaou, 2011), with an exogenously specified pricing kernel. Some of these papers develop general equilibrium models (e.g. Gomes, Kogan, and Zhang (2003)), yet most of them feature a single aggregate shock, implying that the market portfolio conditionally spans the value factor. In contrast, our model features two aggregate risk factors, one of them being driven by embodied technology shocks. Using an empirical measure of embodied technical change, we provide direct evidence for the model mechanism rather than relying only on indirect model implications.

General equilibrium models face additional challenges in replicating properties of asset returns, since dividends and consumption are both endogenous (e.g., Rouwenhorst, 1995; Jermann, 1998; Boldrin, Christiano, and Fisher, 2001; Kaltenbrunner and Lochstoer, 2010).

³For a recent review of this literature, see Kogan and Papanikolaou (2012a)

Equilibrium models with only disembodied shocks often imply that the aggregate payout of the corporate sector is negatively correlated with consumption (e.g., Kaltenbrunner and Lochstoer, 2010). In these models, the fact that firms cut dividends in order to invest following a positive disembodied shock implies that dividends are less pro-cyclical than consumption (see e.g., Rouwenhorst, 1995). In our setup, dividends are much more responsive than consumption to the embodied shock, which helps the model generate realistic moments for stock returns.

Our work is related to the growing literature on embodied technology shocks (e.g., Cooley, Greenwood, and Yorukoglu, 1997; Greenwood, Hercowitz, and Krusell, 1997; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010). Technology is typically assumed to be embodied in new capital goods – new projects in our setting. Several empirical studies document substantial vintage effects in the productivity of plants. For instance, Jensen, McGuckin, and Stiroh (2001) find that the 1992 cohort of new plants was 50% more productive than the 1967 cohort in its entry year, controlling for industry-wide factors and input differences. Further, our paper is related to work that explores the effect of technological innovation on asset returns (e.g., Greenwood and Jovanovic, 1999; Hobijn and Jovanovic, 2001; Laitner and Stolyarov, 2003; Kung and Schmid, 2011; Garleanu, Panageas, and Yu, 2012). The focus of this literature is on exploring the effects of innovation on the aggregate stock market. We contribute to this literature by explicitly considering the effects of heterogeneity in both firms and households in terms of their exposure to embodied technology shocks.

The closest related work is Papanikolaou (2011), Garleanu, Kogan, and Panageas (2012) and Kogan and Papanikolaou (2011, 2012b). Papanikolaou (2011) demonstrates that in a general-equilibrium model, capital-embodied technology shocks are positively correlated with the stochastic discount factor when the elasticity of intertemporal substitution is less than or equal to the reciprocal of risk aversion. However, the price of embodied shocks in his model is too small relative to the data. We generalize the model in Papanikolaou (2011), allowing for both firm and household heterogeneity and imperfect risk sharing among households. Our model delivers quantitatively more plausible estimates of the risk premium associated with innovation, as well as additional testable predictions. Our model shares some of the features in Garleanu et al. (2012), namely intergenerational displacement risk and technological improvements embodied in new types of intermediate goods. We embed these features into a model with capital accumulation, limited market participation, and a richer, more realistic cross-section of firms. In addition, we construct an explicit empirical measure of innovation shocks and use it to directly test the empirical implications of our model’s mechanism. Last, our work is related to Kogan and Papanikolaou (2011, 2012b), who analyze the effect of capital-embodied technical progress in partial equilibrium. The general

equilibrium model in this paper helps understand the economic mechanism for pricing of such innovation shocks, and provides further insights into how these shocks impact the economy.

Our work is related to the literature emphasizing the role of consumption externalities and relative wealth concerns for asset prices and equilibrium investment and consumption dynamics (Duesenberry, 1949; Abel, 1990; Gali, 1994; Roussanov, 2010). Consistent with the presence of consumption externalities, Luttmer (2005) provides micro-level evidence that consumption of neighbouring households has a negative effect on self-reported happiness measures. Further, preferences for relative wealth can arise endogenously. The model of DeMarzo, Kaniel, and Kremer (2007) shares some of the same features of the simple model in Section 1, in that incomplete markets gives rise to relative wealth concerns among agents. In DeMarzo et al. (2007) the relative wealth concerns arises due to competition with existing agents for future resources that are in limited supply. In our setting, relative wealth concerns arise due to inter-generational displacement.

Our model replicates several stylized facts documented in the consumption-based asset pricing literature. First, our model is consistent with the findings of Malloy, Moskowitz, and Vissing-Jorgensen (2009) that the return differential between value and growth firms has a relatively high exposure to the consumption growth of stockholders, especially at lower frequencies. Second, our model is consistent with the evidence in Lustig and Van Nieuwerburgh (2008) and Lustig, Van Nieuwerburgh, and Verdelhan (2008), who report that human wealth – the present value of wages discounted using the stochastic discount factor implied by absence of arbitrage – earns lower risk premia than financial wealth. In our model, embodied innovation shocks raise equilibrium wages while reducing dividends on existing firms, resulting in a low correlation between the growth of dividends and labor income and a lower risk premium for human wealth. Last, our model is consistent with the recently reported empirical evidence on the dynamics of income shares of financial and human capital in Lettau and Ludvigson (2011).

1 A simple model

To illustrate the main intuition behind our mechanism, we first present a simple two-period model. The economy consists of overlapping generations of capital owners and workers. Capital owners have logarithmic preferences over consumption C_0 and C_1

$$U(C_0, C_1) = \ln C_0 + E_0[\ln C_1]. \tag{1}$$

Workers do not participate in the financial markets. There are two technologies available to produce output, $k \in \{o, n\}$, each using old or new capital, respectively.

In the first period, only the old technology is available. Existing capital owners are endowed with a unit of capital K_o that, along with labor $L_{o,t}$, can be used to produce output in each period:

$$Y_{o,t} = K_o^\alpha L_{o,t}^{1-\alpha}, \quad t = 0, 1. \quad (2)$$

For simplicity, we normalize the measure of workers and capital owners to unity in the first period. In the second period, a measure μ of new workers and new capital owners enter the economy. The new capital owners own the entire stock of new capital, K_n , which produces output according to

$$Y_{n,1} = (\xi K_n)^\alpha L_{n,1}^{1-\alpha}, \quad (3)$$

where ξ is a positive random variable with unit mean: $\xi > 0$ and $E[\xi] = 1$. The random variable ξ is the technology shock embodied in the new vintage of capital. A value of $\xi > 1$ implies that the new capital is more productive than the old. In contrast, the new workers are identical to the old workers; labor can be freely allocated to either the old or to the new technology.

In equilibrium, the allocation of labor between the old and the new technology depends on the realization of the embodied shock ξ ,

$$L_{o,1} = \frac{1 + \mu}{1 + \xi\mu} \quad \text{and} \quad L_{n,1} = \xi\mu \frac{1 + \mu}{1 + \xi\mu}. \quad (4)$$

Because of the Cobb-Douglas production technology, equilibrium consumption of existing capital owners is proportional to the output of the old technology. Since $L_{o,1}$ is decreasing in ξ , so does the consumption growth of existing shareholders,

$$\frac{C_1^o}{C_0^o} = \left(\frac{1 + \mu}{1 + \xi\mu} \right)^{1-\alpha}. \quad (5)$$

Equation (5) illustrates the displacive effect of innovation on the owners of existing capital. Unlike workers, who can supply labor to both the new and the old firms, the owners of old capital do not benefit from the embodied shock ξ . Since they compete with the owners of new capital in the market for labor, a positive innovation shock leads to lower consumption for the owners of existing capital.

Now, suppose that a claim on the output of the new technology were available at time 0. For simplicity, assume that this claim is on an infinitesimal fraction of the output of the new technology, so that (5) still characterizes the consumption growth of the old capital owners.

Given the preferences of the existing households (1) and their consumption growth (5), the difference between the realized return to the new and the old technology is

$$R_1^n - R_1^o = \left(\frac{\xi}{E[\xi]} - 1 \right) \left(\frac{1 + \mu}{1 + \xi\mu} \right)^{1-\alpha}. \quad (6)$$

Since the innovation shock ξ is embodied in new capital, a positive innovation shock $\xi > 1$ is associated with a higher return of the new technology relative to the old.

Proposition 1 *In equilibrium, the claim to the new technology has a lower expected return than the claim to the old technology,*

$$E[R_1^n] < E[R_1^o].$$

Proof. Let $f(\xi) = \left(\frac{\xi}{E[\xi]} - 1 \right) \left(\frac{1+\mu}{1+\xi\mu} \right)^{1-\alpha}$. Since $f''(\xi) < 0$, Jensen's inequality implies $E[f(\xi)] < f(E[\xi]) = 0$ ■

Proposition 1 summarizes the intuition behind the main results of the paper. In contrast to labor, capital is tied to a specific technology. Hence, technological improvements embodied in new vintages of capital lower the value of older vintages. Imperfect inter- and intra-generational risk sharing imply that innovation leads to high marginal utility states for the owners of existing capital. Given the opportunity, owners of existing capital are willing to own a claim to the new technology, and accept lower returns on average, to obtain a hedge against displacement. Limited risk sharing across new and old capital owners, as well as shareholders and workers is key for this result. As a result of limited risk sharing, the consumption CAPM fails in the model because the consumption growth of the marginal investor (5) differs from aggregate, per capita, consumption growth

$$\frac{\bar{C}_1}{\bar{C}_0} = \left(\frac{1 + \xi\mu}{1 + \mu} \right)^\alpha. \quad (7)$$

The model in this section is too stylized to allow us to quantify the importance of this mechanism of asset returns and economic quantities. Next, we develop a dynamic general equilibrium model that builds on these basic ideas.

2 The model

In this section we develop a dynamic general equilibrium model that extends the simple model above along several dimensions. First, we endogenize the investment in the capital stock each period. Labor participates in the production of new capital, hence an increase in investment

expenditures leads to an increase in labor income. Labor benefits from the expansion and improvement in the capital stock – as in the simple model above – but also because workers do not share the costs of new capital acquisition with current capital owners. Second, the model features a full cross-section of firms. Existing firms vary in their ability to capture rents from new projects. By investing in existing firms, existing capital owners can hedge their displacement from innovation. Differences in the ability of firms to acquire innovation lead to ex-ante differences in risk premia. Third, we consider a richer class of preferences that separate risk aversion from the inverse of the elasticity of intertemporal substitution and allow for relative consumption effects in the utility function. These extensions allow for a better quantitative fit of the model to the data, but do not qualitatively alter the intuition from the simple model above.

2.1 Firms and technology

There are three production sectors in the model: a sector producing intermediate consumption goods; a sector that aggregates these intermediate goods into the final consumption good; and a sector producing investment goods. Firms in the last two sectors make zero profits due to competition and constant returns to scale, hence we explicitly model only the intermediate-good firms.

Intermediate-good firms

Production in the intermediate sector takes place in the form of projects. Projects are introduced into the economy by the new cohorts of inventors, who lack the ability to implement them on their own and sell these blueprints to existing intermediate-good firms. There is a continuum of infinitely lived firms; each firm owns a finite number of projects. We index individual firms by $f \in [0, 1]$ and projects by j . We denote the set of projects owned by firm f by \mathcal{J}_f , and the set of all active projects in the economy by \mathcal{J}_t .⁴

Active projects

Projects are differentiated from each other by three characteristics: a) their scale, k_j , chosen irreversibly at their inception; b) the level of frontier technology at the time of project creation, s ; and c) the time-varying level of project-specific productivity, u_{jt} . A project j

⁴While we do not explicitly model entry and exit of firms, firms occasionally have zero projects, thus temporarily exiting the market, whereas new entrants can be viewed as a firm that begins operating its first project. Investors can purchase shares of firms with zero active projects.

created at time s produces a flow of output at time $t > s$ equal to

$$y_{jt} = u_{jt} e^{\xi_s} k_j^\alpha, \quad (8)$$

where $\alpha \in (0, 1)$, ξ denotes the level of frontier technology at the time the project is implemented, and u is a project-specific shock that follows a mean-reverting process. In particular, the random process governing project output evolves according to

$$du_{jt} = \theta_u(1 - u_{jt}) dt + \sigma_u \sqrt{u_{jt}} dZ_{jt}, \quad (9)$$

All projects created at time t are affected by the embodied shock ξ , which follows a random walk with drift

$$d\xi_t = \mu_\xi dt + \sigma_\xi dB_{\xi t}. \quad (10)$$

The embodied shock ξ captures the level of frontier technology in implementing new projects. In contrast to the disembodied shock x , an improvement in ξ affects only the output of *new* projects. In most respects, the embodied shock ξ is formally equivalent to investment-specific technological change.

All new projects implemented at time t start at the long-run average level of idiosyncratic productivity, $u_{jt} = 1$. Thus, all projects managed by the same firm are ex-ante identical in terms of productivity, but differ ex-post due to the project-specific shocks. Last, active projects expire independently at a Poisson rate δ .

Firm investment opportunities – new projects

There is a continuum of firms in the intermediate goods sector that own and operate projects. Firms are differentiated by their ability to attract inventors, and hence initiate new projects. We denote by N_{ft} the Poisson count process that denotes the number of projects the firm has acquired. The probability that the firm acquires a new project, $dN_t = 1$, is firm-specific and equal to

$$\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{ft}. \quad (11)$$

The likelihood that the firm acquires a new project λ_{ft} is composed of two parts. The first part λ_f captures the long-run likelihood of firm f receiving new projects, and is constant over time. The second component, $\tilde{\lambda}_{ft}$ is time-varying, following a two-state, continuous time Markov process with transition probability matrix S between time t and $t + dt$ given by

$$S = \begin{pmatrix} 1 - \mu_L dt & \mu_L dt \\ \mu_H dt & 1 - \mu_H dt \end{pmatrix}. \quad (12)$$

We label the two states as $\{\lambda_H, \lambda_L\}$, with $\lambda_H > \lambda_L$. Thus, at any point in time, a firm can be either in the high-growth ($\lambda_{ft} = \lambda_f \cdot \lambda_H$) or in the low-growth state ($\lambda_{ft} = \lambda_f \cdot \lambda_L$). The instantaneous probability of switching to each state is $\mu_H dt$ and $\mu_L dt$, respectively. Without loss of generality, we impose the restriction $E[\tilde{\lambda}_{f,t}] = 1$. Our specification implies that the aggregate rate of project creation $\bar{\lambda} \equiv E[\lambda_{ft}]$ is constant.

Implementing new projects

The implementation of a new project idea requires new capital k purchased at the equilibrium market price q . Once a project is acquired, the firm chooses its scale of production k_j to maximize the value of the project. A firm's choice of project scale is irreversible; firms cannot liquidate existing projects and recover their original costs.

Capital-good firms

Firms in the capital-good sector use labor to produce productive the investment goods needed to implement new projects in the intermediate-good sector

$$I_t = e^{x_t} L_{It}. \quad (13)$$

The labor augmenting productivity shock x follows a random walk with drift

$$dx_t = \mu_x dt + \sigma_x dB_{xt}. \quad (14)$$

Final-good firms

Final consumption good firms using a constant returns to scale technology employing labor L_C and intermediate goods Y_t

$$C_t = Y_t^\phi (e^{x_t} L_{Ct})^{1-\phi}. \quad (15)$$

Production of the final consumption good is affected by the labor augmenting productivity shock x_t .

