Fragility in Money Market Funds: Sponsor Support and Regulation.

Cecilia Parlatore Siritto*†

Department of Finance, Wharton School, University of Pennsylvania

Download Newest Version

July 12, 2013

Abstract

I develop an equilibrium model of U.S. money market funds (MMFs) and use it to analyze how adopting a floating net asset value (NAV) would affect the liquidity provided by MMFs and their fragility. The model captures key institutional features of MMFs such as liquidation after "breaking the buck" and voluntary sponsor support. I show that sponsor support, which is instrumental to attain a stable NAV, may be a source of fragility: it may make the MMF industry prone to runs different from the canonical bank-runs. Moreover, the general equilibrium effects in the model, mostly ignored in the policy discussion, can overturn conventional intuition.

*email: cecip@wharton.upenn.edu

†I am especially indebted to Douglas Gale, Ricardo Lagos, and Thomas Sargent for their advice in this project. I also thank Viral Acharya, Marco Cipriani, Antoine Martin, Marcin Kacperczyk, Michal Szkup, and Tanju Yorulmazer for very helpful comments and suggestions. I thank Anmol Bhandari, Jaroslav Borovicka, Fernando Duarte, Klaus Hellwig, Emiliano Marambio Catan, and David Musto for their insightful remarks, as well as seminar participants at Duke, IESE, McGill, NYU, Philadelphia FRB, UBC Sauder, University of Michigan, University of Texas at Austin, UPF, and Wharton. This paper has been prepared by the author under the Lamfalussy Fellowship Program sponsored by the ECB. Any views expressed are only those of the author and do not necessarily represent the views of the ECB or the Eurosystem. A previous version was circulated as "The Regulation of MMFs: Adding Discipline to the Policy Debate".

1 Introduction

U.S. money market funds (MMFs) are financial intermediaries key to the financial system. By the end of 2011, MMFs managed over 2.7 trillion dollars in assets, almost a quarter of all U.S. mutual fund assets, and over 10 percent of mutual fund assets worldwide.[1]

MMFs are also important suppliers of short term financing, especially for financial institutions. In December of 2011, MMFs owned over 40 per cent of U.S. dollar-denominated financial commercial paper, around a third of dollar-denominated negotiable certificates of deposit, and they were among the biggest category of repo lenders, with an estimated $460 billion in repos.[2] Given the size of the MMF industry, and its importance as a supplier of liquidity to financial institutions, flows into and out of MMFs can affect the whole financial system. In fact, the large outflows experienced by the MMF industry after Reserve Primary Fund "broke the buck" in 2008 contributed substantially to the freezing of the short-term funding market. Later, in 2011, the heavy exposures of MMFs to European financial institutions put the MMF industry at risk of transmitting distress from Europe to the U.S. short-term funding market and the outflows from the MMF industry worsened the situation of the Eurozone banks.[3]

Because of the systemic importance of MMFs, its regulation has been at the center of policy discussion. Today, the need for further regulation is being debated. On June 5, 2013, the Securities and Exchange Commission (SEC) voted to move forward with a proposal to impose new regulations on the MMF industry. The mutual fund industry opposes these changes and argues that further regulations would make the MMF industry less profitable and reduce the availability of short-term funding.[4]

The most important regulations that are being considered are to impose redemption fees on investors, and to force MMFs to abandon the stable net asset value (NAV) in favor of a floating NAV. Redemption fees would increase investors' costs of redeeming shares and,

[1] See ICI (2012).

[2] Source: McCabe et al. (2012), Mackenzie, Financial Times May 3, 2011.

[3] See Board (2009), SEC (2009) , PWG (2010) , FSOC (2011) and Chernenko and Sunderam (2012).

[4] See McCabe (2011), Mendelson and Hoerner (2011), Lacker (2011), Squam Lake Group (2011), Volcker (2011), Hanson et al. (2012), McCabe et al. (2012).and http://www.preservemoneymarketfunds.org/the-impact-on-you/. Last visited November 25, 2012.

therefore, reduce their incentives to run on the MMFs. By adopting a floating NAV, MMFs would not be subject to liquidation after breaking the buck nor would they be allowed to use penny-rounding accounting which can be destabilizing.[5] This change would reduce the relevance of voluntary sponsor support in the MMF industry.

There seems to be no consensus on the effect adopting a floating NAV would have on the economy, in particular on the provision of liquidity by MMFs. In this paper I develop an equilibrium model of MMFs and use it to understand the trade-offs implied by the adoption of a floating NAV.

The model is a 3-period model of financial intermediation that incorporates features that make MMFs unique financial institutions. There are two types of agents: risk averse investors and risk neutral fund managers. A short-term safe asset and a long-term risky asset are traded in competitive markets. Managers are the only agents who can access the risky asset market directly. Investors may only access the risky asset through the managers. The intermediation contract between investors and managers captures the demandable nature of the shares held by investors in MMFs, the eventual liquidation after a fund breaks the buck, and, through the possibility of voluntary sponsor support, the stable NAV. The asset structure incorporates the fluctuation in asset prices faced by MMFs: at the time of purchase there is uncertainty about the quality of the risky asset and its default risk. Asset prices are determined in equilibrium, which captures the size and relevance of the MMF industry in the market for short-term financing.

The managers' support decisions determine the demand for the risky asset in the interim period and its liquidation price. At the same time, the liquidation price of the risky asset in the interim period determines the costs and benefits for managers of offering support. This interdependence may give rise to strategic complementarities in the managers' support decisions and may make the model prone to runs that are different in nature from the canonical bank runs described in Diamond and Dybvig (1983): these runs are not runs of investors on the financial intermediaries but runs of financial intermediaries on the asset market.

The model developed in this paper is consistent with several stylized facts documented

[5] For a description of penny-rounding accounting refer to the next section.

in the literature on MMFs. In particular, the model captures the responsiveness of MMFs' inflows to the funds' past return (positive performance-flow relation), and the lower risk-taking by fund with sponsors that also offered non-money market mutual funds and other financial services.

Finally, I find that, in the model, the adoption of a floating NAV would be successful in decreasing the fragility of the MMF industry. However, in equilibrium, the regulation would also affect intermediation fees, support decisions, and asset prices. All these, in turn, could have countervailing effects on the risk and the return of intermediation for investors and MMF managers, and, hence, on the provision of liquidity and on welfare.

When going from a stable to a floating NAV system, investors in MMFs lose the insurance provided by sponsor support. Everything else equal, risk averse investors would reduce their exposure to MMFs in response to this loss of insurance. Thus -as argued by the industry- the supply of liquidity in the asset market would go down. In equilibrium, the new regulation also changes asset prices and fees, and, hence, the risk and the return of investing in a MMF change beyond the loss of insurance. In numerical examples, I find that the total supply of liquidity increases when adopting a floating NAV. These examples illustrate that general equilibrium effects can be counterintuitive, and highlight the usefulness of looking at the policy discussion through the lens of a model.

To the best of my knowledge, this paper develops the first model of MMFs. Since the regulations mentioned above target particular institutional features of the MMF industry, a model that captures these peculiar features is needed to analyze the impact of those regulations. MMFs are different from banks and from other mutual funds and, therefore, existing models of banking and delegated portfolio management are not suited to perform this task.

Though a model of MMFs has not been developed in the literature, several papers have analyzed the MMF industry empirically. Chen et al. (2010) document the presence of strategic complementarities in the redemption behavior of investors in all mutual funds, including MMFs. These strategic complementarities, which are not the focus of my paper, make MMFs prone to self-fulfilling runs like those considered by Diamond and Dybvig (1983) and analyzed extensively in the banking literature. McCabe (2010), Chernenko and Sunderam

(2012), Kacperczyk and Schnabl (2012) and Wermers (2012) document run-like events in the MMF industry. Moreover, as discussed in the next section, the proneness to runs in MMFs is exacerbated by penny-rounding accounting. McCabe et al. (2012) propose a minimum balance at risk rule to mitigate the vulnerability of MMFs to runs and reduce the investors' incentives to redeem MMF shares quickly when a fund is in distress. The relevance of sponsor support in the MMF industry is documented by Moody's (2010), Brady et al. (2012), and Kacperczyk and Schnabl (2012).

Starting with Bhattacharya and Pfleiderer (1985) there has been a large literature dealing with the optimal compensation structure in delegated portfolio management relationships. In order to analyze the effect of the regulations, the model developed in this paper takes the intermediation contract as given. I do not provide a rationale for the adoption of such contract which, though an interesting task, is beyond the scope of this paper.[6]

The next section presents some institutional features of MMFs. Section 3 introduces the model. Section 4 defines and characterizes the equilibrium. Section 5 discusses the forces at play in the model. Sections 6 contains the policy analysis and section 7 concludes.

2 Money Market Funds: Institutional Features

MMFs are open-ended mutual funds that offer individuals, corporations, and governments access to money market instruments, such as US Treasury bills and commercial paper. MMFs act as intermediaries between investors and borrowers who seek short-term financing. As required by regulation, all mutual funds, including MMFs, issue demandable shares, *i.e.*, they provide "same day" liquidity, allowing investors to redeem their shares at any time at the net asset value of the shares (NAV).[7] What makes MMFs special is that they seek to maintain a stable NAV, usually of $1. To prevent the NAV from going above $1, all positive investment returns are paid out entirely as dividends, with no capital gains or losses to track. If the NAV drops below $1, it is said that the fund "broke the buck". In the event of a fund breaking the buck, regulation states that "the board of directors shall promptly consider

[6] For a discussion on the origin of MMFs see chapter 1 in Carnell et al. (2008).