2.2 Households

There are two types of households, each with a unit mass: hand-to-mouth workers who supply labor; and inventors, who supply ideas for new projects. Both types of households have finite lives: they die stochastically at a rate μ , and are replaced by a household of the same type. Households have no bequest motive and have access to a market for state-contingent life

insurance contracts. Hence, each household is able to perfectly share its mortality risk with other households of the same cohort.

Inventors

Each new inventor is endowed with a measure $\bar{\lambda}/\mu$ of ideas for new projects. Inventors are endowed with no other resources, and lack the ability to implement these project ideas on their own. Hence, they sell these projects to existing firms. Inventors and firms bargain over the surplus created by new projects. Each inventor captures a share η of the value of each project. After they sell their project, inventors invest their proceeds in financial markets. Inventors are only endowed with projects upon entry, and cannot subsequently innovate. As a result, each new successive generation of inventors can potentially displace older cohorts. Inventors have access to complete financial markets, including an annuity market.

Inventor's utility takes a recursive form

$$J_t = E_t \int_t^\infty \tilde{f}(C_s, \bar{C}_s, J_s) ds, \quad (16)$$

where the aggregator \tilde{f} is given by

$$\tilde{f}(C, \bar{C}, J) \equiv \frac{\rho}{1 - \theta^{-1}} \left(\frac{\left(C^{1-h} (C/\bar{C})^h \right)^{1-\theta^{-1}}}{((1-\gamma)J)^{\frac{\gamma-\theta^{-1}}{1-\gamma}}} - (1-\gamma)J \right). \quad (17)$$

Household preferences depend on own consumption C , but also on the consumption of the household relative to the aggregate \bar{C} . Thus, our preference specification nests 'keeping up with the Joneses' and non-separability across time (see e.g. Abel, 1990; Duffie and Epstein, 1992). The parameter h captures the strength of the external habit; $\rho = \hat{\rho} + \mu$ is the effective time-preference parameter, which includes the adjustment for the likelihood of death μ ; γ is the coefficient of relative risk aversion; and θ is the elasticity of intertemporal substitution (EIS).

Workers

Workers inelastically supply one unit of labor that can be freely allocated between producing consumption or investment goods

$$L_I + L_C = 1. \quad (18)$$

The allocation of labor between the investment-good and consumption-good sectors is the mechanism through which the economy as a whole saves or consumes.

Workers are hand-to-mouth; they do not have access to financial markets and consume their labor income every period.

3 Competitive equilibrium

Definition 1 (Competitive Equilibrium) *The competitive equilibrium is a sequence of quantities $\{C_t^S, C_t^W, Y_t, L_{Ct}, L_{It}\}$; prices $\{p_t^Y, p_t^I, w_t\}$; firm investment decisions $\{k_t\}$ such that given the sequence of stochastic shocks $\{x_t, \xi_t, u_{jt}, N_{ft}\}$: i) shareholders choose consumption and savings plans to maximize their utility (16); ii) intermediate-good firms maximize their value according to (19); iii) Final-good and investment-good firms maximize profits; iv) the labor market (18) clears; v) the market for capital clears (21); vi) the market for consumption clears $C_t^S + C_t^W = C_t$; vii) the resource constraints (13)-(15) are satisfied; and viii) market participants rationally update their beliefs about λ_{ft} using all available information.*

We relegate the details of the computation of equilibrium to Appendix A.

3.1 Firm optimization

We begin our description of the competitive equilibrium by characterizing the firms' optimality conditions.

Market for capital

Intermediate good firms choose the scale of investment, k_j , in each project to maximize its net present value, that is, the market value of the new project minus its implementation cost. We guess – and subsequently verify – that the equilibrium price of a new project equals $P_t e^{\xi_t} k^\alpha$, where P is a function of only the aggregate state of the economy. Then, the net present value of a project is

$$\max_k NPV = P_t e^{\xi_t} k^\alpha - p_t^I k. \quad (19)$$

The optimal scale of investment is a function of the ratio of the market value of a new project to its marginal cost of implementation p_t^I ,

$$k_t = \left(\frac{\alpha e^{\xi_t} P_t}{p_t^I} \right)^{\frac{1}{1-\alpha}}. \quad (20)$$

Equation (20) bears similarities to the q-theory of investment (Hayashi, 1982). A key difference here is that the numerator involves the market value of a new project – marginal q – which is distinct from the market value of the firm – average q. Aggregating across firms, the total demand for new capital equals

$$I_t = \int k_{ft} dN_{ft} = \bar{\lambda} k_t. \quad (21)$$

The equilibrium price of investment goods, p_t^I , clears the supply (13) and the total demand for new capital (21)

$$p_t^I = \alpha e^{\xi t} P_t \left(\frac{\bar{\lambda}}{e^{x_t} L_{It}} \right)^{1-\alpha}. \quad (22)$$

A positive innovation shock leads to an increase in the demand for capital, and thus to an increase in its equilibrium price p_t^I .

Market for labor

Labor is used to produce both the final consumption good, and the capital needed to implement new projects. The first order condition of the firms producing the final consumption good with respect to labor input links their labor choice L_C to the competitive wage w_t

$$(1 - \phi) Y_t^\phi e^{(1-\phi)x_t} L_{Ct}^{-\phi} = w_t. \quad (23)$$

The profit maximization in the investment-goods sector implies that

$$e^{x_t} p_t^I = w_t. \quad (24)$$

The equilibrium allocation of labor between producing consumption and investment goods is determined by the labor market clearing condition (18), along with (22)-(24)

$$(1 - \phi) Y_t^\phi e^{(1-\phi)x_t} (1 - L_{It})^{-\phi} = \alpha e^{\alpha x_t + \xi t} P_t \left(\frac{\bar{\lambda}}{L_{It}} \right)^{1-\alpha}. \quad (25)$$

All else equal, an increase in the embodied shock ξ increases the demand for new investment goods. As a result, the economy reallocates resources away from producing consumption goods towards producing investment goods.

Market for intermediate goods

Consumption firms purchase the intermediate good Y at a price p^Y and hire labor L_C at a wage w to maximize their value. Their first order condition with respect to their demand for intermediate goods yields

$$\phi Y_t^{\phi-1} (e^{x_t} L_{Ct})^{1-\phi} = p_t^Y. \quad (26)$$

The price of the intermediate good p_Y is therefore pinned down by the equilibrium allocation of labor to the final good sector L_C and the supply of intermediate goods, Y .

The total output of the intermediate good, Y_t , equals the sum of the output of the individual projects, $Y_t = \int y_{f,t}$, and is equal to the effective capital stock

$$Y_t = K_t \equiv \int_{j \in \mathcal{J}_t} e^{\xi_j} k_j^\alpha dj. \quad (27)$$

adjusted for the productivity of each vintage – captured by ξ at the time the project is created – and for decreasing returns to scale. An increase in the effective capital stock K , for instance due to a positive embodied shock, leads to a lower price of the intermediate good and to displacement for productive units of older vintages.

3.2 Household optimization

Here, we describe the household's optimality conditions.

Inventors

Upon entry, inventors sell the blueprints to their projects to firms and use the proceeds to invest in financial markets. A new inventor entering at time t acquires a share of total financial wealth W_t equal to

$$b_{tt} = \frac{\eta \bar{\lambda} NPV_t}{\mu W_t}, \quad (28)$$

where NPV_t is the maximand in (19), η is the share of the project value captured by the inventor, and W_t is total financial wealth in the economy.

As new inventors acquire shares in financial wealth, they displace older cohorts. The share of total financial wealth W held at time t by an inventor born at time $s < t$ equals

$$b_{ts} = b_{ss} \exp \left(\mu(t-s) - \mu \int_s^t b_{uu} du \right). \quad (29)$$

Agents insure the risk of death with other members of the same cohort; hence surviving

agents experience an increase in the growth rate of per-capital wealth equal to probability of death μ .

We guess – and subsequently verify – that the value function of an inventor born in time s is given by

$$J_{ts} = \frac{1}{1 - \gamma} b_{ts}^{1-\gamma} F_t, \quad (30)$$

where F_t is a function of the aggregate state.

Even though the model features heterogenous households, aggregation is simplified due to homotheticity of preferences. Existing inventors vary in their level of financial wealth, captured by b_{ts} . However, all existing agents at time t share the same growth rate of consumption going forward, as they share risk in financial markets. Hence, all existing inventors have the same intertemporal marginal rate of substitution

$$\frac{\pi_s}{\pi_t} = \exp \left(\int_t^s \tilde{f}_J(C_u, \bar{C}_u, J_u) du \right) \frac{\tilde{f}_C(C_s, \bar{C}_s, J_s)}{\tilde{f}_C(C_t, \bar{C}_t, J_t)}, \quad (31)$$

where J is the utility index defined recursively in equation (16), and \tilde{f} is the preference aggregator defined in equation (17). We also refer to π_s/π_t as the stochastic discount factor.

Workers

Workers inelastically supply one unit of labor and face no investment decisions. Every period, they consume an amount equal to their labor proceeds

$$C_t^W = w_t. \quad (32)$$

3.3 Asset prices

The last step in characterizing the competitive equilibrium involves the computation of financial wealth. Since firms producing capital goods and the final consumption good have constant returns to scale technologies and no adjustment costs, they make zero profits in equilibrium. Hence, we only focus on the sector producing intermediate goods.

Total financial wealth is equal to the sum of the value of existing assets plus the value of future projects

$$W_t = VAP_t + PVGO_t. \quad (33)$$

The value of financial wealth also corresponds to the total wealth of inventors, which enters the denominator of the displacement effect (28). Next, we solve for the two components of financial wealth.

Value of Assets in Place

A single project produces a flow of the intermediate good, whose value in terms of consumption is $p_{Y,t}$. The value, in consumption units, of an existing project with productivity level u_{jt} equals

$$E_t \left[\int_t^\infty e^{-\delta s} \frac{\pi_s}{\pi_t} p_{Y,s} u_{j,s} e^{\xi_j} k_j^\alpha ds \right] = e^{\xi_j} k_j^\alpha \left[P_t + \tilde{P}_t(u_{j,t} - 1) \right], \quad (34)$$

where P_t and \tilde{P}_t are functions of the aggregate state of the economy – verifying our conjecture above. The total value of all existing projects is equal to

$$VAP_t \equiv \int_{j \in \mathcal{J}_t} e^{\xi_j} k_j^\alpha \left[P_t + \tilde{P}_t(u_{j,t} - 1) \right] dj = P_t K_t, \quad (35)$$

where K is the effective capital stock defined in equation (27).

Value of Growth Opportunities

The present value of growth opportunities is equal to the present value of rents to existing firms from all future projects

$$PVGO_t \equiv (1 - \eta) E_t \int_t^\infty \left(\int \lambda_{fs} \frac{\pi_s}{\pi_t} NPV_s df \right) ds = \bar{\lambda} (1 - \eta) \left[\Gamma_t^L + \frac{\mu_H}{\mu_L + \mu_H} (\Gamma_t^H - \Gamma_t^L) \right] \quad (36)$$

where NPV_t is the equilibrium net present value of new projects in (19), $1 - \eta$ represents the fraction of this value captured by existing firms; $\mu_H / (\mu_H + \mu_L)$ is the measure of firms in the high growth state; and Γ_t^L and Γ_t^H determine the value of a firm in the low- and high-growth phase, respectively.

3.4 Dynamic evolution of the economy

The current state of the economy is characterized by the vector $Z_t = [\chi_t, \omega_t]$, where

$$\chi \equiv (1 - \phi)x + \phi \log K \quad (37)$$

$$\omega \equiv \alpha x + \xi - \log K. \quad (38)$$

The dynamic evolution of the aggregate state Z depends on the laws of motion for ξ and x , given by equations (10) and (14), respectively, and the evolution of the effective stock of

capital,

$$dK_t = \left(i(\omega_t) - \delta \right) K_t dt, \quad \text{where} \quad i(\omega_t) \equiv \bar{\lambda} e^{\xi t} k_t^\alpha = \bar{\lambda} e^{\omega t} \left(\frac{L_{It}}{\bar{\lambda}} \right)^\alpha. \quad (39)$$

At the aggregate level, our model behaves similarly to the neoclassical growth model. Growth – captured by the difference-stationary state variable χ – occurs through capital accumulation and growth in the level of labor-augmenting technology x . The effective capital K grows by the average rate of new project creation $\bar{\lambda}$, the equilibrium scale of new projects k , and improvements in the quality of new capital ξ ; the effective capital depreciates at the rate δ of project expiration.

The variable ω captures transitory fluctuations along the stochastic trend. Since $i'(\omega) > 0$, an increase in ω accelerates the growth rate of the effective capital stock, and thus the long-run growth captured by χ . We therefore interpret shocks to ω as shocks to the investment opportunity set in this economy; the latter are affected both by the embodied innovation shocks $d\xi_t$ and the disembodied productivity shocks dx_t . Further, the state variable ω is mean-reverting; an increase in ω leads to an acceleration of capital accumulation K , in the future ω reverts back to its long-run mean. In addition to $i(\omega)$, the following variables in the model are stationary since they depend only on ω : the optimal allocation of labor across sectors L_I and L_C ; the consumption share of workers C^w/\bar{C} ; the rate of displacement of existing shareholders b .

4 Model implications

Here, we calibrate our model and explore its implications for asset returns and aggregate quantities. We then analyze the main mechanisms behind the model's predictions.

4.1 Calibration

The model has a total of 18 parameters. We choose these parameters to approximately match a set of aggregate and cross-sectional moments.

We choose the mean growth rate of the technology shocks, $\mu_x = 0.023$ and $\mu_\xi = 0.005$, to match the growth rate of the economy; and their volatilities $\sigma_x = 0.05$ and $\sigma_\xi = 0.125$ to match the volatility of shareholder consumption growth and investment growth, respectively. We select the parameters of the idiosyncratic shock, $\sigma_u = 1.15$ and $\theta_u = 0.05$, to match the persistence and dispersion in firm output-capital ratios.

We choose the returns to scale parameter at the project level $\alpha = 0.45$ to approximately

match the correlation between investment rate and Tobin’s Q . We choose a depreciation rate of $\delta = 0.05$ in line with typical calibrations of RBC models. We choose the share of capital in the production of final goods $\phi = 0.3$ to match the average level of the labor share. The firm-specific parameter governing long-run growth rates, λ_f is drawn from a uniform distribution [5, 15]; the parameters characterizing the short-run firm growth dynamics are $\lambda_H = 4.25$, $\mu_L = 0.2$ and $\mu_H = 0.05$. We choose these values to approximately match the average investment-to-capital ratio in the economy as well as the persistence, the dispersion and the lumpiness in firm investment rates.

We choose a low value of time preference $\rho = 0.005$, based on typical calibrations. We select the coefficient of risk aversion $\gamma = 45$ and the elasticity of intertemporal substitution $\theta = 0.6$ to match the level of the premium of financial wealth and the volatility of the risk free rate. Our choice of the EIS lies between the estimates reported by Vissing-Jorgensen (2002) for stock- and bondholders (0.4 and 0.8 respectively). We choose the preference weight on relative consumption $h = 1/2$ following Garleanu et al. (2012), so that households attach equal weights to own and relative consumption. Our calibration of relative consumption preferences effectively halves the effective risk aversion with respect to shocks that have symmetric effects on household and aggregate consumption. The degree of intergenerational risk-sharing is affected by the bargaining parameter η ; we calibrate $\eta = 0.8$ to match the volatility of cohort effects. We choose the probability of death $\mu = 0.025$, so that the average length of adult life is $1/\mu = 40$ years. We create returns to equity by levering financial wealth by 2, which is consistent with estimates of the financial leverage of the corporate sector (see e.g. Rauh and Sufi, 2011).

4.2 Model properties

We start by verifying that our model generates implications about macroeconomic quantities that are consistent with the data. Next, we study the implications of our model for asset returns.

Quantities

The model generates realistic moments for aggregate quantities, in addition to the moments we target, as we see in Table 1. Given that the standard RBC model does a reasonable job replicating the behavior of aggregate quantities, we focus our attention on the implications of the non-standard features of our setup relative to the standard RBC model.

The presence of the two aggregate technology shocks – embodied and disembodied – results in a correlation between investment and consumption growth that is substantially less

than one (45%), which is in line with the data (44.1%). Limited stock market participation typically implies that shareholder consumption is more volatile than aggregate consumption. In our case, this is true, but the difference is quantitatively minor (3.7% vs 3.0%), which is in line with the data (3.6% vs 2.8%). Hence, the improved performance of our model in matching asset pricing moments is not a result of higher consumption volatility for financial market participants.

Last, aggregate payout to capital owners – dividends, interest payments and repurchases minus new issuance – are volatile and positively correlated with consumption and labor income (51% and 30% respectively). Obtaining estimates of this number is complicated by difficulties in measuring total payout; however, these numbers are in line with the documented properties of dividends in Bansal and Yaron (2004).

Equity premium and the risk-free rate

The equity premium implied by our model is in line with the data, and realized equity returns are sufficiently volatile. The risk-free rate is smooth, despite the relatively low EIS and the presence of consumption externalities. The level of the risk-free rate is somewhat higher than the post-war average, but lower than the average level in the long sample in Campbell and Cochrane (1999).

We conclude that our model performs at least as well as most general equilibrium models with production in matching the moments of the market portfolio and risk-free rate (e.g., Jermann, 1998; Boldrin et al., 2001; Kaltenbrunner and Lochstoer, 2010).

Cross-section of stock returns

The finance literature has extensively documented the value premium puzzle, that is, the finding that firms with high book-equity to market-equity ratios (value) have substantially higher average returns than firms with low book-to-market (growth) (Fama and French, 1992, 1993; Lakonishok, Shleifer, and Vishny, 1994). This difference in average returns is economically large, and is close in magnitude to the equity premium. The book-to-market ratio is closely related to the inverse of Tobin's Q, as it compares the replacement cost of the firm's assets to their market value.⁵ A closely related finding is that firms with high past investment have lower average returns than firms with low past investment (Titman, Wei, and Xie, 2004).

⁵The difference arises because i) firms are also financed by debt; ii) the denominator in measures of Tobin's Q is the replacement cost of capital rather than the book-value of assets. Nevertheless, Tobin's Q and book-to-market generate very similar dispersion in risk premia.

Previous work has argued that growth firms have higher exposure to embodied shocks than value firms (e.g. Papanikolaou, 2011; Kogan and Papanikolaou, 2010); hence studying this cross-section in the context of our model is informative about the properties of embodied shocks. We follow the standard empirical procedure (see e.g. Fama and French, 1993) and sort firms into decile portfolios on their I/K and B/M ratios in simulated data. Table 2 shows that our model generates a 5.9% spread in average returns between the high-B/M and the low-B/M decile portfolios, compared to 6.4% in the data. Similarly, the model generates difference in average returns between the high- and low-investment decile portfolios is -5.9% , compared to -5.3% in the data.

An important component of the value premium puzzle is that value and growth firms appear to have the roughly the same systematic risk, measured by their exposure to the market portfolio, implying the failure of the Capital Asset Pricing Model (CAPM). Here, we show that our model replicates this failure. As we see in Table 3, firms' market betas are only weakly correlated with their book-to-market ratios, and returns on the high-minus-low B/M portfolio have a positive alpha with respect to the CAPM (3.6% in the model versus 5.9% in the data). Similarly, CAPM betas are essentially unrelated to the firms' past investment rates in the model, and high-minus-low I/K portfolio has a CAPM alpha of -5.01% , compared to -7.09% in the data.

Last, our model also replicates the fact that the high-minus-low B/M and investment rate portfolios are not spanned by the market return, as evidenced by the low R^2 resulting from regressing their returns on the market return. This empirical pattern led Fama and French (1993) to propose an empirical asset pricing model that includes a portfolio of value minus growth firms as a separate risk factor in the time-series of returns, in addition to the market portfolio. Our general equilibrium model provides a theoretical justification for the existence of this value factor.

4.3 Inspecting the mechanism

Here, we detail the intuition behind the main mechanism in our model. We first consider the mechanism for how innovation risk is priced – the relation between the innovation shock and the stochastic discount factor. Then, we discuss the determinants of the cross-sectional differences in exposure to innovation risk among firms, and the resulting differences in expected stock returns.

Equilibrium quantities

Aggregate quantities show different responses to the embodied and disembodied shock. In Figures 1 and 2 we plot the impulse response of consumption, investment, aggregate payout and labor income to a positive embodied and disembodied shock respectively.

A positive embodied shock leads to an improvement in real investment opportunities. Investment increases on impact – leading to an acceleration in capital accumulation – and then reverts to a slightly lower level as the economy accumulates more capital. Aggregate payout by firms declines on impact, as firms cut dividends or raise capital to fund investment in new projects. Since the economy reallocates resources away from consumption towards investment – as we see in panel *a* of Figure 3 – aggregate consumption drops on impact but then sharply accelerates due to faster capital accumulation. Further, similar to the simple model in section 1, a positive embodied shock increases the effective stock of capital K and benefits laborers due to an increase in the equilibrium wage *in the long run*. In the extended model, a positive innovation shock benefits workers relative to capital owners through an additional channel: labor participates in the production of capital, hence equilibrium wages also increase on impact.

In contrast, from the perspective of the existing shareholders, a positive embodied shock leads to a much sharper drop in their consumption – and a much slower acceleration in future consumption growth – relative to the aggregate economy, due to limited risk sharing with workers and future generations. First, the increase in equilibrium wage leads to a temporary reallocation of income from capital to labor, as we see in panel *b* of Figure 3. Second, the embodied shock leads to displacement of existing cohorts by future generations of innovators, which is captured by $b(\omega_t)$ in panel *c* of Figure 3. Both of these effects imply that, in relative terms, a positive embodied shock has a persistent negative impact on existing shareholders.

A positive disembodied shock leads to higher output in both the consumption and the investment sector, leading to positive comovement in investment, consumption and output growth. Further, as in the standard RBC model, dividends respond less than consumption as firms cut payout to finance investment (see e.g. Rouwenhorst, 1995). Last, since the disembodied shock affects the real investment opportunities ω , a positive disembodied shock leads to a reallocation of wealth from existing shareholders to workers and future generations; however, this effect is qualitatively minor.

Comparing the response of dividends to an embodied and disembodied shock, we see that dividends respond more than consumption in the first case, and less than consumption in the second case. Hence, the presence of the embodied shock is a key part of the mechanism that leads to an equilibrium dividend process that is more pro-cyclical – with respect to consumption – relative to existing models (e.g. Rouwenhorst, 1995; Kaltenbrunner and

Lochstoer, 2010).

Equilibrium price of technology shocks

The price of risk of technology shocks $\gamma_i(\omega)$ – which equals the sharpe ratio of a security that is perfectly correlated with the shock – can be recovered from investors’ inter-temporal marginal rate of substitution

$$\frac{d\pi_t}{\pi_t} = -r_{ft} dt - \gamma_x(\omega_t) dB_t^x - \gamma_\xi(\omega_t) dB_t^\xi. \quad (40)$$

The equilibrium stochastic discount factor is proportional to the gradient of the utility function of the stock holders in the model, therefore

$$\frac{d\pi_t}{\pi_t} = [\dots] dt - \theta^{-1} \left(\frac{dC_{ts}}{C_{ts}} - h(1-\theta) \frac{d\bar{C}_{ts}}{\bar{C}_{ts}} \right) - \frac{\gamma - \theta^{-1}}{1-\gamma} \frac{dJ_{ts}}{J_{ts}}, \quad (41)$$

At time t , the change in the marginal utility of consumption for an existing stockholder of cohort s , $s < t$, is related to the change in her own consumption C_t ; the change in aggregate consumption \bar{C}_t , due to relative consumption concerns parameterized by h ; and the change in continuation utility J_{ts} . As a result, the price of risk of each technology shock γ_i depends on how it affects each of these three objects.

In panel *e* of of Figure 3 we plot the conditional market price of innovation risk, $\gamma_\xi(\omega)$. The embodied shock is positively correlated with marginal utility; the price of innovation risk is negative, and approximately equal to -0.8 at the mean of the stationary distribution of ω . A positive embodied shock leads to lower instantaneous consumption growth for existing shareholders, both in absolute as well as in relative terms, as we see in Figure 1, leading to an increase in marginal utility. The effect of the embodied shock in the value function J is in general ambiguous; following a positive embodied shock, shareholders capture a smaller slice of a larger pie. The net effect on utility depends on preference parameters, including the weight on relative consumption. In our calibration the displacement effect dominates, hence the value function J of asset holders is negatively exposed to the innovation shock – panel *d* of Figure 3 – resulting in a further increase in marginal utility following innovation.

The conditional market price of the disembodied shock $\gamma_x(\omega)$ is positive, and approximately equal to 0.5 at the mean of the stationary distribution of ω , as we see in panel *f* of Figure 3. The disembodied technology shock also affects the stochastic discount factor through several channels. Some of these channels are the same as for the embodied shock. Since increased labor productivity makes it cheaper to produce new capital, the disembodied shock also affects real investment opportunities ω in (38). However, the key difference in comparison

to the embodied shock is that a positive disembodied technology shock also has a large positive effect on consumption of stockholders, since increased labor productivity raises the productivity of assets in place and future investments, as we see in Figure 2. This positive effect dominates, hence the equilibrium price of risk of the disembodied shock is positive.

Firm risk premia

Equilibrium risk premia are determined by the covariance of stock returns with the equilibrium stochastic discount factor. At the firm level, expected returns are heterogeneous because firms have different exposures to technology shocks. Consider the decomposition of the firm value in the intermediate-good sector:

$$\begin{aligned} V_{ft} &= VAP_{ft} + PVGO_{ft} \\ &= \int_{j \in \mathcal{J}_{ft}} e^{\xi_j} k_j^\alpha \left[P_t + \tilde{P}_t(u_{j,t} - 1) \right] dj + \lambda_f(1 - \eta) \left[\Gamma_t^L + p_{ft} (\Gamma_t^H - \Gamma_t^L) \right]. \end{aligned} \quad (42)$$

The first term captures the value of assets in place and depends on the firm's current portfolio of projects, \mathcal{J}_f . The second term captures the value of growth opportunities. This term depends on the current growth state of the firm, captured by the indicator function p_{ft} , which equals one if the firm is in the high-growth state ($\tilde{\lambda}_{ft} = \lambda_H$). Importantly, the two components of the firm value, assets in place and growth opportunities, have different exposures to technology shocks.

We derive the firms' exposures to the fundamental shocks dB_t^x and dB_t^ξ from (42) using Ito's lemma:

$$\frac{dV_{ft}}{V_{ft}} = [\dots] dt + (1 - \phi) \sigma_x dB_t^x + B_{ft} \left(\sigma_\xi dB_t^\xi + \alpha \sigma_x dB_t^x \right), \quad (43)$$

where

$$B_{ft} = \left(\zeta'_\nu(\omega_t) + \tilde{\zeta}'_\nu(\omega_t) \frac{A_{ft}^v}{1 + A_{ft}^v} \right) \frac{VAP_{ft}}{V_{ft}} + \left(\zeta'_g(\omega_t) + \tilde{\zeta}'_g(\omega_t) \frac{A_{ft}^g}{1 + A_{ft}^g} \right) \frac{PVGO_{ft}}{V_{ft}}, \quad (44)$$

and $\zeta_\nu, \zeta_g, \tilde{\zeta}_\nu$, and $\tilde{\zeta}_g$ are functions of the aggregate state of the economy ω_t ; the functions A_{ft}^v and A_{ft}^g depend on the deviations of the firm's project portfolio from the average productivity ($u = 1$) and the firm's growth state $p = \mu_H / (\mu_H + \mu_L)$ respectively.

The first stochastic term in (43), $(1 - \phi) \sigma_x dB_t^x$, is identical across firms, and is driven solely by the disembodied productivity shocks. Variation in firm risk premia arises solely due to the second term, $B_{ft} \left(\sigma_\xi dB_t^\xi + \alpha \sigma_x dB_t^x \right)$, capturing firm exposures to unanticipated

changes in aggregate investment opportunities.

In Figure 5, we plot the firm's innovation risk exposure B_f , as well as the innovation exposure of each of the two firm value components, as functions of the firm's state. We do the same for the risk premia. The value of assets in place is negatively exposed to innovation shocks, $\zeta'_v(\omega) < 0$. The value of growth opportunities is less exposed to displacement, since firms' investment opportunities improve as a result of innovation. Hence, assuming the firm is in its steady state average ($A_f^v = 0$, $A_f^g = 0$), the firm's ratio of growth opportunities to firm value $PVGO/V$ is a primary determinant of the firm's exposure to the embodied shock: firms that derive larger fraction of their value from growth opportunities have higher loading on the innovation shock, as we illustrate in panel *a*.

However, the firm's ratio of growth opportunities to value, $PVGO/V$, is not a sufficient statistic for the firm's systematic risk. The firm's current profitability A_f^v and current investment opportunities, A_f^g , play a role. The timing of cash flows matters for risk exposures, and firms' idiosyncratic productivity shocks and their current growth state, $\tilde{\lambda}_{ft}$, are transient in nature. These firm-specific risk exposures are summarized by a firm specific exposure that depends on the deviation from the average productivity ($u = 1$) and growth state $p = \mu_H/(\mu_H + \mu_L)$. In panel *b*, we see that, holding the share of growth opportunities constant, more productive firms have higher exposure to innovation shocks. Last, in panel *c*, we see that firms with better current investment opportunities benefit disproportionately more from aggregate innovation, hence ceteris paribus, B_f is increasing in λ_{ft} . However, both of these effects are qualitatively minor.

Risk exposure of human capital

In our calibrated model human capital earns lower risk premium than financial capital. This lower risk premium results from the fact that labor income is positively correlated with the embodied shock. As we show in Figure 1, a positive innovation shock leads to an increase in the equilibrium wage and a decline in firm payouts and the level of financial wealth. This prediction is consistent with existing evidence. In recent work, Lustig and Van Nieuwerburgh (2008) and Lustig et al. (2008) document that returns to human wealth are lower than returns to financial wealth. Lustig et al. (2008) calculate the risk premium of financial and human wealth to be 3.77% and 2.17% respectively.