[7] Title 17 - Commodity and Securities Exchanges - 17 CFR § 270.22c-1 (b)(c)

what action, if any, should be initiated by the board of directors." (Rule 2a-7(b), 17 C.F.R. § 270.2a-7(b)) In practice, in the only two cases in which a MMF's NAV dropped below $1, the fund were eventually liquidated.[8]

Even though a MMF could become a floating NAV fund after breaking the buck, adopting a floating NAV does not reduce investors incentives to redeem their shares and it decreases the NAV even more. Since a fund would likely sell the more liquid assets to meet excess redemptions, investors who chose not to redeem their shares would be left holding a portfolio of less-liquid, longer-dated securities. This increases the incentives of investors to withdraw quickly, even at a reduced NAV, and drives the NAV even lower.[9] This downward spiral mechanism makes it impossible for a MMF to transition to a floating NAV fund and makes liquidation the only viable option after breaking the buck. For example, on September 16, 2008, a day after Lehman Brothers declared bankruptcy, the Reserve Primary Fund (RPF) delayed share redemptions for up to seven days, abandoned the stable NAV, and became a floating NAV fund. On September 18, 2008, RPF suspended the redemption of shares and started the orderly liquidation of the fund's assets after experiencing massive share redemptions. RPF, which had approximately $62 billion in assets under management on September 15, 2008, experienced redemptions of $60 billion between September 15 and September 18, 2008.[10]

In order to maintain a stable NAV, MMFs rely on two mechanisms: the computation of the NAV and sponsor support. The SEC, via Rule $2a - 7$, allows MMFs to use amortized cost valuation and penny-rounding pricing to compute the NAV.[11] Many of the assets held by MMFs lack market price quotations. This makes it difficult for MMFs to price their assets accurately. Amortized cost valuation allows MMFs to value their assets as if held to maturity. Penny-rounding pricing allows MMFs to report a NAV of $1 as long as the

[8] See Fisch and Roiter (2011) for a detailed description of the current regulation of MMFs.

[9] See Hanson et al. (2012).

[10] See http://www.primary-yieldplus-inliquidation.com/pdf/PressRelease2008_0916.pdf and http://www.primary-yieldplus-inliquidation.com/pdf/PressReleasePrimGovt2008_0919.pdf . Last accessed February 15, 2013.

[11] Rule 2a-7 allows MMFs to use amortized cost valuation "only so long as the [fund] board of directors believes that it fairly reflects the market-based net asset value per share".

calculated value is between \$0.995 and \$1.005. Using amortized cost valuation and penny rounding makes MMFs prone to runs by shareholders. For example, if the NAV is just below \$1, shareholders who redeem their shares first get \$1 and by doing so reduce the value of the fund's assets, imposing costs on non-redeeming shareholders who might not get \$1 for their shares. Analogously, if the NAV calculated using amortized cost valuation differs from the market value of the asset, investors might be better off redeeming their shares. Since the liquidation value of the assets differs from the NAV computed by the MMF, investors who do not redeem their shares bear the cost of paying redeeming investors \$1 for something which is worth less than \$1 in the market. These phenomena resemble the mechanism behind the canonical bank runs described by Diamond and Dybvig (1983), and dealt with in a very large literature.

In my model, I abstract from the possibility of runs on MMFs by assuming that all assets are traded in frictionless competitive markets, and that the shares in MMFs are priced-to-market. In my model, there is no need to use amortized cost accounting since price quotations are always available.

The other mechanism MMFs rely on to maintain a stable NAV is voluntary sponsor support. Breaking the buck is a costly event for sponsors of MMFs, not only because of the forgone future profits from operating the MMF, but also because of possible negative spillovers to other activities in which the sponsor participates.[12] Therefore, companies that sponsor MMFs may choose to offer support to their MMFs to prevent the funds from breaking the buck. Brady et al. (2012) find that at least 21 MMFs would have broken the buck if they had not received sponsor support during the last financial crisis. Moreover, their data suggests that sponsor support was frequent and significant between 2007 and 2011: 78 MMFs (out of a total of 341 MMFs) received sponsor support during this period in 123 instances for a total amount of at least \$4.4 billion. In fact, as shown by Moody's Investors Service (2010), voluntary sponsor support has been a common feature throughout the history of the MMF industry even prior to the recent financial crisis. Figure 1 shows the number of funds

[12] Kacperczyk and Schnabl (2012) find evidence of these spillovers.

For example, a sponsor company that offers other mutual fund and financial services may suffer losses from these activities after one of his money market funds breaks the buck.

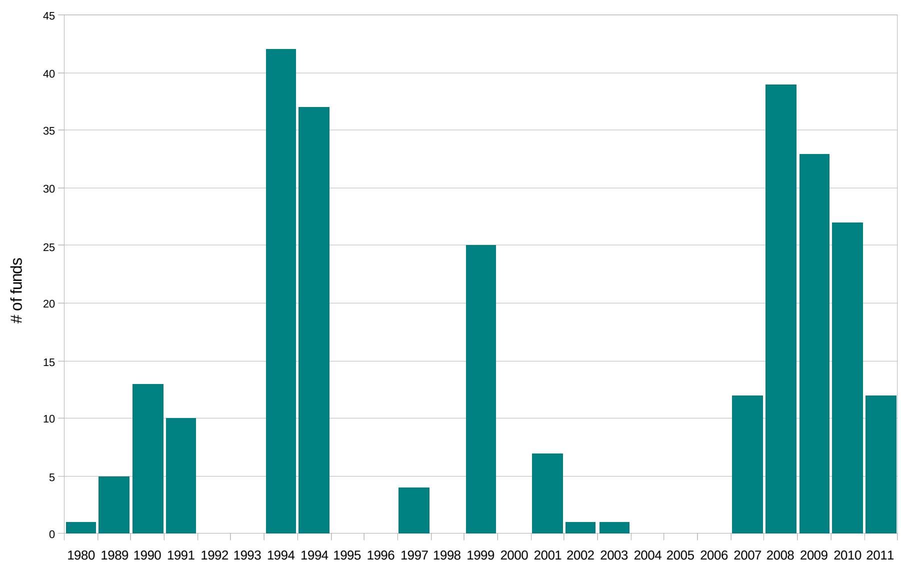

Figure 1: Number of Funds Receiving Support: 1980-2011. Sources: Brady et al. (2012) and Moody's (2010).

receiving support from 1980 to 2011.

The features described above make MMF unique financial institutions. In particular, as illustrated in figure 2, the payoff received by investors in MMFs can be seen as a hybrid between that received by investors in other mutual funds, and that received by depositors in banks. If no sponsor support is offered, investors in MMFs are true shareholders and, as for investors in other mutual funds, the value of their shares coincides with the market NAV. If sponsor support is offered, the value of the shares for investors in MMFs is the same independently of the value of the fund's assets. This flat portion of the investors' payoff makes shares in MMFs resemble debt. Nevertheless, sponsor support is voluntary, and, though it is anticipated by investors it is not mandated by the intermediation contract. The intermediation contract assumed in the model developed in the next section captures these features.

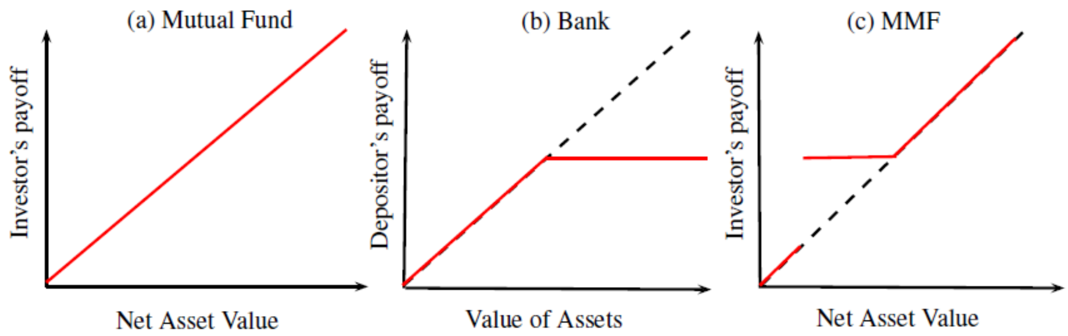

Figure 2: The net asset value for mutual funds and MMFs is calculated as the total value of the fund's assets divided the total number of outstandig shares. The asset value of assets for banks is calculated as the total value of the bank's assets divided the total amount of deposits.

3 Model

The model has three-periods, $t = 0, 1, 2$, and one good. There are two types of agents in this economy: investors and managers. There is a continuum of measure 1 of each type of agent. Investors are risk averse with preferences given by $\mathbb{E}\left[\log\left(W_2^I\right)\right]$ where W_2^I is the investor's wealth at $t = 2$, and they are endowed with W_0^I units of the good at $t = 0$. Managers are risk neutral with preferences given by $\mathbb{E}\left[W_2^M\right]$ where W_2^M is the manager's wealth at $t = 2$. Managers are endowed with W_0^M units of the good at $t = 0$ and $E > 0$ at $t = 1$. There are two types of assets: a short-term, safe asset, and a long-term, risky asset. The safe asset is a one-period bond supplied perfectly elastically at price q_t in periods $t = 0, 1$. One unit of the safe asset bought at t pays 1 unit of the good at $t + 1$. The risky asset is a two-period asset. A unit of the asset bought at $t = 0$ has a random payoff d at $t = 2$. The payoff structure is as follows:

$$d = \begin{cases} \bar{d} > 0 & \text{with probability } \pi \\ 0 & \text{with probability } (1-\pi) \end{cases}.$$

The probability π is a random variable whose realization is observed by everyone at $t = 1$. π is uniformly distributed over $[\underline{\pi}, \bar{\pi}]$ with $0 \leq \underline{\pi}, \bar{\pi} < 1$. The probability π can be interpreted as the quality of the risky asset.

The risky asset is traded in frictionless competitive markets at $t = 0, 1$, at prices p_0

and $p_1(\pi)$ respectively. At $t = 0$, the supply of the risky asset is given by $S(p_0)$, where $S'(p_0) > 0$. The supply of the asset in period 1 is fixed and equal to the amount of the risky asset traded at $t = 0$. I assume that no short-selling is allowed.

To include motives for intermediation, I follow the limited market participation literature and assume that investors can only invest directly in the safe asset. As in He and Krishnamurthy (2012) I allow managers to invest in the risky asset on behalf of investors and act as financial intermediaries.

Managers can choose whether to become intermediaries. If they choose not to become intermediaries, they manage their own wealth. If they choose to offer intermediation services, they can manage assets on behalf of one investor, at most, by opening a fund. The intermediation relation is long-term: once an investor chooses a manager with whom to invest at $t = 0$, he cannot invest with other managers at $t = 1$. Managers incur in a fixed cost $C > 0$ if they manage funds for an investor. Each period t, a fund consists of the manager's wealth, W_t^M, and the amount the manager is managing for the investor, A_t^I. The manager makes all portfolio decisions on behalf of the fund and these decisions are non-contractible. For simplicity, I assume that the manager has to invest his own wealth in the same way in which he invests the investor's wealth, *i.e.*, managers can only manage one portfolio at a time.

The intermediation contract is as follows. Managers charge a fraction f of the assets under management each period in return for intermediation services. For example, if a manager manages A_0^I for an investor in period 0 and A_1^I in period 1, he will collect fees fA_0^I at $t = 0$ and fA_1^I at $t = 1$. An investor with A_t^I under the management of a manager with wealth W_t^M in period t is entitled to a fraction $(1-f)\,A_t^I/\left(W_t^M + A_t^I\right)$ of the value of the fund's assets at $t + 1$. Hence, by investing with a manager, investors become shareholders of the fund owning $(1-f)\,A_t^I$ shares out of a total of $\left(W_t^M + A_t^I\right)$ outstanding shares of the fund. At $t = 1$, investors are paid the value of their shares and choose how much of their wealth to invest with their manager and in the safe asset. The value of a share, or net asset value (NAV), is the value of the fund's assets divided the total number of outstanding shares.

To capture the "breaking the buck" feature of MMFs, I assume that if the value of the investor's share goes below x in period 1, the fund is liquidated, where $x < 1/q_0$. Upon early liquidation of the fund, the manager suffers a loss $B\pi$, $B \geq 0$, and he cannot offer intermediation services at $t = 1$. To prevent the fund from being liquidated at $t = 1$, the manager can choose to offer support to the investor and keep the value of the investor's shares at x.

I assume that the intermediation industry is competitive and that all the surplus of a match between an investor and a manger goes to the investor.

The timing of the model is as follows. At $t = 0$, each manager chooses whether to offer intermediation services or not. If they choose to become intermediaries, managers choose the fund's portfolio. Otherwise they choose how to invest their own wealth. Managers choose which fraction of the fund's assets under management to invest in the risky asset, a_0^M, and which to invest in the safe asset $\left(1 - a_0^M\right)$. At the same time, investors make their portfolio decision: they choose the fraction of their wealth to invest with their manager, a_0^I, and the fraction to invest in the safe asset, $\left(1 - a_0^I\right)$. Simultaneously, the price of the risky asset, p_0, is determined in a centralized, competitive market.

At $t = 1$, the probability of success of the risky asset, π, is realized. After observing this probability, portfolio and support decisions are made simultaneously and, at the same time, the price of the risky asset is determined. Each manager chooses the fraction of his fund's assets to invest in the risky asset, a_1^M, and which to invest in the safe asset, $\left(1 - a_1^M\right)$, and the probability with which to offer support to investors, $s(\pi) \in [0, 1]$. If the fund is not liquidated early, investors choose the fraction of their wealth they want to invest with their manager, a_1^I, and the fraction they want to invest in the safe asset, $\left(1 - a_1^I\right)$. Finally, at $t = 2$, the payoff of the risky asset is realized and the fund is liquidated. Figure 3 depicts this timeline.

3.1 Discussion of assumptions

The model presented in the previous section can be viewed as a delegated portfolio management model with particular features that are meant to capture key characteristics of MMFs. In particular, the demandable nature of shares and the open-endedness of MMFs are captured by the ability of investors to adjust their portfolio in the interim period. The liquidation threshold, x, introduced as part of the intermediation contract, gives incentives

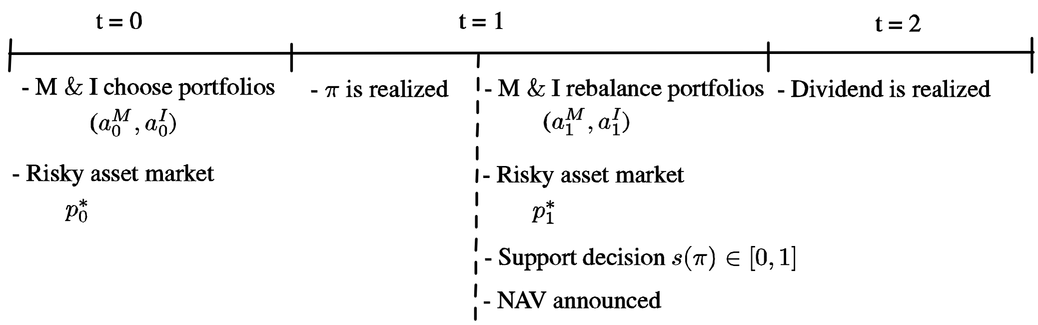

Figure 3: Timeline.

to managers to keep the net asset value stable and captures the eventual liquidation of a fund after a "breaking the buck" event. The cost of early liquidation of a fund imposed on managers, $B\pi$, is meant to capture the spillover losses a sponsor may suffer if one of his MMFs breaks the buck. Finally, the possibility of sponsor support is introduced by allowing the managers, who fulfill both the role of fund managers and sponsor companies in the model, to transfer resources to investors to prevent the fund from being liquidated.

The main difference between the model presented here and other banking and delegated portfolio models lies in the intermediation contract assumed, which incorporates key features of MMFs.

The asset structure is meant to capture the maturity mismatch problem inherent to MMFs and the trade-off between maturity, risk and return. The short-term safe asset can be interpreted as a treasury bill while the long-term, risky asset can be though of as commercial paper. The uncertainty structure embedded in the payoff of the risky asset implies that there are two sources of risk for commercial paper. On one hand, the quality of commercial paper is subject to shocks before the maturity date (downgrades). On the other, at maturity, commercial paper is subject to default. The assumption of no short-selling is consistent with the regulation of MMFs.

The functional form of the utility functions assumed makes the model tractable. The managers' risk neutrality makes the calculation of the support decision independent of wealth levels other than through the managers' ability to offer support. In the same way, investors having constant relative risk aversion implies that the portfolio choice in the interim period depends only on the quality of the risky asset, π, and on its price. This investment decision is independent of the investors' wealth and, therefore, of the support decision and the choices at $t = 1$. These simplifications make it possible to have closed form solutions for prices and support thresholds at $t = 1$.

The assumption that $x < 1/q_0$ implies that a manager can always choose a risky portfolio which is still conservative enough to always avoid breaking the buck. If the only way for the manager to avoid breaking the buck in all states π was to invest in a safe portfolio, managers would have to choose between breaking the buck with positive probability or not opening a fund at all. Since investors can invest in the safe asset on their own, they would not pay fees to a manager that chose a safe portfolio.

4 Equilibrium

The return on a unit invested with a manager depends on the asset prices, on the manager's portfolio choice and on the manager's support decision. Hence, to decide how much to invest with a manager, investors need to anticipate not only the equilibrium asset prices, but also their managers' actions. In the same way, a manager's payoffs depend on the amount of resources he manages for investors. Therefore, managers need to anticipate asset prices and their investors' actions when making choices. This is captured in the definition of equilibrium.

Definition 1 *A symmetric equilibrium is a set of price functions $\{p_0^*, p_1^*(\pi)\}$, functions for portfolio choices $\left\{a_0^{M*}, a_0^{I*}, a_1^{M*}(\pi), a_1^{I*}(\pi)\right\}$, a support probability function $s^*(\pi) \in [0,1]$, and a fee f^* such that: $\left(a_0^{I*}, a_1^{I*}(\pi)\right)$ solve each investors' problem taking prices and his manager's decisions as given, $\left(a_0^{M*}, a_1^{M*}(\pi), s^*(\pi)\right)$ solve each manager's problem taking prices and his investor's decisions as given, the risky asset market clears at $t = 0, 1$, and managers are indifferent between managing an investor's funds and not doing so.*

The equilibrium can be computed using backwards induction. I will divide the characterization of equilibrium in two parts: time 0 and time 1. In the following subsection I characterize the agents' problems at $t = 1$ for a given realization of π, and given portfolio choices a_0^M and a_0^I and fees f. Then, I move to the decisions at $t = 0$ which take into account the equilibrium choices at $t = 1$.

4.1 t=1

All decisions at $t = 1$ depend on the choices and equilibrium objects determined at $t = 0$, that is, on a_0^M, a_0^I, f and p_0. To simplify the notation, I will mostly ignore this dependence when writing down equilibrium objects at $t = 1$.

4.1.1 Manager's portfolio choice at t=1

At $t = 1$, a manager with wealth W_1^M who receives fees fA_1^I solves

$$\max_{a_{1,i}^M \in [0,1]} \mathbb{E}_d \left[\left(a_1^M \frac{d}{p_1} + \left(1 - a_1^M\right) \frac{1}{q_1} \right) \left(W_1^M + fA_1^I\right) \middle| \pi \right].$$

The manager's portfolio choice depends only on the expected returns of the assets the manager can invest in: $a_{1,i}^{M*} = 1$ if $\pi \bar{d}/p_1 > 1/q_1$, $a_{1,i}^{M*} = 0$ if $\pi \bar{d}/p_1 < 1/q_1$, and he will be indifferent between investing in the safe and risky assets if $\pi \bar{d}/p_1 = 1/q_1$. The manager's portfolio choice at $t = 1$ is independent of his own wealth and of the amount investors invest with him.