To facilitate a comparison with the results in Lustig and Van Nieuwerburgh (2008) and Lustig et al. (2008), we define human capital in the model as the present value of aggregate

labor income,

$$H_t = E_t \int_t^\infty \frac{\pi_s}{\pi_t} w_s ds. \quad (45)$$

The ratio of human capital to total wealth $H/(H + W)$ in our benchmark calibration is 83%, which is close to the 90% ratio reported in Lustig et al. (2008). Our model implies that the equilibrium risk premium on human capital is equal to 1.98%, compared to 4.11% for an unlevered claim on the stock market.

The role of imperfect risk sharing

Three features of our model are non-standard relative to standard RBC models: a) limited stock market participation by workers; b) limited intergenerational risk sharing; and c) preferences over relative consumption. In this section, we explore the quantitative effect of these features on the model’s predictions. In Table 10, we consider seven alternative specifications of the model, where we switch off one or two of these features and summarize the main properties of asset prices in the model.

We find that these three features have a minor effect on the behavior of aggregate quantities. The first two moments of consumption growth are similar across specifications. Further, the behavior of the risk-free rate and the volatility of stock returns are largely similar across all the specifications. The major differences across specifications are in the equilibrium prices of risk, which lead to different predictions for risk premia.

In columns “Alt 1” to “Alt 4” of Table 10, we summarize the key moments of the model without relative-consumption effects in preferences. The version of the model with full stock market participation and risk sharing across generations “Alt 1” produces a lower equity premium relative to the benchmark model, 3.8% vs 8.3%. More importantly, the average return on the value factor and its CAPM alpha are both negative and approximately equal to -8%. Comparing specifications “Alt 2” and “Alt 3” to “Alt 4”, we see that both limited stock market participation and limited intergenerational risk sharing are necessary to produce a positive value premium. However, in this specification, the CAPM works quite well, since the CAPM alpha of the value factor is close to zero.

As we see from column “Alt 5”, the relative-consumption feature of preferences does not by itself generate the main properties of asset prices in the model. Without limited risk sharing, the model with relative-consumption concerns produces a relatively low equity premium of 3.1% and a negative value premium, -3.8%. In columns “Alt 6” and “Alt 7”, we see that preferences over relative consumption magnify the effects of market incompleteness. As we see in the last column of Table 10, relative consumption preferences ensure that the

CAPM fails in the model. Investors’ desire to hedge changes in relative consumption leads to a version of the ICAPM (Merton, 1973).

In summary, the interaction of imperfect risk sharing and agents’ preferences is critical for the cross-sectional asset pricing implications of the model – the value premium. In contrast, even though the magnitude of the equity premium varies across specifications, it is consistently positive. We conclude that the cross-section of asset returns is informative about whether innovation leads to displacement of financial market participants.

5 Testing new empirical predictions

In this section we analyze the new testable predictions of the model that are directly tied to its core economic mechanism.

5.1 Constructing a proxy for the embodied shock

Our empirical analysis relies on an observable measure of the state variable ω that captures the state of real investment opportunities. We exploit the fact that, in the model, the total net present value of new projects scaled by the aggregate stock market wealth, is a strictly increasing function of the state variable ω ,

$$A_t \equiv \frac{1}{W_t} \int NPV_t dN_{ft} \propto b(\omega_t), \quad (46)$$

where $b(\omega_t) = b_{tt}$ is the share of wealth captured by new inventors (28). As we see in panel A of Figure 5, $\ln A_t$ is almost a linear function of the state variable ω in the model.

In constructing our empirical proxy for (46), we use patents as the empirical equivalent to the projects in our model economy. To assess their value, we use the methodology of Kogan et al. (2012), who construct an estimate of the dollar value of patents granted to public firms using their stock market reaction around the day that news of the patent issuance becomes public. First, we obtain a dollar measure of innovation at the firm level, A_{ft}^v , corresponding to the net present value of all new projects created by firm f in year t . Second, we aggregate across firms and scale by total market capitalization to obtain the empirical equivalent \hat{A}_t of (46). See Appendix B and Kogan et al. (2012) for more details on the empirical procedure.

To assess the effectiveness of the Kogan et al. (2012) procedure in the context of our model, we replicate the construction of \hat{A}_t in simulated data, defining the event day d as the time when a firm acquires a new project. As we see in panel B of Figure 5, the innovation measure \hat{A}_t is highly correlated with the state variable ω , both in levels (93.4%) and in first

differences (80.1%). In terms of the primitive technology shocks, changes in $\ln A_t$ in the model are primarily driven by the innovation shock ξ ; the median correlation between $\Delta \ln A$ and $\Delta \xi$ and Δx in simulated samples is 75.3% and 1.3% respectively.

We plot the aggregate innovation measure $\ln \hat{A}_t$ in panel *c* of Figure 5. We see that this measure of innovative activity lines up well with the three major waves of technological innovation: the 1930s, consistent with the views expressed in Field (2003); the 1960s and early 1970s – a period commonly recognized as a period of high innovation (e.g. Laitner and Stolyarov, 2003); and the 1990s and 2000s.

In Table 4, we compare the properties of the innovation measure in the data and in the model. First, we focus on the firm-level measure A^v , scaled by the firm’s market capitalization V . As we see in Panel A, both in the data and in the model, the cross-sectional distribution of A^v/V is highly skewed. Approximately half of the firms do not innovate, and most of the innovative activity is concentrated in the right tail of the distribution. In Panel B, we see that the relation between changes in the aggregate measure $\ln \hat{A}$ and the stock market (or Tobin’s Q) is negative and comparable in magnitude across the data and the model.

5.2 Innovation and consumption displacement

Here we test the model’s predictions about the relation between innovation and consumption.

Household-level evidence

Our model implies that the consumption of shareholders of cohort s , as a share of aggregate consumption, equals

$$\frac{C_{ts}}{C_t} = b(\omega_s) \exp \left(\mu(t-s) - \mu \int_s^t b(\omega_u) du \right) \tilde{l}(\omega_t). \quad (47)$$

We estimate the empirical equivalent of equation (47) using data on non-durable household consumption from the CEX. We define the household’s cohort as the year in which the head of the household turns 25. Varying this age by plus or minus two years leads to similar results. Absent measurement error, our innovation measure \hat{A} is linearly related to $b(\omega)$, hence we form the econometric specification by taking logs of both sides of (47)

$$\ln C_{its} - \ln C_t = \beta_0 \ln \hat{A}_s + \beta_1 \sum_{u=s+1}^{t-1} \hat{A}_u + \beta_2 \ln \hat{A}_t + a(t) + c(t-s) + c_2 Z_i + \varepsilon_{its} \quad (48)$$

where, i indexes households; t is the observation year; s is the cohort year; C denotes log non-durable consumption expenditures; $a(t)$ is a time trend; A is our innovation measure;

$c(t - s)$ is a quadratic term parameterizing household age effects; and Z_i is a vector of household-level controls including years of education and number of earning members. We cluster standard errors at the cohort level. We estimate (48) separately for stockholders and non-stockholders.⁶ We include a deterministic time trend to account for the secular trend in CEX data relative to aggregate consumption.

We focus on the coefficients β_0 , β_1 and β_2 . The estimate of β_0 captures the effect of innovation on the consumption of the entering cohort – corresponding to the term $b(\omega_s)$. Our model implies that the coefficient β_0 should be positive for stockholders. The estimate of β_1 captures the effect of displacement, which corresponds to the integral term inside the exponential in (47). A higher level of innovation results in the displacement of stockholders from earlier cohorts, hence our model predicts that β_1 should be negative for stockholders. Last, the coefficient β_2 captures both the displacement of the stockholders from cohort s by the time- t entrants and the contemporaneous consumption distribution between the workers and the owners of capital. In the model, higher recent innovation results in a higher consumption share of the workers. Thus, our model predicts that β_2 should be negative for stockholders and positive for non-stockholders.

The results in Panel I of Table 5 are consistent with the model. The coefficient β_0 is positive and statistically significant across specifications for both stockholders and non-stockholders, suggesting that the level of technological innovation at the time households enters the market has a lasting positive impact on their lifetime consumption. Consistent with our model, the coefficient β_1 is negative and statistically significant for stockholders, and statistically insignificant for non-stockholders. Hence, our results imply that existing generations of stockholders get displaced by subsequent innovation activity, while there is no corresponding effect for non-stockholders. Last, the coefficient β_2 is positive and statistically significant for non-stockholders, but not significant for the stockholders.

As a robustness test, we repeat the exercise but we normalize by the mean consumption level of stockholders in the CEX, rather than aggregate consumption. As we see in Panel II of Table 5, results are similar. Relative to the total consumption of stockholders, consumption of the stockholders from cohort s is positively affected by the innovation at the time of their entry and negatively affected by subsequent innovation activity.

⁶We define stockholders as households that report owning stocks, bonds or mutual funds. Since many households often do not report their bond and stock holdings in their retirement accounts, restricting the sample in this way is a conservative way of restricting the sample to stockholders.

Aggregate evidence

Here, we provide further supporting evidence using time series data on the consumption growth rate of stockholders c^S and non-stockholders c^{NS} . We estimate the following specification,

$$(c_{t+k}^S - c_t^S) - (c_{t+k}^{NS} - c_t^{NS}) = a + \beta(T)\Delta \ln A_t + \varepsilon_{tT}, \quad (49)$$

where we study horizons from $k = 1$ to $k = 4$ years. We use the series constructed in Malloy et al. (2009), which covers the 1982-2004 period.⁷ We compute Newey-West adjusted standard errors in (49), setting the maximum number of lags equal to 3 plus the number of overlapping years.

We show the empirical results in Panel A of Table 6. We find a negative relation between our innovation measure and the consumption growth rate of stockholders relative to non-stockholders. Despite the short length of the sample, the relation is statistically significant at the 10% level at the one to three year horizon. To assess the economic magnitude of the empirical estimates, we replicate the same procedure in simulated data. As we see in Panel B, the empirical magnitudes are consistent with our calibrated model.

5.3 Innovation and firm displacement

Here, we test the prediction of the model that firms with few growth opportunities are more vulnerable to displacement than firms with high growth opportunities.

Output

First, we show that consistent with the model, firms with high growth opportunities are less subject to displacement by innovation activity of their competitors.⁸ To test this prediction, we study the response of firm output – sales plus change in inventories – to the firm’s own innovation activity, A_f , and the innovation activity of its competitors, A_{If} ,

$$\ln y_{ft+k} - \ln y_{ft} = a_0 + a_1 A_{ft} + a_2 A_{If,t} + a_3 A_{If,t} D(G_{ft})_H + b Z_{ft} + e_{t+k}, \quad (50)$$

⁷We follow Jagannathan and Wang (2007) and construct annual consumption growth rates by using end-of-period consumption. In particular, we focus on the sample of households that are interviewed in December of every year, and use the average 4 to 16 quarter consumption growth rate of non-stockholders, stockholders and top-stockholders, defined as in Malloy et al. (2009). Our results remain quantitatively similar when we instead construct annual growth rates by an equal-weighted average of the k -period consumption growth of all households interviewed in year t .

⁸In the model, a positive innovation shock ξ leads to an increase in the total production of the intermediate good Y , and therefore a reduction in its price p_Y . Firms that did not innovate and thus extended their production capacity will experience a reduction in sales. In the medium run, firms with high growth opportunities are less sensitive to this displacement effect because they are likely to acquire projects.

where y is firm output; $A_{ft} \equiv A_{ft}^v/V_{ft}$ is innovation by the firm, scaled by its market capitalization; A_{If_t} is a value-weighted average of innovation A_{ft} by the firm’s competitors (other firms in the same 3-digit SIC industry); $D(G)_H$ is a dummy variable taking the value 1 if the firm is ranked higher than the industry median in terms of growth opportunities – proxied either by Tobin’s Q or by the investment rate. The vector of controls Z includes industry effects; time effects; firm size; and lagged output growth. We examine horizons of $k = 1$ to $k = 7$ years. To facilitate comparison between the data and the model, we scale the variables A_f and A_{If} to unit standard deviation. We cluster the standard errors by year.

As we see in Panel A of Table 7, innovation by competitors leads to displacement of firms with low growth opportunities – measured either using Tobin’s Q or past investment – as evidenced by the negative estimate of a_2 . In contrast, the interaction effect a_3 is positive, implying that firms with above-median growth opportunities are displaced less. This difference in displacement is economically meaningful. A one-standard deviation increase in the amount of innovation by firm’s competitors is associated with a 4.4-4.6% drop in output over the next seven years for the firm’s that are below the median industry in terms of growth opportunities, compared to 2.6-3.3% for the firms above the median. To assess the empirical magnitudes in the context of our calibration, we replicate the analysis in simulated data from the model. As we see in Panel B, the empirical magnitudes are close to those implied by the model.

Return exposure

Next, we show that differences in firm characteristics related to growth opportunities are related to differences in firms’ exposures to the aggregate innovation shock. Using our empirical measure of innovation A we study whether portfolios of stocks, sorted on either their past investment rate, or their book-to-market ratio, have differential exposure to our innovation measure \hat{A} , controlling for their market exposure

$$R_{pt} - r_{ft} = a_p + \beta_p (R_{mt} - r_{ft}) + \gamma_p \Delta \ln \hat{A}_t + \varepsilon_{pt} \quad (51)$$

As we see in panel A of Table 8, firms with high (low) growth opportunities have positive (negative) stock return exposure to innovation shocks $\Delta \ln A_t$, controlling for excess returns to the stock market, $R_{mt} - r_{ft}$. The empirical magnitudes are comparable to the magnitudes in simulated data, as we see in panel B.

5.4 Asset pricing tests

Having established that firms with different growth opportunities have differential return exposure to our empirical proxy for the embodied shock, we use the cross section of asset returns to test the model-implied relation between innovation and investors' intertemporal rates of substitution or stochastic discount factor (SDF). The SDF implied by the model (A.28) is not available in analytic form, hence we estimate a linearized version

$$m = a - \gamma_x \Delta x - \gamma_\xi \Delta \xi. \quad (52)$$

We proxy for the innovation shock $\Delta \xi$ by changes in our log innovation measure $\Delta \ln \hat{A}$. We proxy for the disembodied technology shock x by the change in the (log) total factor productivity from Basu, Fernald, and Kimball (2006). In addition, since the disembodied shock x accounts for most of the short-run variation in aggregate consumption growth, we test an alternative version of the model where, we replace Δx by aggregate consumption growth.

We estimate (52) using the generalized method of moments (GMM), using the model pricing errors as moment restrictions (Hansen and Singleton, 1982, 1983).⁹ As test assets, we use deciles 1,2, 9 and 10 from the book-to-market and investment rate portfolios. We report first-stage GMM estimates using the identity matrix to weigh moment restrictions, and adjust the standard errors using the Newey-West procedure with a maximum of three lags. As a measure of fit, we report the cross-sectional R^2 and the mean absolute pricing errors. We replicate the same procedure in simulated data.

The specifications of the stochastic discount factor without the innovation shock result in large pricing errors, both in the data and in the model. As we see in columns A1 and A2 of Table 9, differences in exposure with either total factor productivity or consumption growth are not related to differences in risk premia across portfolios. In Panel B, we show that the same pattern holds in simulated data. In particular, column B2 shows that the consumption CAPM does not hold in the model, since there is no relation between return exposures to aggregate consumption growth and risk premia.