4.1.2 Investor's problem at t=1

An investor's portfolio decision in period 1 depends on the behavior he anticipates for his manager, which in turn depends only on the state π. An investor with wealth W_1^I who anticipates his manager will choose $a_{1,i}^M$ chooses a_1^I to solve

$$\max_{a_{1,i}^I \in [0,1]} \mathbb{E}_d \left[\log \left(\left(a_{1,i}^I \left(1 - f\right) \left(a_{1,i}^M \frac{d}{p_1} + \left(1 - a_{1,i}^M\right) \frac{1}{q_1} \right) + \left(1 - a_{1,i}^I\right) \frac{1}{q_1} \right) W_1^I \right) \middle| \pi \right]$$

An investor will never invest with a manager who only invests in the safe asset. If he did, he would be paying fees for something he could do himself. Therefore, investors will choose not to invest with their manager if $\pi \bar{d}/p_1 \leq 1/q_1$. Furthermore, they will only invest with their manager if the expected return of doing so is greater than the return of the safe asset, *i.e.*, if $\pi \left(1 - f\right) \bar{d}/p_1 > 1/q_1$. Therefore, the optimal portfolio choice for investors is

$$a_{1,i}^I \left(p_1, \pi\right) = \frac{\max \left\{ \pi \left(1 - f\right) \frac{\bar{d}}{p_1} - \frac{1}{q_1}, 0 \right\}}{\left(1 - f\right) \frac{\bar{d}}{p_1} - \frac{1}{q_1}}.$$

As it is usual with CRRA preferences, the share of wealth invested with the manager is independent of the wealth level. As I mentioned above, this assumption makes the problem easier to track. To keep notation simple, I will define $A_{0,i}^{i} \equiv a_{0,i}^{I} W_0^I$ and $A_{1,i}^{I}(p_1, \pi) \equiv a_{1,i}^{I}(p_1, \pi) W_{1,i}^{I}(p_1)$.

4.1.3 Demand for risky assets at t=1

The demand for the risky asset at $t = 1$ of an individual manager i with wealth $W_{1,i}^{M}(p_1)$ is

$$D^i(p_1, \pi) = \begin{cases} a_{1,i}^{M}(p_1, \pi)\left(A_{1,i}^{I}(p_1, \pi) + W_{1,i}^{M}(p_1)\right)/p_1 & \text{if the fund continues} \\ a_{1,i}^{M}(p_1, \pi) W_{1,i}^{M}(p_1)/p_1 & \text{if the fund is liquidated} \end{cases}$$

where $W_{1,i}^{I}(p_1)$ is the wealth level for manager $i's$ investor when the price is p_1.[13] If the fund continues at $t = 1$, the manager will manage his own wealth and whatever his investor gives him, $A_1^I(p_1, \pi)$. If the fund is liquidated, a manger will only manage his own wealth.

Given these individual demands for the risky asset, the equilibrium liquidation price $p_1^*(\pi)$ will be such that

$$p_1^*(\pi) \int \left[D^i(p_1^*(\pi), \pi) - a_{0,\,i}^{M}\left(a_{0,i}^{I} W_0^I + W_0^M\right)/p_0\right] di = 0$$

where the last term inside brackets is the supply of the risky asset which is fixed and equal to the total amount traded at $t = 0$.

The wealth levels of managers and investors will depend, not only on whether the fund continues, but also on whether sponsor support is offered. Therefore, to compute the equilibrium price as a function π one needs to consider four possible cases: π such that no sponsor support is needed, π such that all sponsors offer support if needed, π such that sponsors offer support with probability s when needed, and π such that sponsor support is needed to prevent the liquidation of the fund, but it is not provided. Then,

$$p_1^*(\pi) = \begin{cases} p_1^{NS}(\pi) & \text{if no support is needed} \\ p_1^{S}(\pi) & \text{if sponsors offer support always } s^*(\pi) = 1 \\ p_1^{SS}(\pi) & \text{if sponsors offer support with probability } s^*(\pi) \in (0,1) \\ p_1^{L}(\pi) & \text{if support is needed but not offered} \end{cases}$$

[13] Expressions for these wealth levels can be found in the appendix.

.

Characterizations of the price functions $p_1^{NS}(\pi)$, $p_1^{S}(\pi)$, $p_1^{SS}(\pi)$, and $p_1^{L}(\pi)$ can be found in the appendix.

4.1.4 Equilibrium Support Decision

In a symmetric equilibrium, all managers will choose the same support strategy and the same portfolios. Therefore, if the manager does not need to offer support the relevant price is p_1^{NS}, if he strictly prefers to offer support it is p^S, if he is indifferent between offering support and liquidating the fund it is $p_1^{SS}(\pi)$, and if the fund is liquidated it is p^L. Taking this into consideration, the equilibrium aggregate support and liquidation decisions are calculated as follows.

A manager's support decision will depend on the equilibrium price through the market value of the shares or net asset value (NAV). The net asset value, $n\left(p_1, a_0^M\right)$, is the return on the portfolio of a manager that chose to invest a fraction a_0^M of the fund in the risky asset when the liquidation price of the risky asset is p_1, *i.e.*, $n\left(p_1, a_0^M\right) = a_0^M p_1/p_0 + \left(1 - a_0^M\right)/q_0$. As long as $n\left(p_1, a_0^M\right) \geq x$, the manager will not have any need to offer support for investors.

Proposition 1 *Given a portfolio choice for managers at $t = 0$ a_0^M, there exists a threshold $\pi_x\left(a_0^M\right)$ such that in a symmetric equilibrium managers will not need to offer support to investors if $\pi \geq \pi_x\left(a_0^M\right)$.*

The proof of this proposition follows from the fact that the relevant price function when no support is offered is $p_1^{NS}(\pi)$ and from monotonicity of $p_1^{NS}(\pi)$ in π.

If support is needed for a fund to continue, *i.e.*, if $\pi < \pi_x\left(a_0^M\right)$, given p_1 a manager will make his support decision to solve

$$\begin{aligned}&\max_{s\in[0,1]} \frac{\pi \bar{d}}{p_1}\left(n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right) + (1-s)(-B\pi)\\&+s\frac{\pi \bar{d}}{p_1}\left(fA_1^I(p_1,\pi) - \left(x - n\left(p_1, a_0^M\right)\right)(1-f)A_0^I\right)\end{aligned}$$

s.t.

$$n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right) + E + fA_1^I(p_1,\pi) \geq \left(x - n\left(p_1, a_0^M\right)\right)(1-f)A_0^I, \tag{1}$$

where $s(\pi)$ is the probability with which a manager chooses to offer support in state π. If the manager offers support in state π, he invests the value of his shares, $n\left(p_1, a_0^M\right)\left(W_0^M + fA_0^I\right)$, his endowment in period 1, E, the incoming fees from keeping the fund open, fA_1^I, minus the cost of offering support, $\left(x - n\left(p_1, a_0^M\right)\right)(1-f)\,A_0^I$. The expected return of this investment is $\pi\bar{d}/p_1(\pi)$ since $p_1(\pi) \leq \pi\bar{d}q_1$. If he does not offer support he invests the return on his shares in the fund and his endowment but he suffers a loss $B\pi$ from liquidating the fund early. The amount of support a manager can offer is determined by the amount of resources he has available at $t = 1$. This is captured by constraint (1).

The maximization presented above implies that the manager will choose to offer support to the investors if the benefit of offering support offsets the costs of doing so. That is if support is needed and

$$\begin{aligned} & \frac{\pi\bar{d}}{p_1(\pi)}\left(\left(x - n\left(p_1(\pi), a_0^M\right)\right)(1-f)\,A_0^I - fA_1^I\left(p_1(\pi), \pi\right)\right) \\ \leq\ & \min\left\{B\pi, \frac{\pi\bar{d}}{p_1(\pi)}\left(n\left(p_1\left(\pi, a_0^M\right), a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right)\right\}. \end{aligned} \tag{2}$$

The left hand side of this expression represents the opportunity cost of offering support to the investor, net of incoming fees. The right hand side includes both the manager's willingness and ability to offer support. The first term inside the curly brackets captures the manager's willingness to offer support. Since the manager loses $B\pi$ if the fund is liquidated early, he will be willing to offer support, on top of the incoming fees, to up to an amount equal to his loss, *i.e.*, $B\pi$. The second term, captures the manager's ability to offer support. The manager will only be able to offer support up to the amount of resources he owns. To offer support, the manager has to be both willing and able to offer support. This is captured by the minimum operator.

Proposition 2 *Given a portfolio choice for managers at $t = 0$ a_0^M, there exists a threshold $\pi^*\left(a_0^M\right)$ such that there is a symmetric equilibrium in which all managers offer support, i.e., $s^*(\pi) = 1$, iff $\pi^*\left(a_0^{M*}\right) \leq \pi < \pi_x\left(a_0^{M*}\right)$.*

Proposition 3 *Given a portfolio choice for managers at $t = 0$ a_0^M, there exists a threshold $\pi^{**}\left(a_0^M\right)$ such that there is a symmetric equilibrium in which all managers choose not to offer support, i.e., $s^*(\pi) = 0$, iff $\pi \in \left[\pi_x\left(a_0^{M*}\right), \bar{\pi}\right] \cup \left[\underline{\pi}, \pi^{**}\left(a_0^M\right)\right)$.*

The proof of these propositions relies on the monotonicity and affinity of $p_1^S(\pi)$ and $p_1^L(\pi)$ in π. The appendix characterizes the thresholds $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, and $\pi^{**}\left(a_0^M\right)$, and using these characterizations, the following proposition can be shown.

Corollary 1 *If $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$ there are multiple equilibria when $\pi \in \left[\pi^*\left(a_0^M\right), \pi^{**}\left(a_0^M\right)\right]$. In particular, there is an equilibrium in which all managers offer support, one in which all managers choose not to offer support, and one in which managers choose to offer support with probability $s \in (0,1)$.*

The corollary follows from propositions 2 and 3. If all managers choose (not) to offer support, the net demand for the risky asset is high (low) and the equilibrium liquidation price for the risky asset is high (low). Given this high (low) price, the cost of offering support is low (high) for an individual managers and, thus, he has more (less) incentives to offer support. This strategic complementarity between the managers support decision through the equilibrium liquidation price of the asset is only present in the model because support is voluntary. For the remainder of this paper, I will assume that whenever there are multiple equilibria, managers will coordinate in the equilibrium in which all managers offer support.

Proposition 4 *If $\pi^{**}\left(a_0^M\right) < \pi^*\left(a_0^M\right)$ there is a unique equilibrium for each value of π and $s(\pi) \in (0,1)$ for $\pi \in \left(\pi^{**}\left(a_0^M\right), \pi^*\left(a_0^M\right)\right)$.*

If there is a unique equilibrium for all values of π, i.e., if $\pi^{**} \leq \pi^*$, managers may choose to offer support with probability $s \in (0,1)$. For $\pi \in (\pi^{**}, \pi^*)$ an individual manager chooses to offer support when he expects no other manager to offer support and he chooses not to offer support when he expects all other managers to offer support. Therefore, there is no symmetric equilibrium in pure strategies for the support decision. Nevertheless, there is an equilibrium for these values of π in which managers are indifferent between offering support and not doing so if they expect all other managers to offer support with positive probability but not for sure and, therefore, he finds it optimal to do follow the same strategy other managers follow.

Runs on the risky asset market The managers' support decisions determine the demand for the risky asset in the interim period and, through it, the liquidation price of the risky asset. If a manager chooses not to offer support when that is needed to keep the fund open, his fund breaks the buck and is liquidated. Upon liquidation, the manager cannot offer intermediation services at $t = 1$ and his investor is excluded from the risky asset market. This implies that the manager's demand for the risky asset is going to be lower if he chooses not to offer support than if he kept the fund open and managed funds for his investor.[14] If all managers choose not to offer support, the total demand for the risky asset in the interim period and, therefore, the liquidation price of the risky asset will be lower than if all funds remained open.

Moreover, the managers' support decisions depend on the liquidation price of the risky asset. This liquidation price will determine the costs and the benefits of offering support (as one can see from 2). To illustrate this mechanism, suppose there are no spillover losses, *i.e.*, $B = 0$. On the one hand, a low liquidation price of the risky asset translates into a low NAV and a high cost of offering support. On the other hand, a low price of the risky asset also implies a high expected return of investing in the risky asset and increases investors' incentives to invest with managers. A higher volume of intermediation increases the fees earned by the managers and the benefits of offering support and avoiding the liquidation of the fund. If the increase in the cost dominates the increase in the benefit of offering support, there may be strategic complementarities in the managers' support decisions and multiple equilibria may arise. As seen from corollary 1, these complementarities arise when $\pi^* < \pi^{**}$.

When $\pi^* < \pi^{**}$, if an individual manager expects all other managers (not) to offer support and liquidate their funds, he expects a high (low) demand for the risky asset in the interim period, a high (low) liquidation price of the risky asset, and a low (high) cost of offering support which will increase (decrease) the manager's incentives to offer support. This source of complementarity gives rise to self fulfilling equilibria that may lead to runs on the risky asset market. These runs are different from the classical bank runs: they are not runs of investors on intermediaries but runs of intermediaries on each other through he risky asset

[14] Assuming that the amount of support does not exceed the total amount investors invest with their managers if they receive support.

market and fire sales.[15]

4.1.5 Equilibrium price

Given the equilibrium support decision for the managers and the portfolio choice a_0^M one can compute the equilibrium liquidation price of the risky asset.

Assumption B is such that the amount of support offered does not exceed the total amount investors invest with their manager when they receive support, i.e.,

$$A_1^I < fA_1^I + \min\left\{B\frac{p_1^L}{\bar{d}}, n\left(p_1^L, a_0^M\right)\left(W_0^M + fA_0^I\right) + E\right\}.$$

This assumption holds for $B = 0$ and for values of B low enough. Moreover, it implies that the demand for the risky asset is lower when sponsors decide not to offer support than when they do and it guarantees the monotonicity of the price function.[16] For the remainder of the paper, I will assume this assumption holds.

Proposition 5 *Under assumption* 4.1.5*, the price function* $p_1^*\left(\pi, a_0^M\right)$ *is non-decreasing* π *for all* $\pi \in [\underline{\pi}, \bar{\pi}]$.

The liquidation price of the risky asset will be higher the higher the probability of success π. A higher realization of π implies a higher expected return of investing in the risky asset and a higher demand for it. Since the supply of the risky asset at $t = 1$ is fixed, the equilibrium price has to increase with π for the market to clear.

Figure 4 depicts the price function when (a) $\pi^{**} < \pi^*$ and when (b) $\pi^* < \pi^{**}$ (assuming that the support equilibrium is played whenever there are multiple equilibria). When all funds are liquidated, *i.e.*, $\pi < \min\{\pi^{**}, \pi^*\}$, only managers participate in the risky asset market. Since managers are risk neutral, they will either invest everything they have in the risky asset or they will be indifferent between investing in the risky asset and in the safe

[15] The possibility of fire sales arises due to the *cash in the market pricing* in the risky asset market as in Allen and Gale (1994). See Shelifer and Vishny (2011) for a review of fire sales in the finance and macroeconomics literature.

[16] This assumption doesn't seem unrealistic. Between 2007 and 2010, the maximum amount of support offered by a sponsor accounted for 3% of the fund.

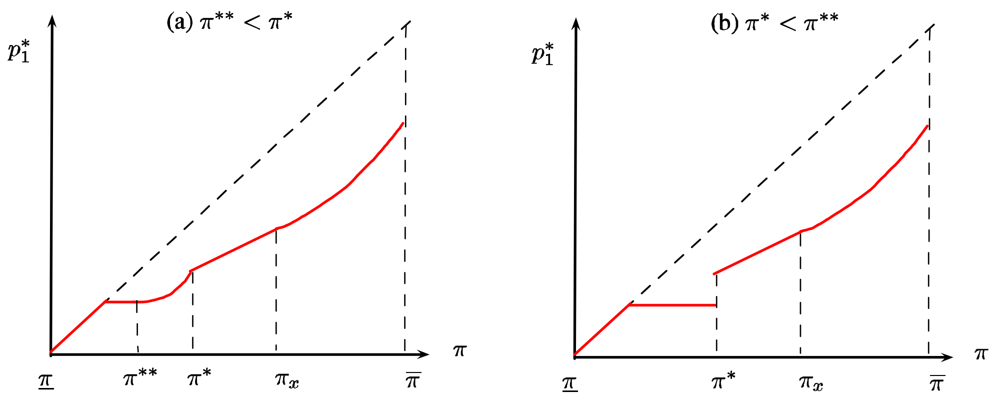

Figure 4: The red solid line is the liquidation price of the risky asset at $t = 1$ as a function of π. The black dotted line is the expected discounted dividend payed by the risky asset.

asset. If the demand for the risky asset is determined by the cash held by managers, *i.e.*, by $\left(1 - a_0^M\right)\left(W_0^M + fA_0^I\right) + E$, the market clearing price will be given by p_1^L, where

$$\frac{\left(1 - a_0^M\right)\left(W_0^M + fA_0^I\right) + E}{p_1^L} = (1 - f)\, a_0^M A_0^I \tag{3}$$

where the right hand side is the amount of risky asset held by investors indirectly through their shares. Moreover, in equilibrium $p_1^*(\pi) \leq \bar{d}\pi q_1$ otherwise the demand for the risky asset at $t = 1$ would be 0 and the market would not clear since the supply of the risky asset at $t = 1$ is fixed and positive. Thus, if the funds are liquidated, the equilibrium liquidation price will be $p_1^*(\pi) = \min\left\{\bar{d}\pi q_1, p_1^L\right\}$. If $p_1^*(\pi) = \bar{d}\pi q_1$ when liquidation occurs, $a_1^M(p_1, \pi)$ adjusts to clear the market.

If at least some funds continue operating at $t = 1$, *i.e.*, $\pi \geq \min\{\pi^{**}, \pi^*\}$, investors can invest in the risky asset, albeit indirectly. In this case, the demand for the risky asset also depends on the investors' propensity to invest with their managers, $a_1^I(p_1, \pi)$. Since this propensity is increasing in π, the demand for the risky asset is increasing in π, and so is the equilibrium liquidation price of the risky asset.

When $\pi^* < \pi^{**}$ there may be a jump in the price function at π^* which captures the sharp decrease in the demand for the risky asset: for $\pi > \pi^*$ all funds are open whereas for $\pi < \pi^*$ all funds are liquidated.

4.1.6 Individual support decisions

In order to characterize the manager's problem completely at $t = 0$, one needs to understand the individual manager's support decision and how this decision depends on the manager's portfolio choice in the initial period.

Proposition 6 *There exist unique thresholds $\pi_{x,i}\left(a_{0,i}^M\right)$ and $\pi_i^*\left(a_{0,i}^M\right)$ such that an individual managers will strictly prefer to offer support, iff $\pi \in \left(\pi_i^*\left(a_{0,i}^M\right), \pi_{x,i}\left(a_{0,i}^M\right)\right)$.*

This proof of this proposition follows from the monotonicity of the price function and can be found in the appendix. The intuition for these results is analogous to the one presented for the aggregate thresholds π_x and π^*.

4.2 t=0

4.2.1 Investor's problem

An investor will make his portfolio choice at $t = 0$ to maximize his expected utility anticipating the equilibrium that will be played at $t = 1$ for each realization of π, and taking his manager's portfolio choices and prices as given.

An investor with manager i solves

$$\max_{a_0^I \in [0,1]} \mathbb{E}_\pi \left[\log\left(W_2^I\left(a_0^I; a_{0i}^M, \pi_i^*, \pi_{x,i}, \bar{s}\left(\pi\right), \pi\right)\right)\right]$$

where $\bar{s}_i\left(\pi\right) \in \{0,1\}$ is a manager's realized support decision and $W_2^I\left(a_0^I;\cdot\right)$ is his wealth at $t = 2$. This wealth depends on the investor's portfolio choice at $t = 0$, on manager i's portfolio choice at $t = 0$ and support decisions at $t = 1$, and on the realized quality of the long term asset, π. The investor's problem is concave in a_0^I and has a unique solution.