Specifications of the SDF with the innovation measure $\Delta \ln \hat{A}$ do well in pricing these portfolios, as we see in columns A3 and A4. The price of risk associated with innovation ranges from -0.83 to -1.03 and is statistically significant at the 1% level. These estimated

⁹We impose that the SDF in equation (52) should price the cross-section of test asset returns in excess of the risk-free rate. Hence, the mean of the stochastic discount factor is not identified. Without loss of generality, we choose the normalization $E(m) = 1$, which leads to the moment restrictions $E[R_i^e] = -cov(m, R_i^e)$, where R_i^e denotes the excess return of portfolio i over the risk-free rate (see Cochrane, 2005, pages 256-258 for details.)

risk prices are very close to those implied by our calibrated model in columns B3 and B4, which range from -1.01 to -1.15 across specifications. Last, both in the data and in the model, the estimated price of the disembodied shock is γ_x is not statistically different from zero, likely due to the lack of dispersion in x -shock exposures across portfolios. We conclude that the relation between embodied shocks and intertemporal rates of substitution implied by our calibrated model is quantitatively consistent with the data.

6 Conclusion

We develop a general equilibrium model to study the effects of innovation on asset returns. A distinguishing feature of innovation is that its benefits are not shared symmetrically across all agents in the economy. Hence, focusing on aggregate moments obscures the effects of innovation in the cross-section of both firms and households. Specifically, technological improvements embodied in new capital benefit workers employed in their production, while displacing existing firms and their shareholders. This displacement process is uneven for two reasons. First, newer generations of shareholders benefit at the expense of existing cohorts. Second, firms well-positioned to take advantage of these opportunities benefit at the expense of firms unable to do so. Existing shareholders value firms rich in growth opportunities despite their low average returns, as they provide insurance against displacement.

Our model delivers rich cross-sectional implications about the effect of innovation on the cross-section of firms and households that are supported by the data. We test the model's predictions using a direct measure of innovation constructed by Kogan et al. (2012) using data on patents and stock returns. Consistent with our model, we find that innovation is associated with a reallocation of wealth from existing shareholders to workers and future generations.

Our work suggests several avenues for future research. Quantifying the impact of wealth reallocation associated with innovation on inequality among households is a promising direction for future work, especially given the availability of a direct measure of technology. Another important topic to consider would be the role of government policies in mitigating intergenerational displacement. Last, we only focus on one particular type of innovation, that is technological change embodied in new capital. Analyzing the effects of more general types of embodied technical change on financial markets and macroeconomy is potentially fruitful.

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Tables

Table 1: Calibration moments

A. Aggregate Quantities	Model	Data
Consumption growth, aggregate; mean* (%)	1.7	1.9
Consumption growth, aggregate; standard deviation (%)	3.0	2.8
Consumption growth, aggregate; serial correlation (%)	24.5	41.2
Consumption growth, stockholders; standard deviation* (%)	3.7	3.6
Dividend growth; standard deviation (%)	4.86	11.49
Labor Income growth; standard deviation (%)	3.2	1.9
Investment growth; standard deviation* (%)	11.6	14.8
Investment to capital; mean* (%)	8.5	8.7
Labor Share; mean* (%)	70.8	68.5
Labor Share; standard deviation (%)	1.8	1.6
First difference of consumption cohort effect, standard deviation* (%)	8.8	8.5
Correlation between consumption and dividend growth (%)	50.6	55.0
Correlation between consumption and investment (%)	45.0	44.1
Correlation between consumption and stock market (%)	44.7	18.4
B. Asset returns	Model	Data
Market portfolio, excess returns; mean* (%)	8.0	7.6
Market portfolio, excess returns; standard deviation (%)	12.0	18.5
Risk-free rate; mean (%)	2.8	0.9-2.9
Risk-free rate; standard deviation* (%)	0.7	0.9
C. Firm-level variables	Model	Data
Investment rate, IQR-to-Median*	1.19	1.21
Investment rate, serial correlation* (%)	45.3	47.8
Investment rate > 1, fraction of firm-year obs* (%)	1.3	1.4
Tobin's Q , IQR-to-Median	0.89	1.27
Tobin's Q , serial correlation (%)	90.4	79.5
Correlation between investment and lagged Tobin's Q * (%)	22.9	23.7
Output to Assets, IQR-to-Median*	0.55	0.80
Output to Assets, serial correlation* (%)	88.0	92.3

Starred moments are targeted in our calibration. Investment, capital and consumption data are from NIPA over the 1926-2010 period; investment is non-residential private domestic investment; stock of capital is current-cost from the NIPA Fixed Assets Table; consumption is non-durables plus services; nominal variables are deflated by population and the CPI. Population is from the Census. Moments of shareholder consumption growth are from the unpublished working paper version of Malloy et al. (2009) and includes adjustment for measurement error. Correlation between dividends and consumption are from Bansal and Yaron (2004). Moments of labor income are from Lustig et al. (2008). The volatility of consumption cohorts is computed as in Garleanu et al. (2012), but we restrict the sample to households that are shareholders. Stock market data are from CRSP. Firm-level accounting data are from Compustat. Labor share is constructed from Flow of Funds data following Sekyu and Rios-Rull (2009). The moments of the real risk-free rate are from Campbell and Cochrane (1999) and Bansal and Yaron (2004); the range refers to the pre- versus post-war sample.

Table 2: Cross-section of expected returns

A. Data							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	6.45 (2.32)	6.98 (3.18)	7.62 (3.33)	11.29 (4.00)	11.38 (4.22)	12.83 (3.90)	6.38 (2.46)
$\sigma(\%)$	21.38	17.64	17.74	22.01	21.38	25.74	20.43
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	10.13 (3.42)	8.20 (3.24)	8.32 (4.03)	7.19 (2.68)	7.86 (2.40)	4.87 (1.26)	-5.26 (-2.05)
$\sigma(\%)$	23.52	20.19	17.37	21.01	25.50	28.93	17.11
B. Model							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	4.15 (3.61)	5.53 (4.78)	6.57 (5.48)	9.04 (6.67)	9.31 (6.74)	10.01 (6.96)	5.86 (7.74)
$\sigma(\%)$	10.39	10.56	10.99	12.29	12.53	12.99	6.36
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$E(R) - r_f(\%)$	8.67 (6.13)	7.28 (5.57)	6.16 (4.96)	5.97 (4.45)	4.86 (3.74)	2.79 (2.21)	-5.92 (-11.49)
$\sigma(\%)$	12.83	11.80	11.26	12.04	11.66	11.12	4.61

Table shows excess returns and standard deviation for portfolios sorted on two measures of growth opportunities: book-to-market and past investment. Data is from CRSP/Compustat. Book to market is book value of common equity divided by CRSP market capitalization in December. Investment rate is growth in property-pant and equipment. Data period is 1950-2008. We form portfolios in June every year. We exclude financial firms (SIC6000-6799), and utilities (SIC4900-4949). When computing investment rates and book to market in simulated data, we measure the book value of capital as the historical cost of firm's capital $\sum_{\mathcal{J}_{ft}} k_j q_{\tau(j)}$ ($\tau(j)$ denotes the time of creation of project j) divided by its current market value V_{ft} .

Table 3: The failure of the CAPM

A. Data							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
α	-1.51 (-1.38)	0.11 (0.16)	0.87 (1.21)	3.74 (2.44)	4.03 (2.92)	4.35 (2.20)	5.86 (2.07)
β_{mkt}	1.08 (21.65)	0.92 (22.91)	0.95 (29.54)	1.03 (8.95)	0.99 (10.08)	1.17 (11.14)	0.09 (0.64)
R^2	83.89	90.53	93.09	74.97	73.79	70.80	0.66
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
α	2.64 (1.85)	1.48 (1.48)	2.75 (2.39)	0.10 (0.12)	-0.60 (-0.47)	-4.45 (-2.29)	-7.09 (-2.97)
β_{mkt}	1.12 (16.38)	1.00 (17.79)	0.83 (13.39)	1.06 (21.26)	1.26 (18.68)	1.39 (16.5)	0.27 (2.79)
R^2	77.72	84.86	79.10	87.27	84.41	79.59	8.77
B. Model							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
α	-2.33 (-4.19)	-1.44 (-3.36)	-0.77 (-2.29)	0.60 (3.44)	0.74 (4.00)	1.26 (5.17)	3.55 (5.17)
β_{mkt}	0.79 (21.25)	0.84 (28.23)	0.89 (38.66)	1.02 (79.78)	1.04 (76.16)	1.07 (61.12)	0.28 (5.74)
R^2	84.99	91.43	95.13	95.80	93.65	88.08	29.46
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
α	2.01 (5.17)	1.05 (3.56)	0.25 (0.93)	-0.43 (-1.66)	-1.30 (-5.47)	-3.03 (-9.62)	-5.01 (-9.34)
β_{mkt}	1.07 (36.53)	1.00 (44.65)	0.95 (42.45)	1.02 (51.30)	0.99 (53.62)	0.94 (37.58)	-0.14 (-3.40)
R^2	92.84	94.72	94.71	92.71	91.95	83.67	12.43

Table shows excess returns and standard deviation for portfolios sorted on two measures of growth opportunities: book-to-market and past investment. See notes to Table 2 for details of the portfolio construction. Data for the market portfolio and risk-free rate are from Kenneth French's data library.

Table 4: Descriptive statistics of innovation measure

A. Moments of firm-level measure – A^v/V		
	Model	Data
Mean	0.044	0.029
Standard deviation	0.129	0.057
50-percentile	0.000	0.000
75-percentile	0.024	0.012
90-percentile	0.129	0.113
95-percentile	0.250	0.145
99-percentile	0.623	0.256
B. Moments of aggregate measure – $\Delta \ln \hat{A}$		
	Model	Data
Standard deviation	8.57	33.22
Correlation with market excess returns	-60.12	-55.40
Correlation with changes in Tobin's Q	-73.21	-57.63

Table compares descriptive statistics for our firm-level and aggregate innovation measure in the model and in the data; A_{ft}^v refers to the dollar value of innovation generated by firm f in year t ; V refers to stock market capitalization; \hat{A} refers to the aggregate innovation measure. See Appendix B and Kogan et al. (2012) for details. Sample period is 1950-2008.

Table 5: Innovation and consumption displacement

A. Stockholders				
$c_{its} - \bar{c}_t$	I. Relative to total consumption		II. Relative to group mean	
	$\ln A_s$	0.1600 (3.00)	0.0284 (2.22)	0.1613 (3.25)
$\sum_{u=s+1}^{t-1} A_u$	-0.0606 (-3.51)	-0.0588 (-2.36)	-0.0597 (-3.76)	-0.0374 (-2.18)
$\ln A_t$	0.0357 (1.60)	0.0138 (0.91)		
R^2	0.128	0.265	0.052	0.185
Observations	13787	12305	13787	12305
B. Non Stockholders				
$c_{its} - \bar{c}_t$	I. Relative to total consumption		II. Relative to group mean	
	$\ln A_s$	0.1640 (2.75)	0.0236 (2.22)	0.1834 (3.32)
$\sum_{u=s+1}^{t-1} A_u$	-0.1311 (-7.58)	-0.0023 (-0.13)	-0.1255 (-7.83)	0.0086 (0.60)
$\ln A_t$	0.0769 (3.57)	0.0344 (3.07)		
R^2	0.208	0.317	0.132	0.226
Observations	36050	29191	36050	29191
Time Trend	Y	Y	-	-
Household controls	-	Y	-	Y

Table reports results of relating our innovation measure A to household consumption data (see equation (48) in main text). Household-level consumption data are from the CEX family-level extracts by Harris and Sabelhaus (2000), available through the NBER website. Data covers the period 1980-2003. See Kogan et al. (2012) for details on the construction of A . Consumption is non-durables, defined as in Harris and Sabelhaus (2000). Stockholders are classified as households reporting ownership of stocks, bonds or mutual funds. Cohort age s is defined as the age the household turns 25. In panel I we normalize household consumption by per-capital aggregate consumption of non-durables. In Panel II we normalize by group (stockholder versus non-stockholder) means. Depending on the specification, we include a vector of household controls which contains: linear and quadratic age effects; number of earning members; years of education. All specifications in Panel I include a time trend to control for the secular trend in the CEX dataset. Standard errors are clustered by cohort.

Table 6: Innovation and stockholder consumption growth

A. Data				
$(c_{t+T}^S - c_t^S) - (c_{t+T}^{NS} - c_t^{NS})$	T=1	T=2	T=3	T=4
$\Delta \ln \hat{A}_t$	-0.013 (-1.76)	-0.026 (-2.28)	-0.025 (-1.74)	-0.029 (-1.09)
R^2	0.075	0.101	0.067	0.070
B. Model				
$(c_{t+T}^S - c_t^S) - (c_{t+T}^{NS} - c_t^{NS})$	T=1	T=2	T=3	T=4
$\Delta \ln \hat{A}_t$	-0.022 (-2.14)	-0.018 (-1.73)	-0.017 (-1.23)	-0.013 (-0.98)
R^2	13.36	3.28	2.23	1.97

Table reports results of relating our innovation measure A to the differential growth rate of stockholders vs non-stockholders $(c_{t+T}^S - c_t^S) - (c_{t+T}^{NS} - c_t^{NS})$ (see equation (49) in main text) in the data (Panel A) and the model (Panel B). Sample period is 1980-2004. Standard errors are computed using Newey-West with T+1 lags. We standardize right-hand side variables to unit standard deviation.

Table 7: Innovation and firm displacement

A. Data							
$y_{t+T} - y_t$	T=1	T=2	T=3	T=4	T=5	T=6	T=7
i. Tobin's Q							
A_{Ift}	-0.022 (-3.84)	-0.026 (-2.21)	-0.034 (-2.92)	-0.035 (-2.66)	-0.036 (-2.62)	-0.038 (-2.67)	-0.044 (-3.10)
$A_{Ift} \times D(Q_{ft})_H$	0.010 (2.79)	0.010 (3.01)	0.019 (6.23)	0.018 (4.04)	0.013 (2.59)	0.015 (3.21)	0.019 (2.54)
ii. Investment rate							
A_{Ift}	-0.020 (-3.61)	-0.026 (-2.59)	-0.032 (-2.68)	-0.034 (-2.26)	-0.039 (-2.33)	-0.041 (-2.45)	-0.046 (-2.73)
$A_{Ift} \times D(IK_{ft})_H$	0.003 (1.28)	0.006 (1.63)	0.009 (2.92)	0.008 (1.98)	0.010 (1.89)	0.011 (2.05)	0.013 (2.74)
B. Model							
$y_{t+T} - y_t$	T=1	T=2	T=3	T=4	T=5	T=6	T=7
i. Tobin's Q							
A_{Ift}	-0.007 (-2.19)	-0.012 (-2.30)	-0.016 (-2.50)	-0.020 (-2.66)	-0.024 (-2.83)	-0.027 (-2.90)	-0.029 (-2.93)
$A_{Ift} \times D(Q_{ft})_H$	0.002 (2.38)	0.005 (4.25)	0.009 (5.58)	0.013 (6.37)	0.017 (6.69)	0.021 (6.77)	0.024 (6.68)
ii. Investment rate							
A_{Ift}	-0.012 (-3.58)	-0.018 (-3.47)	-0.022 (-3.42)	-0.025 (-3.31)	-0.028 (-3.27)	-0.029 (-3.17)	-0.030 (-3.06)
$A_{Ift} \times D(I_{ft})_H$	0.010 (12.14)	0.017 (10.33)	0.021 (9.22)	0.023 (8.40)	0.025 (7.82)	0.026 (7.36)	0.026 (7.05)

Table presents results on the differential rate of firm displacement following innovation by competitors (A_{Ift}) depending on the firm's measure of growth opportunities (Tobin's Q or past investment rate). We estimate equation (50) in the data (Panel A) and in simulated data from the model (Panel B). Sample period is 1950-2008. Accounting data are from Compustat; investment rate is growth rate in property, plant and equipment (ppeg); Tobin's Q is CRSP market capitalization, plus book value of debt (dltt), plus book value of preferred shares (pstkrv), minus deferred taxes (txdb) divided by book assets (at); output y is sales (sale) plus change in inventories (invt). We include a vector of controls Z containing industry effects; time effects; firm size; lagged output growth; and firm and industry stock returns. We cluster the standard errors by year. We scale the variables A_f and A_{Ift} to unit 90-50 range and unit standard deviation respectively.