4.2.2 Manager's problem

Since for $\pi < \pi^*\left(a_0^M\right)$ the manager either prefers not to offer support or is indifferent between offering support and not doing so, I can solve the manager's problem by assuming that he will liquidate the fund if $\pi < \pi^*\left(a_0^M\right)$. Then, manager i solves $V_0^M\left(W_0^M; f\right) =$

$$\max_{a_{0,i}^M \in [0,1]} \mathbb{E}_\pi \left(W_2^M\left(a_{0,i}^M; \pi_{x,i}\left(a_{0,i}^M\right), \pi_i^*\left(a_{0,i}^M\right), a_0^I, \pi\right)\right) - \mathbb{E}_\pi\left(\mathbf{1}\left\{\pi \leq \pi_i^*\left(a_{0,i}^M\right)\right\} B\pi\right)$$

where $\mathbf{1}$ is the indicator function, and $W_2^M\left(a_{0,i}^M;\cdot\right)$ is the manager's wealth at $t=2$. W_2^M depends on the manager's portfolio choice at $t=0$ directly, and indirectly through the support thresholds. Moreover, it depends on the investor's portfolio choice at $t=0$ and on π. The manager makes his portfolio choice taking his investor's decisions and prices as given.

As it is usual in the presence of threshold decisions, the manager's objective function is not well-behaved. The liquidation rule and the possibility of offering support change the manager's attitude towards risk for different portfolio choices, $a_{0,i}^M$, from risk neutral to risk averse to risk lover. Nevertheless, the manager's problem can be fully characterized: there are two possible candidates at which the maximum would be attained: the highest $a_{0,i}^M$ such that there is no risk of liquidating the fund, and $a_0^M=1$. To avoid dealing with cases, I will use numerical examples in the policy section to illustrate the mechanisms at play. The manager's objective function is characterized in the appendix as is W_2^M .

4.2.3 Risky asset market at t=0

The equilibrium price in the risky asset market at $t=0$, p_0^*, is determined by

$$S\left(p_0^*\right)=\frac{a_0^M}{p_0^*}\left(W_0^M+A_0^I\right)$$

where the right hand side is the total amount invested in the risky asset at $t=0$, *i.e.*, the fraction managers choose to invest in the risky asset, a_0^M, times the size of the funds, $W_0^M+a_0^I W_0^I$, divided by the price of the risky asset, p_0.

4.2.4 Free Entry

Finally, to close the model, the equilibrium fees are determined by the indifference condition for managers. If a manager chooses to open a fund, he incurs in operating costs $C>0$ and gets utility $V_0^M\left(W_0^M;f^*\right)$. If he chooses not to open a fund, he invests his own wealth to maximize his utility. Therefore, in equilibrium, intermediation fees f^* are such that

$$V_0^M\left(W_0^M;f^*\right)-C=\max_{a\in[0,1]}\int_{\underline{\pi}}^{\bar{\pi}}\left(\frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(\left(a\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)W_0^M+E\right)\right)d\pi.$$

5 Results

As I mentioned above, the equilibrium at $t = 1$ can be characterized in closed form for any $t = 0$ choices, but the managers' problem at $t = 0$ is not well-behaved. In this section I provide theoretical results for the special case in which $B = 0$. The appendix contains numerical examples for cases in which $B > 0$ that illustrate the mechanisms at play. The proofs for all propositions in this section are in the appendix.

Proposition 7 *If $B = 0$, $a_0^{M*} = 1$ in a symmetric equilibrium.*

Increasing the position in the risky asset has the following three effects for the manager: it increases the expected return on the manager's wealth, it increases the expected collected fees from intermediation at $t = 1$, and it also increases the probability of breaking the buck. Without any extra cost from liquidating the fund early other than the forgone fees, in equilibrium, the benefits from taking risk offset the costs, and managers are better off taking as much risk as they can. This is shown by proposition 7.

The model is consistent with several stylized facts documented in the literature. Chernenko and Sunderam (2012), Christoffersen and Musto (2002), and Kacperczyk and Schnabl (2012) document a strong performance-flow relation in the MMF industry: they show that MMFs inflows are highly responsive to the fund's past returns. As shown in the next proposition, the model developed in this paper is consistent with this finding.

Proposition 8 *The amount of assets managed for investors at $t = 1$, A_1^I, is increasing in the net asset value, $n\left(p_1, a_0^M\right)$.*

The total amount investors invest with managers at $t = 1$, A_1^I, is a stock and it is correlated 1 to 1 with the flow into the funds, which is given by $A_1^I(\pi) - A_0^I$. Moreover, the funds' performance is measured by the return on the investors' shares, which is $n\left(p_1, a_0^M\right)$ when no support is offered, and x otherwise. Therefore, the proposition above shows that the model captures the positive performance-flow relation present in the data.

Moreover, Kacperczyk and Schnabl (2012), using data from 2007 to 2010, document that MMFs sponsored by companies that also offered non-money market mutual funds and other financial services took on less risk, and that funds sponsored by financial intermediaries with limited financial resources took on less risk. The next two propositions show that the model is consistent with these two stylized facts.

Proposition 9 *Suppose $B = 0$ for all managers but for manager j. Then, in equilibrium, the risk taken by manager j at $t = 0$ will be decreasing in B_j and he will choose $a^M_{0,j} \in \left\{a^{MS}_0, 1\right\}$ where $a^{MS}_0 = \max a^M_{0,i}$ s.t. $\pi_{x,i}\left(a^M_{0,i}\right) = \underline{\pi}$.*

Larger B_j implies a higher cost of liquidating the fund early for the manager. Investing in the risky asset has two effects on the manager's expected wealth. On one hand, a higher exposure to the risky asset increases the manager's portfolio expected return and, from proposition 8, the expected fees collected from managing the investor's funds at $t = 1$. On the other hand, a higher $a^M_{0,j}$ increases the probability of early liquidation of the fund which increases the expected losses suffered by the manager due to this early liquidation. When $B = 0$, the latter effect is not present and the manager is always better off by investing everything in the risky asset. If B_j is high enough, the loss from breaking the buck is too high and managers are better off choosing a portfolio such that the fund is never liquidated early. In this last case, managers prefers to forgo expected return to avoid losses from breaking the buck, which are too high.

6 Policy analysis: Floating vs. Stable NAV

The main regulation proposed by the SEC is to abandon the stable NAV in favor of a floating one. Adopting a floating NAV would diminish the risk of sudden redemptions by making MMFs like any other mutual fund and eliminating the possibility of breaking the buck.

In the context of the model developed in this paper, going from a stable to a floating NAV system is equivalent to going from $x > 0$ to $x = 0$. If $x = 0$, the funds managed by the managers become regular mutual funds and there is no breaking the buck rule. In this case, no support is ever offered: 1 unit transferred to investors represents $fa^I_1 < 1$ worth of fees for the manager. The manager does not have any incentive to offer support.

The portfolio decisions at $t = 1$ presented in the benchmark model are independent on x, and, thus, remain unchanged when going from $x > 0$ to $x = 0$. In terms of the thresholds presented in the previous section, $\pi_{x,i}\left(a_{0,i}^M; a_0^M\right) = \pi_i^*\left(a_{0,i}^M; a_0^M\right) = 0$ for all $\left(a_{0,i}^M; a_0^M\right) \in [0,1]^2$ when $x = 0$. The manager's problem at $t = 1$ now becomes

$$\max_{a_{0,i}^M \in [0,1]} \mathbb{E}_\pi \left[\frac{\pi \bar{d}}{p_1^*(\pi)} \left(n\left(p_1(\pi), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E\right)\right]$$
$$+\mathbb{E}_\pi \left[\frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(n\left(p_1(\pi), a_{0,i}^M\right)(1-f) + \left(1 - a_0^{I*}\right)\frac{1}{q_0}\right) W_0^I\right]$$

This implies that there is no downside to taking risk, and therefore $a_0^{M*f} = 1$.

The payoff for investors from investing with the manager is affected in many different ways. If all decisions at $t = 0$ remained unchanged and the liquidation price of the risky asset was kept fixed, investors would lose insurance going from a stable NAV system to a floating one, *i.e.*, they would lose the transfer they were receiving from their managers in all the states in which support was offered. Figure 5 illustrates this argument. However, when all funds go from a stable to a floating NAV system, the liquidation price of the risky asset changes. In particular, keeping all decisions at $t = 0$ unchanged, the equilibrium liquidation price of the risky asset is (weakly) higher when $x = 0$. To see this, first note that the demand for the risky asset increases in those states in which some support was offered when $x > 0$. When support is offered, for each unit managers transfer to investors, only a fraction a_1^I goes to the risky asset market, whereas if managers kept this for themselves they would invest it all in the risky asset. Moreover, for those states in which the fund was liquidated but investors would still have liked to invest in the risky asset via the managers, demand also goes up. When $x > 0$, they were prevented from investing because the funds were closed, but now they can do so. These effects drive the demand for the risky asset up, and increase the equilibrium liquidation price. This change is illustrated in figure 6.

Therefore, when one considers the change in the liquidation price of the risky asset, one can see that investors prefer $x > 0$ in those states in which support is offered with high probability but are better off when $x = 0$ in the states in which the fund is very likely to be liquidated. Figure 7 illustrates this tradeoff.

So far, this analysis keeps all decisions at $t = 0$ fixed. Figure 8 compares the equilibria for different values of W_0^M for $x > 0$ and $x = 0$. When going from a stable to a floating NAV system, managers need to be compensated less to offer intermediation services because they don't risk facing losses B. Putting this together with the partial equilibrium analy-

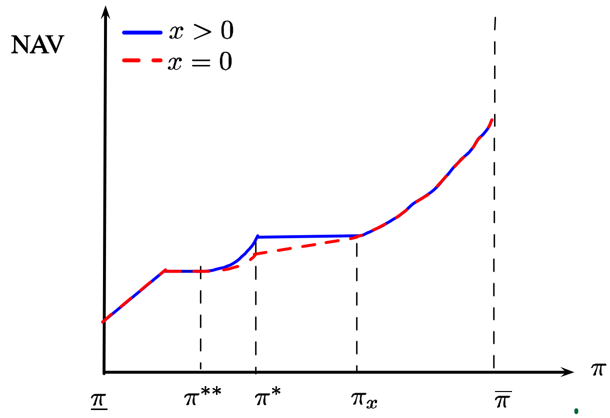

Figure 5: Investor's payoff: The solid (dotted) line represents the payoff for an investor who invests with a manager that offers (does not offer) support when everyone else is offering support. Suport is offered for all values of π between the vertical dotted lines.

sis presented above, one can see that there are countervailing effects on the incentives for investors to invest with managers. In the example computed here, both the risk and the expected return of investing with a manager go down for investors. What happens with the total amount intermediated depends on which effect is larger. In the example shown here, the intermediation level, and thus the supply of liquidity, increase when going from a stable to a floating NAV.

Finally, welfare levels for managers and investors measured in consumption equivalence. In this example managers are worse off when adopting a floating NAV while investors are better off. This is consistent with the industry opposing abandoning the stable NAV and the SEC arguing for it.

7 Conclusion

In this paper, I developed a novel model of MMFs. The model incorporates several features that are characteristic of MMFs: investors' ability to redeem their shares on demand, the stability of the NAV and the provision of sponsor support, the liquidation of the funds after breaking the buck, and the fluctuation in the value of the funds' assets.

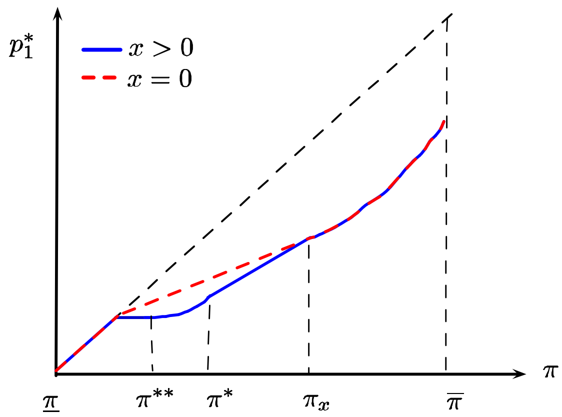

Figure 6: Liquidation price of the risky asset when going from a stable NAV ($x > 0$) to a floating NAV ($x = 0$) fixing all decisions at $t = 0$.

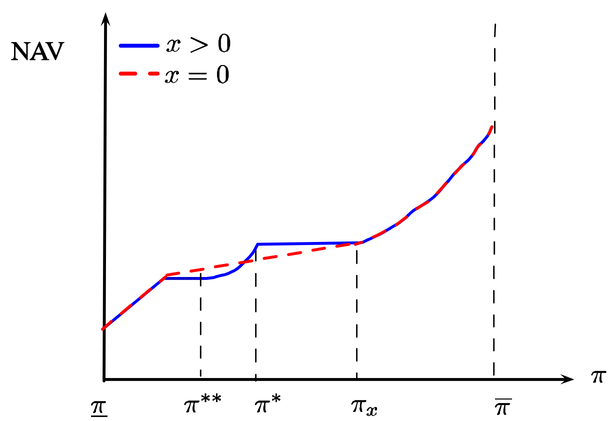

Figure 7: Payoff for investors from investing with a manager when going from a stable NAV ($x > 0$) to a floating NAV ($x = 0$) fixing all decisions at $t = 0$.

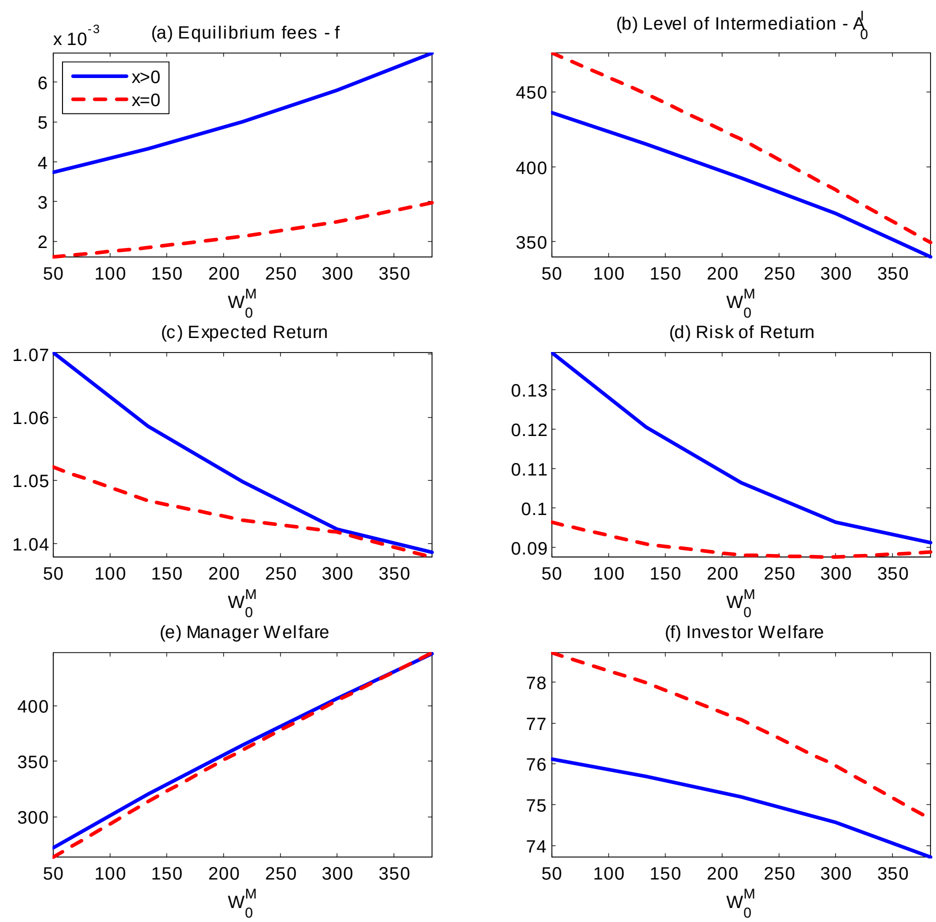

Figure 8: Stable vs. Floating NAV.

The model shows that, even in the absence of penny-rounding and amortized cost accounting, the MMF industry may be fragile. Sponsor support may subject the MMF insdustry to a source of fragility that differs from the canonical bank run: there may be strategic complementarities in the sponsors' support decisions that may give rise to multiple equilibria and to runs of the MMFs on the asset market.

This model is consistent with stylized facts documented in the literature on MMFs. It captures the positive performance-flow sensitivity in the MMF industry and the difference in incentives for risk taking for funds with different sponsors. In particular, it is consistent with the fact that MMFs sponsored by companies that also offer non-money market mutual funds and other financial services tend to take on less risk.

I use this model to analyze the tradeoffs involved in the adoption of a floating NAV. The consequences of this regulation depend on the interaction between potentially countervailing effects. Changing the institutional setup of the MMF industry would affect the risks and returns of intermediation for investors and MMF managers not only directly, but also through the change in equilibrium outcomes such as intermediation fees, the sponsors' support decision, and asset prices. The model allows me to take into account these general equilibrium effects, which seem particularly important given the relative size of the MMF industry in the market for short term financing.

In an example, I show that adopting a floating NAV drives down both the risk and the return of investing in a MMF for investors. Contrary to what one may think at first, the overall effect on the intermediation level is positive, and, thus, in these examples, if MMFs were forced to adopt a floating NAV the provision of liquidity in the money market would increase. This highlights the usefulness of looking at the policy discussion through the lens of a model.

References

Allen, Franklin and Douglas Gale (1994), "Limited participation and volatility of asset prices." *The American Economics Review*, 84 (4), 933–955.

Allen, Franklin and Douglas Gale (2005), "From cash-in-the-market pricing to financial fragility." *Journal of the European Economic Association*, 3 (2-3), 535–546.

Bhattacharya, Sudipto and Paul Pfleiderer (1985), "Delegated portfolio management." *Journal of Economic Theory*, 36(1), 1–25.

Board of Governors of the Federal Reserve System (2009), "Monetary policy report to the congress." 24 February 2009.

Brady, Steffanie A., Ken E. Anadu, and Nathaniel R. Cooper (2012), "The stability of prime money market mutual funds: Sponsor support from 2007 to 2011." *Federal Reserve Bank of Boston, Risk and Policy Analysis Working Papers*, RPA 12-3.

Carnell, Richard Scott, Jonathan R. Macey, and Geoffrey P. Miller (2008), *The Law of Banking and Financial Institutions*. Aspen Publishers.

Chen, Qi, Itay Goldstein, and Wei Jiang (2010), "Payoff complementarities and financial fragility: Evidence from mutual fund outflows." *Journal of Financial Economics*, 97, 239–262.

Chernenko, Sergey and Adi Sunderam (2012), "Frictions in shadow banking: Evidence from the lending behavior of money market funds." *mimeo*.

Christoffersen, Susan E. K and David K. Musto (2002), "Demand curves and the pricing of money management." *Review of Financial Studies*, 15, 1499–1524.

Diamond, Douglas W. and Philip H. Dybvig (1983), "Bank runs, deposit insurance, and liquidity." *Journal of Political Economy*, 91(3), 401–419.

Financial Stability Oversight Council (FSOC) (2011), "Annual report."

Fisch, Jill and Eric Roiter (2011), "A floating nav for money market funds: Fix or fantasy?" *Scholarship at Penn Law.*, Paper 390.

Hanson, Samuel G., David S. Scharfstein, and Adi Sunderam (2012), "An evaluation of money market fund reform proposals." *mimeo*.

He, Zhiguo and Arvind Krishnamurthy (2012), "A model of capital and crises." *The Reveiw of Economics Studies*, 79 (2), 735–777.

Investment Company Institute (ICI) (2012), "2012 investment company fact book."

Kacperczyk, Marcin and Philipp Schnabl (2012), "How safe are money market funds?" *mimeo*.

Lacker, Jeffrey M. (2011), "Letter to the securities and exchange commission re: File no. 4-619; release no. ic-29497 presidentŠs working group report on money market fund reform." 10 January 2011.

Lorenzoni, Guido (2008), "Inefficient credit booms." *Review of Economic Studies*, 75 (3), 809–833.

Mackenzie, Michael (2011), "Prospect of shortage drives us t-bill yields down." *Financial Times*, May 3, 2011.

McCabe, Patrick (2010), "The cross section of money market fund risks and financial crises." *Federal Reserve Board Finance and Economics Discussion Series*, Working Paper 2010-51.