Table 8: Innovation and return comovement

A. Data							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	0.17 (4.49)	0.03 (0.84)	-0.03 (-1.35)	-0.08 (-1.07)	-0.12 (-2.14)	-0.22 (-3.91)	-0.39 (-4.74)
$R_{mt} - r_f$	1.21 (23.61)	0.94 (21.98)	0.92 (27.87)	0.97 (7.33)	0.90 (7.86)	0.99 (9.24)	-0.22 (-1.58)
R^2	86.87	90.65	93.23	75.61	75.46	74.40	18.69
B. Model							
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	-0.13 (-2.35)	-0.05 (-1.44)	-0.04 (-1.01)	0.09 (2.03)	0.17 (3.55)	0.12 (1.75)	0.25 (3.17)
$R_{mt} - r_f$	1.02 (14.07)	0.96 (15.03)	0.80 (13.48)	1.13 (15.54)	1.40 (17.26)	1.49 (13.95)	0.47 (4.33)
R^2	79.15	85.22	79.42	88.18	86.66	80.46	19.16
B/M sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	0.43 (5.74)	0.31 (5.32)	0.20 (4.18)	-0.07 (-2.54)	-0.11 (-3.57)	-0.17 (-4.75)	-0.61 (-6.27)
$R_{mt} - r_f$	0.96 (22.83)	0.96 (29.18)	0.97 (36.59)	0.99 (62.88)	1.00 (60.24)	1.01 (49.89)	0.05 (0.92)
R^2	89.50	93.68	96.07	98.89	98.85	98.49	54.33
I/K sort	Lo	2	3	8	9	Hi	Hi-Lo
$\Delta \ln A_t$	-0.17 (-5.00)	-0.07 (-2.15)	0.02 (0.62)	-0.06 (-2.07)	0.03 (1.04)	0.21 (4.59)	0.38 (5.75)
$R_{mt} - r_f$	0.99 (51.11)	0.99 (54.84)	0.98 (52.80)	1.00 (60.31)	1.00 (58.34)	0.99 (40.34)	-0.00 (-0.01)
R^2	98.52	98.65	98.48	98.87	98.72	97.05	44.98

Table relates our innovation measure A to stock returns of portfolios sorted on book to market (Part I) and past investment (Part II). We estimate equation (51) in the data (Panel A) and in the model (Panel B). Sample period is 1950-2008. See notes to Table 2 for details on portfolio construction. See main text and Kogan et al. (2012) for details on the construction of A . Standard errors are computed using Newey-West with 3 lags.

Table 9: Asset pricing tests

4 B/M + 4 I/K portfolios								
Factor	A. Data				B. Model			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$\Delta \ln X_t$	3.19 [3.99]		-0.54 [-1.08]		0.69 [4.22]		-0.01 [-0.76]	
$\Delta \ln C_t$		1.97 [3.70]		-0.92 [-1.02]		0.88 [4.36]		0.11 [0.89]
$\Delta \ln A$			-0.83 [-3.64]	-1.03 [-3.88]			-1.01 [-6.90]	-1.15 [-6.52]
R^2	-110.61	-63.68	64.70	74.61	-28.79	-8.57	82.77	81.96
MAPE	3.10	2.53	1.06	1.00	2.86	2.45	0.86	0.87

Table presents results of estimating the stochastic discount factor implied by the model (equation (52) in main text) in the data (Panel A) and in simulated data from the model (Panel B). Sample period is 1950-2008. See notes to Table 2 for details on portfolio construction. See main text and Kogan et al. (2012) for details on the construction of innovation measure A . Total factor productivity X is from Basu et al. (2006). Consumption C is non-durables plus services from NIPA, deflated by CPI and population growth. Standard errors are computed using Newey-West with 3 lags.

Table 10: Comparative statics

Calibration	(Alt 1)	(Alt 2)	(Alt 3)	(Alt 4)	(Alt 5)	(Alt 6)	(Alt 7)	(Bench)
Limited stock market participation	-	-	Y	Y	-	-	Y	Y
Limited inter-generational risk sharing	-	Y	-	Y	-	Y	-	Y
Relative consumption preferences	-	-	-	-	Y	Y	Y	Y
Consumption growth, mean	1.75	1.74	1.73	1.72	1.73	1.72	1.72	1.71
Consumption growth, volatility	3.13	3.07	3.01	3.01	3.02	3.02	3.01	3.02
Market portfolio, mean excess return	3.75	8.04	4.07	12.57	3.10	6.99	4.65	8.07
Market portfolio, volatility	10.63	13.50	16.59	13.79	10.03	11.27	12.75	11.98
Risk-free rate, mean	0.78	0.16	1.61	0.11	2.24	1.97	1.89	2.82
Risk-free rate, volatility	2.03	1.60	2.49	1.12	1.09	0.78	1.62	0.71
Value factor, mean	-7.79	-4.80	-3.66	2.92	-3.77	-1.04	-1.09	5.88
Value factor, CAPM alpha	-8.57	-8.20	-4.34	-0.47	-2.94	-3.84	-1.97	3.54
Average market price of dB_{xt}	2.12	2.04	1.97	1.57	1.10	1.01	0.86	0.51
Average market price of $dB_{\xi t}$	0.53	0.37	0.24	-0.49	0.30	0.13	-0.12	-0.78

Table presents comparative statics with respect to three elements of our model. Depending on the column, Alt1-Alt7, we allow for full stock market participation by workers, full intergenerational risk-sharing ($b(\omega) = 1$) and set the preference weight on relative consumption to zero ($h = 0$). In each case, we present median moment estimates across 100 simulations. Value factor refers to the difference in realized returns between the top and bottom book-to-market decile portfolio.

Figure 1: Response to embodied shock

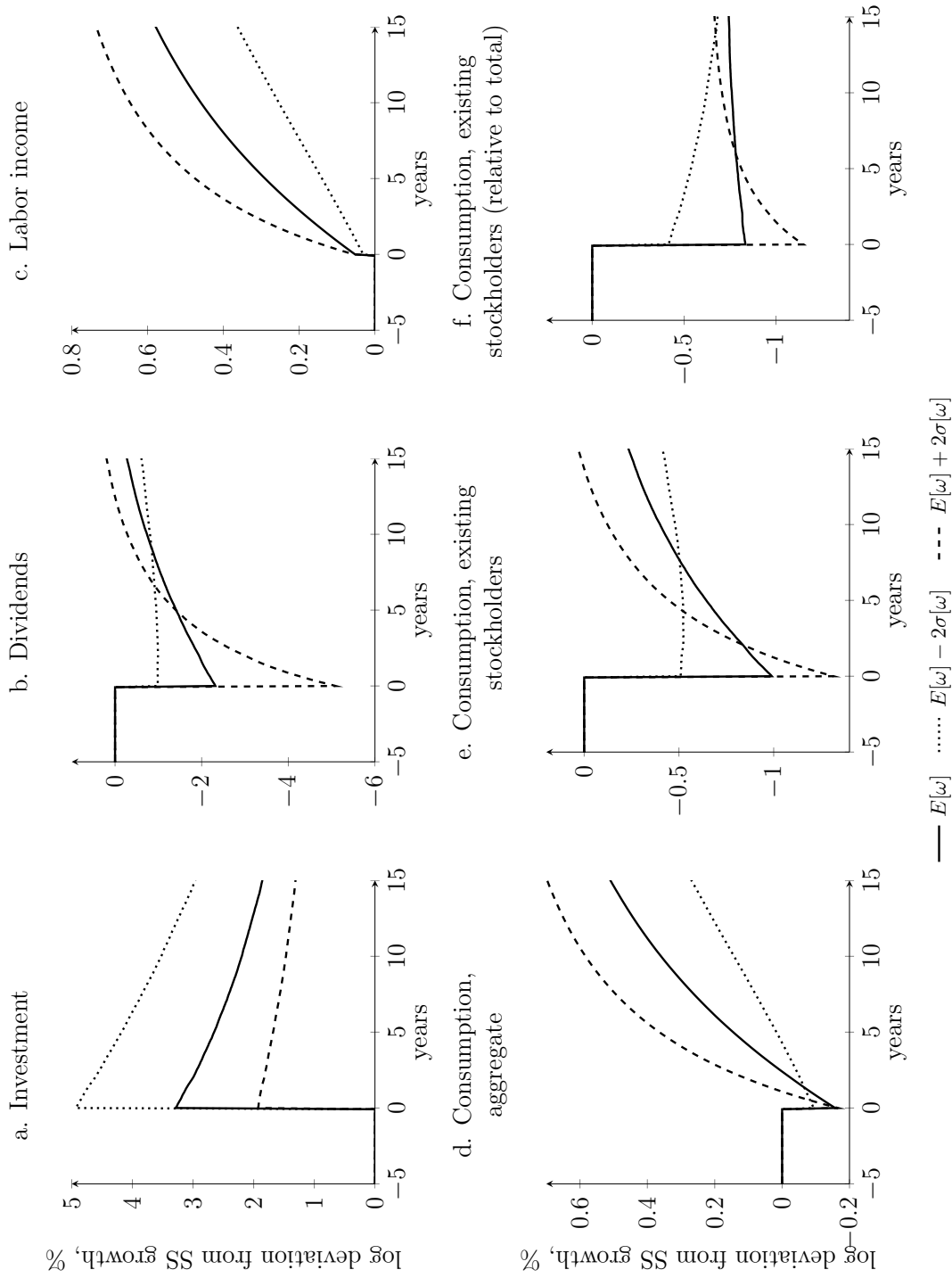


Figure plots the impulse response of quantities to a one-standard deviation increase in the level of the embodied shock ξ . We compute impulse responses by first fixing a sequence of stochastic shocks; we then perturb that sequence by a $\sigma_\xi \sqrt{\Delta t}$ positive shock to $\Delta \xi$ at $t = 0$. We do the perturbation at the mean of the ergodic distribution of ω , as well as at plus/minus two standard deviations. We simulate at monthly frequencies, $dt = 1/12$. We repeat the process 1,000 times, and average the impulse response across simulations.

Figure 2: Response to disembodied shock

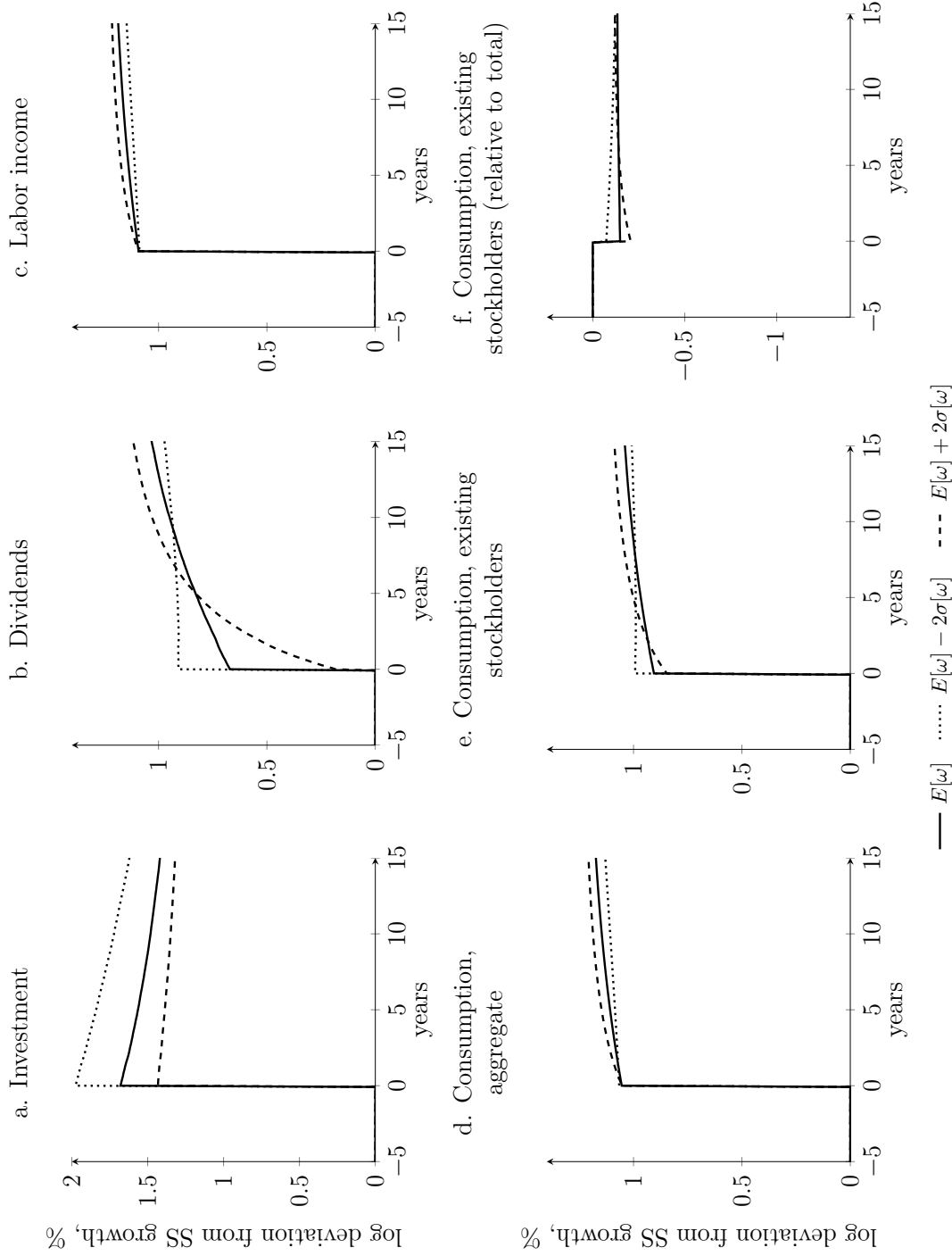


Figure plots the impulse response of quantities to a one-standard deviation increase in the level of the disembodied shock x . We compute impulse responses by first fixing a sequence of stochastic shocks; we then perturb that sequence by a $\sigma_x \sqrt{\Delta t}$ positive shock to Δx at $t = 0$. We do the perturbation at the mean of the ergodic distribution of ω , as well as at plus/minus two standard deviations. We simulate at monthly frequencies, $dt = 1/12$. We repeat the process 1,000 times, and average the impulse response across simulations.

Figure 3: Model Solution

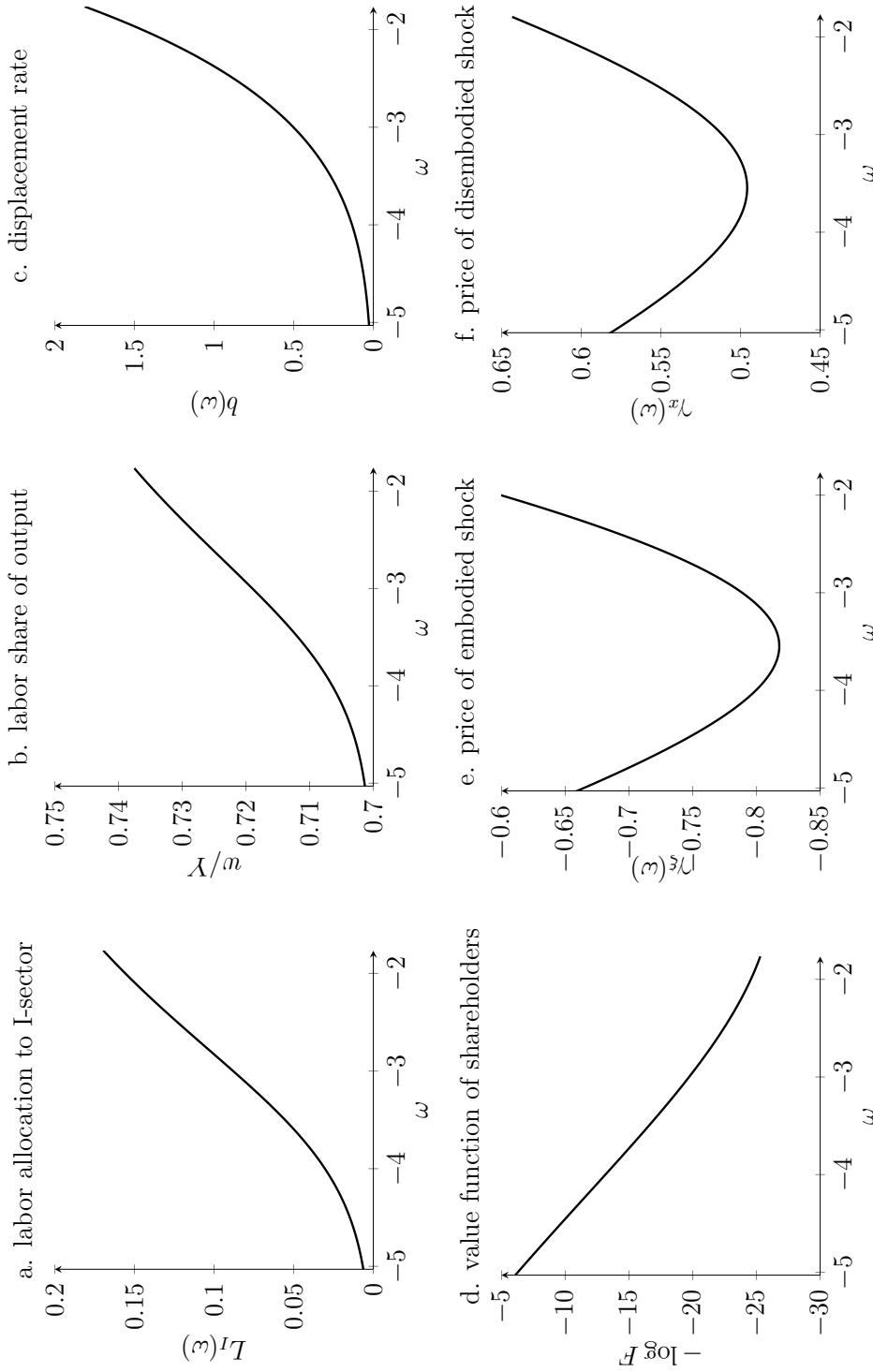


Figure plots the numerical solution to the model as a function of the state variable ω . We plot the solution in the relevant range of ω based on its stationary distribution.

Figure 4: Firm return sensitivity to innovation shock

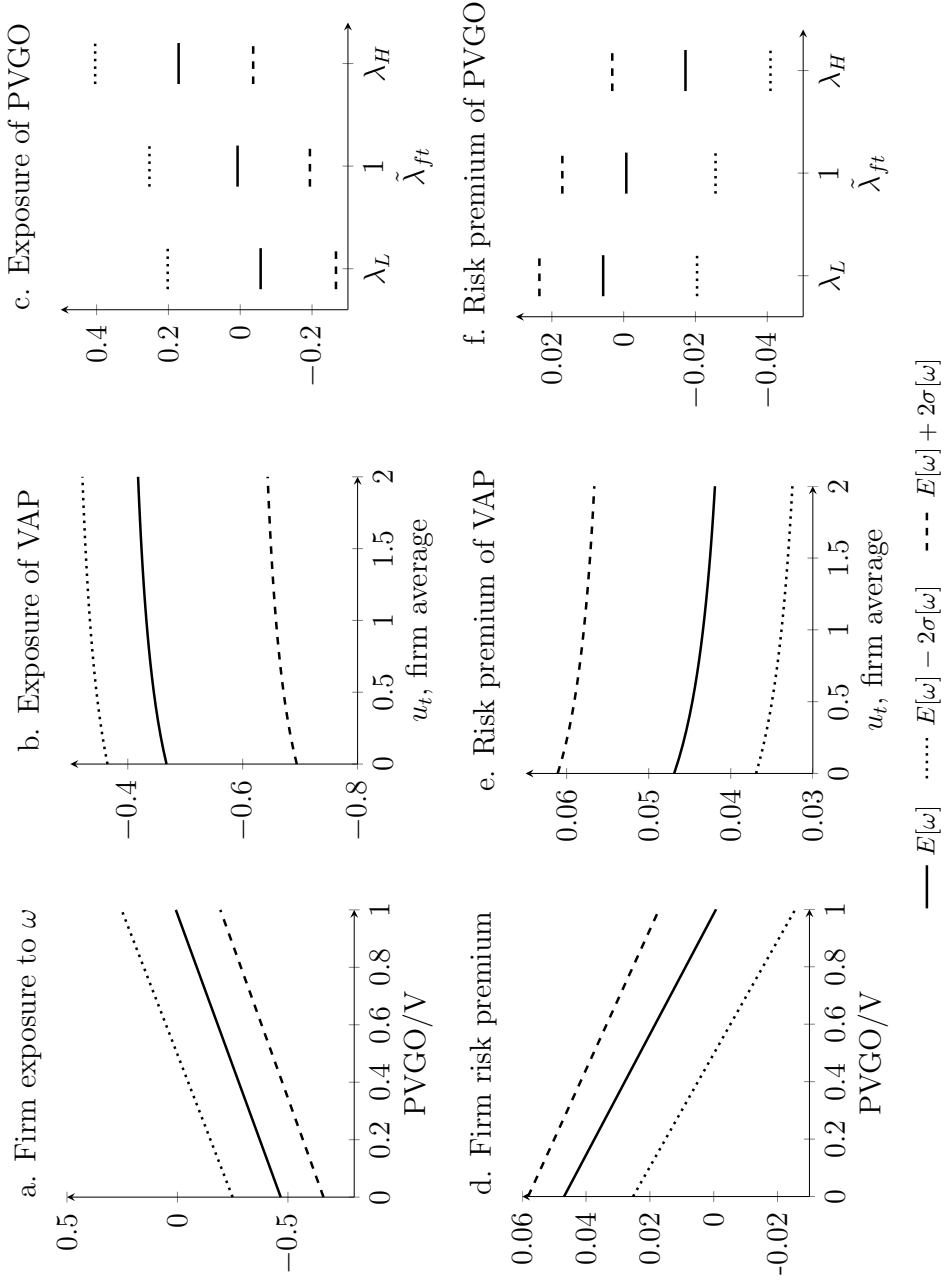
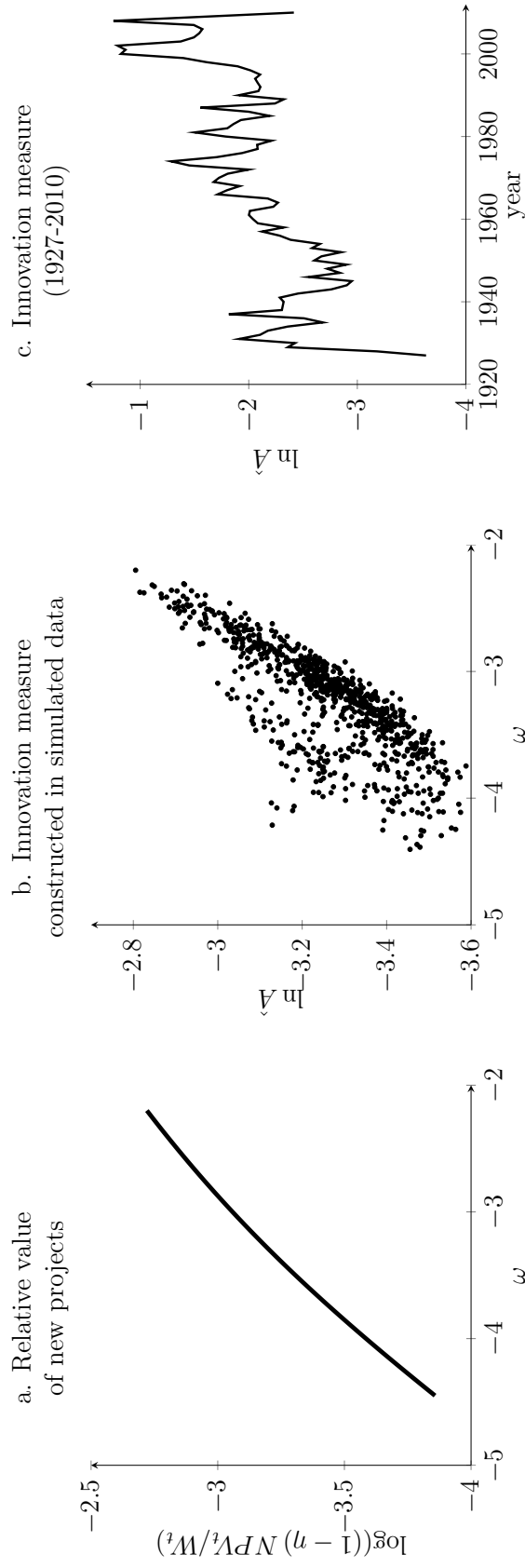


Figure plots the exposure of the firm to changes in the state variable ω as a function of its characteristics. We plot the sensitivity of firm value to ω as a function of the share of growth opportunities to firm value (panel a), as a function of the share of growth opportunities to firm value $PVGO/V$ assuming that the firm is in its steady state: ($A_f^v = 0, A_f^g = 0$); the sensitivity of assets in place as a function of the average project-specific shock at the firm level, u (panel b); and the sensitivity of growth opportunities as a function of the current growth state of the firm (panel c). In panels d to f we compute conditional risk premia by multiply firms' conditional risk exposures to x and ξ with the conditional market prices of risk $\gamma_x(\omega)$ and $\gamma_\xi(\omega)$.

Figure 5: Measuring innovation



In panel *a* we plot the value of new projects, relative to the stock market, as a function of the state variable ω . In panel *b* we compare the innovation measure of Kogan et al. (2012) constructed in simulated data versus realizations of the state variable ω over a long simulation of 500 years. In panel *c* we plot the innovation measure of Kogan et al. (2012) in the data.

A Analytical Appendix

In order to solve the fixed point problem, we conjecture that the equilibrium allocation of labor L_I is only a function of the stationary variable ω . We verify that this is indeed the case below.

First, we characterize the consumption allocation. Workers consume their wage (see equation (32)), and shareholders consume the residual. Furthermore, all inventors have the same consumption-to-wealth ratio. As a result, the inventor's share of financial wealth b_{ts} defined in (29) also determines the fraction of total consumption available to shareholders that he consumes

$$\begin{aligned} C_{ts}^S &= b_{ts} (C_t - C_t^W) \\ &= b_{ts} e^{\chi t} \left((1 - L_I(\omega))^{1-\phi} - (1 - \phi)(1 - L_I(\omega))^{-\phi} \right), \end{aligned} \quad (\text{A.1})$$

since

$$\bar{C}_t = e^{\chi t} (1 - L_I(\omega_t))^{1-\phi} \quad (\text{A.2})$$

we can write

$$C_{ts}^{1-h} \left(\frac{C_{ts}}{\bar{C}_t} \right)^h = b_{ts} e^{(1-h)\chi t} \hat{l}(\omega_t)$$

where

$$\hat{l}(\omega) \equiv \left((1 - L_{It})^{1-\phi} - (1 - \phi)(1 - L_{It})^{-\phi} \right) (1 - L_I(\omega_t))^{-h(1-\phi)}. \quad (\text{A.3})$$

Given the equilibrium consumption process A.1, the value function of an inventor born in time s is given by

$$J_{ts} = \frac{1}{1 - \gamma} b_{ts}^{(1-\gamma)} e^{(1-\gamma)(1-h)\chi t} f(\omega_t), \quad (\text{A.4})$$

where the function f satisfies the ODE

$$0 = \rho \frac{1 - \gamma}{1 - \theta - 1} \hat{l}(\omega)^{1-\theta-1} f(\omega)^{\frac{\gamma-\theta-1}{\gamma-1}} + \rho_f(\omega) f(\omega) + \mathcal{A}f(\omega) \quad (\text{A.5})$$

where the operator \mathcal{A} is defined as

$$\begin{aligned} \mathcal{A}f(\omega) &\equiv f'(\omega) \left(\mu_\xi + \delta + \alpha \mu_x + (1 - \gamma)(1 - \phi)\alpha \sigma_x^2 - \lambda e^\omega \left(\frac{L_I(\omega)}{\lambda} \right)^\alpha \right) \\ &\quad + \frac{1}{2} f''(\omega) (\sigma_\xi^2 + \alpha^2 \sigma_x^2), \end{aligned} \quad (\text{A.6})$$

and

$$\begin{aligned} \rho_f(\omega) &= -\frac{\rho(1 - \gamma)}{1 - \theta - 1} + (1 - \gamma)(\mu - \kappa(\omega)) + (1 - h)(1 - \gamma) \left((1 - \phi)\mu_x - \phi\delta + \phi \lambda^{1-\alpha} e^\omega L_I(\omega)^\alpha \right) \\ &\quad + \frac{1}{2} (1 - \phi)^2 \sigma_x^2 (1 - \gamma)^2 (1 - h)^2. \end{aligned} \quad (\text{A.7})$$

Given the consumption allocation (A.1) and the inventor's value function (A.4), we compute the

stochastic discount factor,

$$\pi_t = \exp \left(\int_0^t \tilde{f}_J(C_s, \bar{C}_s, J_s) ds \right) \tilde{f}_C(C_t, C_t, J_t),$$

where

$$h_{C,ts} = \rho(e^{\lambda t})^{-\hat{\gamma}} b_{ts}^{-\gamma} l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \quad (\text{A.8})$$

$$l(\omega_t) \equiv \left(\left((1 - L_I(\omega_t))^{1-\phi} - (1 - \phi)(1 - L_I(\omega_t))^{-\phi} \right) (1 - L_I(\omega_t))^{-s(1-\phi)} \right)^{\theta-1} \hat{l}(\omega_t) \quad (\text{A.9})$$

$$\hat{\gamma} \equiv \gamma(1 - h) + 1 \quad (\text{A.10})$$

$$h_J(C, J) = -\frac{\rho}{1 - \theta^{-1}} \left((\gamma - \theta^{-1}) \left(\hat{l}(\omega_t) \right)^{1-\theta^{-1}} \left(f(\omega_t) \right)^{\frac{1-\theta^{-1}}{\gamma-1}} + (1 - \gamma) \right). \quad (\text{A.11})$$

Next, we determine the value of assets in place and growth opportunities. First, we solve for the two functions P and \tilde{P} that determine the value of existing projects (34)

$$P_t = \phi e^{\lambda t} K_t^{-1} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \nu(\omega_t) \quad (\text{A.12})$$

$$\tilde{P}_t = \phi e^{\lambda t} K_t^{-1} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \tilde{\nu}(\omega_t), \quad (\text{A.13})$$

where $\nu(\omega)$ and $\tilde{\nu}(\omega)$ solve the ODEs

$$0 = (1 - L_I(\omega))^{1-\phi} l(\omega)^{-\theta-1} f(\omega)^{\frac{\gamma-\theta-1}{\gamma-1}} + \rho_\nu(\omega)\nu(\omega) + \mathcal{A}\nu(\omega) \quad (\text{A.14})$$

$$0 = (1 - L_I(\omega))^{1-\phi} l(\omega)^{-\theta-1} f(\omega)^{\frac{\gamma-\theta-1}{\gamma-1}} + (\rho_\nu(\omega) - \theta_u)\tilde{\nu}(\omega) + \mathcal{A}\tilde{\nu}(\omega), \quad (\text{A.15})$$

and the function ρ_ν is given by

$$\begin{aligned} \rho_\nu(\omega) = & -\frac{\rho}{1 - \theta^{-1}} \left((\gamma - \theta^{-1}) \hat{l}(\omega_t)^{1-\theta^{-1}} f(\omega_t)^{\frac{1-\theta^{-1}}{\gamma-1}} + (1 - \gamma) \right) + \gamma(\kappa(\omega) - \mu) + \\ & + ((1 - \gamma)\phi - 1)\lambda^{1-\alpha} e^\omega L_I(\omega)^\alpha + (1 - h)(1 - \gamma)((1 - \phi)\mu_x - \phi\delta) + \frac{1}{2}(1 - \gamma)^2(1 - h)^2(1 - \phi)^2\sigma_x^2. \end{aligned} \quad (\text{A.16})$$

Using (A.12) and (A.13), the value of a firm's existing assets can be written as

$$\begin{aligned} VAP_{ft} = & \phi e^{\lambda t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \\ & \times \left(\nu(\omega_t) \sum_{j \in \mathcal{J}_{ft}} \varepsilon^{\xi_j} k_j^\alpha / K_t + \tilde{\nu}(\omega_t) \sum_{j \in \mathcal{J}_{ft}} \varepsilon^{\xi_j} k_j^\alpha (u_{j,t} - 1) / K_t \right). \end{aligned} \quad (\text{A.17})$$

The relative contribution of the functions ν and $\tilde{\nu}$ in the value of assets in place depends on the size and profitability of existing projects, as we can see from the last term in (A.17). Second, we solve

for the two functions Γ^H and Γ^L that determine the value of growth opportunities

$$\Gamma_t^H = (1 - \alpha) e^{\lambda t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \left(g(\omega_t) + (\lambda_H - \lambda_L) \frac{\mu_L}{\mu_L + \mu_H} \tilde{g}(\omega_t) \right) \quad (\text{A.18})$$

$$\Gamma_t^L = (1 - \alpha) e^{\lambda t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \left(g(\omega_t) - (\lambda_H - \lambda_L) \frac{\mu_H}{\mu_L + \mu_H} \tilde{g}(\omega_t) \right) \quad (\text{A.19})$$

where $g(\omega)$ and $\tilde{g}(\omega)$ solve the ODEs

$$0 = \nu(\omega) e^\omega \left(\frac{L_I(\omega)}{\lambda} \right)^\alpha + \rho_g(\omega) g(\omega) + \mathcal{A} g(\omega) \quad (\text{A.20})$$

$$0 = \nu(\omega) e^\omega \left(\frac{L_I(\omega)}{\lambda} \right)^\alpha + (\rho_g(\omega) - \mu_L - \mu_H) \tilde{g}(\omega) + \mathcal{A} \tilde{g}(\omega), \quad (\text{A.21})$$

and the function ρ_g is given by

$$\rho_g(\omega) \equiv \rho_\nu(\omega) + \lambda^{1-\alpha} e^\omega L_I(\omega)^\alpha. \quad (\text{A.22})$$

Using (A.18)-(A.19) the value of the firm's growth opportunities (36) equals

$$\begin{aligned} PVGO_{ft} &= \lambda_f (1 - \eta) (1 - \alpha) e^{\lambda t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \\ &\quad \times \left[g(\omega_t) + \left(p_{ft} - \frac{\mu_H}{\mu_L + \mu_H} \right) (\lambda_H - \lambda_L) \tilde{g}(\omega_t) \right], \end{aligned} \quad (\text{A.23})$$

so the contribution of the functions g and \tilde{g} to the value of growth opportunities depends on current growth state of the firm p_{ft} . Aggregating (A.17) and (A.23) across firms, the aggregate value of assets in place and growth opportunities is

$$VAP_t = \phi e^{\lambda t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \nu(\omega_t) \quad (\text{A.24})$$

$$PVGO_t = \bar{\lambda} (1 - \eta) (1 - \alpha) e^{\lambda t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} g(\omega_t). \quad (\text{A.25})$$

Given (A.24) and (A.25), we next determine the amount of inter-generational displacement

$$b_{tt} = b(\omega) \equiv \frac{\bar{\lambda} \eta (1 - \alpha) \phi \nu(\omega) e^\omega \left(\frac{L_I(\omega)}{\lambda} \right)^\alpha}{\phi \mu \nu(\omega) + \bar{\lambda} \mu (1 - \eta) (1 - \alpha) g(\omega)}. \quad (\text{A.26})$$

The last step is to determine the equilibrium allocation between the two sectors L_I and verify that it depends only on ω . The first order condition (25) simplifies to

$$(1 - \phi) (1 - L_I)^{-\phi} = \alpha \phi e^{\omega t} \left(l(\omega_t)^{-\theta-1} f(\omega_t)^{\frac{\gamma-\theta-1}{\gamma-1}} \right)^{-1} \nu(\omega_t) \left(\frac{\bar{\lambda}}{L_I} \right)^{1-\alpha}. \quad (\text{A.27})$$

The competitive equilibrium involves the solution of the five differential equations (A.5), (A.14), (A.15), (A.20), (A.21) and the first order condition in (A.27). We solve for these equations using finite differences on a grid with 2,000 points.

We simulate the model at a weekly frequency $dt = 1/52$ and then aggregate the data to form

annual observations. We simulate 1,000 model histories of 3,000 firms and 120 years each. We drop the first third of each history to eliminate the impact of initial conditions. When we compare the output of the model to the data, we report the median parameter estimate across simulations.

The equilibrium stochastic discount factor is given by

$$\frac{d\pi_t}{\pi_t} = -r_{ft} dt - \gamma_x(\omega_t) dB_t^x - \gamma_\xi(\omega_t) dB_t^\xi, \quad (\text{A.28})$$

$$\text{where } \gamma_x(\omega) = \left[(\gamma(1-h) + 1)(1-\phi) + \alpha \left(\theta^{-1} \frac{l'(\omega)}{l(\omega)} - \frac{\gamma - \theta^{-1}}{\gamma - 1} \frac{f'(\omega)}{f(\omega)} \right) \right] \sigma_x,$$

$$\gamma_\xi(\omega) = \left(\theta^{-1} \frac{l'(\omega)}{l(\omega)} - \frac{\gamma - \theta^{-1}}{\gamma - 1} \frac{f'(\omega)}{f(\omega)} \right) \sigma_\xi.$$

Last, the functions characterizing firm's exposure to changes in aggregate growth opportunities are

$$\zeta_\nu(\omega) = \ln \left(\left(l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} \nu(\omega) \right), \quad \zeta_{\tilde{\nu}}(\omega) = \ln \left(\frac{\tilde{\nu}(\omega)}{\nu(\omega)} \right), \quad (\text{A.29})$$

$$\zeta_g(\omega) = \ln \left(\left(l(\omega_t)^{-\theta^{-1}} f(\omega_t)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \right)^{-1} g(\omega) \right), \quad \zeta_{\tilde{g}}(\omega) = \ln \left(\frac{\tilde{g}(\omega)}{g(\omega)} \right), \quad (\text{A.30})$$

The functions A_{ft}^v and A_{ft}^g depend on the current state of the firm,

$$A_{ft}^v = \frac{\sum_{j \in \mathcal{J}_{ft}} e^{\xi_j} k_j^\alpha (u_{j,t} - 1)}{\sum_{j \in \mathcal{J}_{ft}} e^{\xi_j} k_j^\alpha} e^{\zeta_{\tilde{\nu}}(\omega_t)}, \quad \text{and} \quad A_{ft}^g = \left(p_{ft} - \frac{\mu_H}{\mu_L + \mu_H} \right) (\lambda_H - \lambda_L) e^{\zeta_{\tilde{g}}(\omega_t)}. \quad (\text{A.31})$$

In our comparative statics, we allow for inter-generational risk sharing through the following transfer scheme: the social planner exchanges the financial wealth of entering cohorts with a fraction of the existing wealth in the economy μW_t , implying that $b(\omega) = 1$ always.

B Measurement Appendix

Aggregate quantities

Investment, capital and consumption data are from NIPA. Investment is non-residential private domestic investment; stock of capital is current-cost from the NIPA Fixed Assets Table; consumption is non-durables plus services; nominal variables are deflated by population and the CPI. Population is from the Census Bureau. We construct the labor share using Flow of Funds data following Sekyu and Rios-Rull (2009). We compute aggregate Tobin's Q using NIPA and Flow of Funds data following Laitner and Stolyarov (2003). Data on total factor productivity is from Basu et al. (2006).

We simulate the model at monthly frequencies and time-aggregate the data to form annual observations. In simulated data, we measure consumption as the output of the consumption sector C ; we measure investment as the value of investment in terms of consumption units, $p^I I$; we measure output as the sum of consumption, investment and the inventors' share of the net present value of new projects. When constructing aggregate Tobin's Q , we measure the book value of capital as the historical cost of firm's capital $\sum_{J_{ft}} k_j q_{\tau(j)}$, where $\tau(j)$ denotes the time of creation of project j .

Innovation

We closely follow Kogan et al. (2012) in constructing the aggregate and firm-level innovation measure. The construction of the aggregate measure A proceeds in two steps. First, we infer the value of a patent based the stock market reaction around the day when a patent has been granted to a firm. We then aggregate our innovation measure across firms to infer the realizations of ω in the data.

We infer the value of patents from the stock market reaction around the day when a patent has been granted to a firm. We decompose the idiosyncratic stock return r of firm f around the day d that a patent is issued as

$$r_{fd} = x_{fd} + \varepsilon_{fd}, \quad (\text{B.1})$$

where x_{fd} denotes the value of patent as a fraction of the firm's market capitalization; and ε_{fd} denotes the component of the firm's stock return that is unrelated to the intrinsic value of the patent. Following Kogan et al. (2012), we choose a three-day window over which we compute returns. To recover the filtered value of the patent $E[x_{fd}|r_{fd}]$, we follow the procedure of Kogan et al. (2012), which involves assumptions about the distribution of x and e , and use their estimated parameters.

We construct the conditional expectation of the dollar value of each patent j issued to firm f in day d as

$$\hat{A}_j = \frac{1}{P_{fd}} E[x_{fd}|r_{fd}] V_{fd-1}, \quad (\text{B.2})$$

where V_{fd-1} is the market capitalization of the patenting firm on the day prior to the announcement. If multiple patents P are issued to the same firm on the same day, we assign each patent a fraction $1/P$ of the total value.

To construct the firm-level measure of innovation, we aggregate the dollar values across all patents on their grant and application days for firm f in year t :

$$A_{ft}^v = \sum_{j \in J_{ft}} \hat{A}_j, \quad (\text{B.3})$$

where J_{ft} denotes the sets of patents issued to firm f in year t . To avoid scale effects, we normalize

the dollar value of innovation (B.3) by the end-of-year firm market capitalization in year t ,

$$A_{ft} \equiv \frac{A_{ft}^v}{V_{ft}}. \quad (\text{B.4})$$

To construct the aggregate measure of innovation, we aggregate the firm-level dollar measure (B.3) across the set N_t of firms in the entire economy, and scale by the sum of their end of year total market capitalization S

$$A_t \equiv \frac{\sum_{f \in N_t} A_{ft}^v}{\sum_{f \in N_t} S_{ft}}. \quad (\text{B.5})$$

We follow the same procedure in constructing (B.4) and (B.5) in simulated data.

Firm accounting data

Firm accounting data is from Compustat. Book to market is book value of common equity (ceq) divided by CRSP market capitalization in December. Investment rate is growth rate in property, plant and equipment (ppeg). Tobin's Q is CRSP market capitalization, plus book value of debt (dltt), plus book value of preferred shares (pstkrv), minus deferred taxes (txdb) divided by book assets (at). Output y is sales (sale) plus change in inventories (inv).

In simulated data, we measure the book value of capital as the historical cost of firm's capital $\sum_{\mathcal{J}_{ft}} k_j q_{\tau(j)}$, where $\tau(j)$ denotes the time of creation of project j ; we measure firm investment as the accumulated investment expenses over the year; we construct dividends as profits minus investment expenses minus payment to inventors for the acquisition of project blueprints.

Household consumption data

Data on household consumption is from the Consumption Expenditure Survey (CEX) Family-level extracts by Harris and Sabelhaus (2000), available through the NBER website. We follow the variable definitions in Harris and Sabelhaus (2000). The data contain observations of households of different cohorts taken at different points in time. Consumption is non-durables, defined as in Harris and Sabelhaus (2000). Stockholders are classified as households reporting ownership of stocks, bonds or mutual funds. Cohort age is defined as the age the household turns 25. The volatility of consumption cohorts is computed as in Garleanu et al. (2012), but we restrict the sample to households that are shareholders. In the model, we measure consumption cohort effects by $b(\omega_t)$.

Consumption growth of shareholders and non-shareholders are from Malloy et al. (2009) using their definitions. We construct annualized growth rates using Dec-Dec growth, following Jagannathan and Wang (2007).

Stock returns

Firm stock return data are from CRSP. We form portfolios in June every year. We exclude financial firms (SIC6000-6799), and utilities (SIC4900-4949). Data on the market portfolio and the risk-free rate are from Kenneth French's website.