McCabe, Patrick (2011), "An a-/b- capital buffer proposal for money market funds." *mimeo*.

McCabe, Patrick, Marco Cipriani, Michael Holscher, and Antoine Martin (2012), "The minimum balance at risk: A proposal to mitigate the systemic risks posed by money market funds." *Federal Reserve Bank of New York Staff Reports*, Staff Report No. 564.

Mendelson, Simon and Richard Hoerner (2011), "Blackrock letter to the securities and exchange commission re: Release no. ic-29497; file no. 4-619." 10 January 2011.

Moody's Investors Service (2010), "Sponsor support key to money market funds." 8 August 2010.

President's Working Group on Financial Markets (PWG) (2010), "Report of the president's working group on financial markets: Money market fund reform options."

Shelifer, Andrei and Robert Vishny (2011), "Fire sales in finances and macroeconomics." *Journal of Economic Perspectives*, 25 (1), 29–48.

Squam Lake Group (2011), "Reforming money market funds." January 14, 2011.

U.S. Securities and Exchange Commission (2009), "Securities and exchange commission v. reserve management company, inc. et al." 8 June 2009, Case 1:09–cv–04346–PGG.

Volcker, Paul A. (2011), "Letter to the securities and exchange commission re: File no. 4-619; release no. ic-29497 president's working group report on money market fund reform." 11 February 2011.

Wermers, Russ (2012), "Runs on money market mutual funds." *mimeo.*

8 Appendix

8.1 Equilibrium threshold characterization

This section finds closed form solutions for the thresholds $\pi_x\left(a_0^M\right)$ and $\pi^*\left(a_0^M\right)$.

$\pi_x\left(a_0^M\right)$ is given by

$$\frac{p_1^{NS}\left(\pi_x\left(a_0^M\right)\right)}{p_0}a_0^M+\left(1-a_0^M\right)\frac{1}{q_0}=x.$$

using that $p_1^{NS}\left(\pi_x\left(a_0^M\right)\right)=p_1^S\left(\pi_x\left(a_0^M\right)\right)$ and the definition of $p_1^S\left(\pi\right)$,

$$\pi_x\left(a_0^M\right)=\begin{cases}\left(\left(x-\frac{1}{q_0}\right)+a_0^M\frac{1}{q_0}\right)\frac{p_0}{a_0^M}\frac{\left(1-a_1^{IS}\right)}{\bar{d}(1-f)q_1}+a_1^{IS} & a_1^{IS}>0\\ \left(\left(x-\frac{1}{q_0}\right)+a_0^M\frac{1}{q_0}\right)\frac{p_0}{a_0^M}\frac{1}{\bar{d}q_1} & \text{else}\end{cases}.$$

The equilibrium support threshold is determined by

$$\begin{aligned}&\frac{\pi^*\bar{d}}{p_1^S\left(\pi^*\right)}\left(\left(x-n\left(p_1^S\left(\pi^*\right),a_0^M\right)\right)(1-f)A_0^I-fA_1^I\left(p_1^S\left(\pi^*\right),\pi^*\right)\right)\\ =\ &\min\left\{B\pi^*,\frac{\pi^*\bar{d}}{p_1^S\left(\pi^*\right)}n\left(p_1^S\left(\pi^*\right),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right\}.\end{aligned}$$

Then, $\pi^*\left(a_0^M\right)$ is determined by

$$p_1^S\left(\pi^*\left(a_0^M\right)\right)=\begin{cases}\dfrac{\left(\left(1-fa_1^{IS}\right)x(1-f)a_0^I-fa_1^{IS}\frac{\left(1-a_0^I\right)}{q_0}-\frac{\left(1-a_0^M\right)}{q_0}(1-f)a_0^I\right)W_0^I}{\frac{B}{\bar{d}}+\frac{a_0^M}{P_0}(1-f)a_0^IW_0^I} & \chi\left(a_0^M\right)=1\\ \dfrac{\left(\left(1-fa_1^{IS}\right)x(1-f)a_0^I-fa_1^{IS}\frac{\left(1-a_0^I\right)}{q_0}\right)W_0^I-\frac{\left(1-a_0^M\right)}{q_0}\left(W_0^M+a_0^IW_0^I\right)-E}{\frac{a_0^M}{P_0}\left(W_0^M+a_0^IW_0^I\right)} & \chi\left(a_0^M\right)=0\end{cases}$$

where $a_1^{IS}=0$ if $p_1^S\left(\pi^*\left(a_0^M\right)\right)=\bar{d}\pi^*\left(a_0^M\right)q_1$ and

$$\chi\left(a_0^M\right)=\begin{cases}1 & \frac{B}{\bar{d}}p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right)<\left(n\left(p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right),a_0^M\right)\left(W_0^M+fa_0^{I*}W_0^I\right)+E\right)\\ 0 & \frac{B}{\bar{d}}p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right)\geq\left(n\left(p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right),a_0^M\right)\left(W_0^M+fa_0^{I*}W_0^I\right)+E\right)\end{cases}.$$

Finally, $\pi^{**}\left(a_0^M\right)$ is given by

$$\begin{aligned}&\frac{\pi^{**}\bar{d}}{p_1^L\left(\pi^{**}\right)}\left(\left(x-n\left(p_1^L\left(\pi^{**}\right),a_0^M\right)\right)(1-f)A_0^I-fA_1^I\left(p_1^L\left(\pi^{**}\right),\pi^{**}\right)\right)\\ =\ &\min\left\{B\pi^{**},\frac{\pi^{**}\bar{d}}{p_1^L\left(\pi^{**}\right)}n\left(p_1^L\left(\pi^{**}\right),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right\}.\end{aligned}$$

Proposition 10 *The thresholds* $\pi_x\left(a_0^M\right)$, $\pi^*\left(a_0^M\right)$, *and* $\pi^{**}\left(a_0^M\right)$ *are increasing in* a_0^M.

The net asset value, $n\left(p_1, a_0^M\right)$, is decreasing in a_0^M for all $p_1 < p_0/q_0$. Since $x < \frac{1}{q_0}$ by assumption, $p_1^S(\pi) < p_0/q_0$ for all $\pi < \pi_x\left(a_0^M\right)$, *i.e.*, when support is needed. Therefore, the higher the exposure to the risky asset at $t = 0$, the lower the net asset value when support is needed for any given price. This implies that a higher a_0^M requires a higher liquidation price for the manager not to need to offer support. Since the price function is increasing in π for $\pi \geq \pi_x\left(a_0^M\right)$, higher a_0^M requires higher realizations of π not to need to offer support, *i.e.*, a higher π_x. The same argument can be applied to the thresholds π^* and π^{**}.

The following proposition characterizes the support region, and shows that managers will offer support in some states π as long as they have something to lose from closing the fund, either from forgone fees of from spillover losses.

Proposition 11 $\pi^*\left(a_0^M\right) \leq \pi_x\left(a_0^M\right)$ *and* $\pi^*\left(a_0^M\right) < \pi_x\left(a_0^M\right)$ *iff* $A_1^I\left(p_1\left(\pi_x\left(a_0^M\right)\right), \pi_x\left(a_0^M\right)\right) > 0$ *or* $B > 0$.

Proof. From the definition of $\pi_x\left(a_0^M\right)$ it is easy to see that (2) always holds for $\pi_x\left(a_0^M\right)$. Therefore $\pi_x\left(a_0^M\right) \geq \pi^*\left(a_0^M\right)$. Moreover, if $A_1^I\left(p_1\left(\pi_x\left(a_0^M\right)\right), \pi_x\left(a_0^M\right)\right)$ or $B > 0$, (2) holds with strict inequality at $\pi_x\left(a_0^M\right)$ which implies that $\pi_x\left(a_0^M\right) > \pi^*\left(a_0^M\right)$. ■

8.2 Equilibrium Price

When the return on the manager's portfolio is above x, no support is needed to keep the fund operating. In this case, if a_1^{I*} is interior, the equilibrium price is $p_1^{NS}(\pi) =$

$$p_1^{NS}(\pi) = \min\left\{ \frac{(1-f)\bar{d}\left(\pi\left(\left(1-a_0^M\right)(1-f)a_0^I + \left(1-a_0^I\right)\right)\frac{1}{q_0}W_0^I + \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + f a_0^I W_0^I\right) + E\right)}{\frac{a_0^M}{p_0} a_0^I (1-f)^2 (1-\pi)\bar{d}W_0^I + \frac{1}{q_1}\frac{1}{q_0}\left(\left(1-a_0^I\right)W_0^I + \left(1-a_0^M\right)\left(W_0^M + a_0^I W_0^I\right)\right) + \frac{E}{q_1}}, \bar{d}\pi q_1 \right\}.$$

If support is provided by the manager, and a_1^{I*} is interior, the equilibrium price, $p_1^S(\pi)$, is determined by

$$\begin{aligned} &\left(x(1-f)a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^I W_0^I\right) - E\right) \\ = \;& a_1^{I*}\left(p_1(\pi), \pi\right)\left(x(1-f)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)W_0^I. \end{aligned}$$

If $x(1-f)a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^I W_0^I\right) - E < 0$, there is an excess demand for the risky asset at every price $p_1(\pi) < \bar{d}\pi q_1$. Therefore, in this case, $p_1(\pi) = \bar{d}\pi q_1$, $a_1^{I*}(p_1(\pi),\pi) = 0$ and the share invested by managers in the risky asset, $\bar{a}_1^M(\pi)$, is given by

$$\begin{aligned} 0 &= \left(1-\bar{a}_1^M(\pi)\right)\frac{\bar{d}\pi q_1}{p_0}a_0^M\left(W_0^M + a_0^I W_0^I\right) \\ &\quad + \bar{a}_1^M(\pi)\left(x(1-f)a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^I W_0^I\right) - E\right). \end{aligned}$$

If $x(1-f)a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^I W_0^I\right) - E > 0$ the equilibrium price is given by

$$a_1^{I*}(p_1(\pi),\pi) = \frac{\left(x(1-f)a_0^I W_0^I - \left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + a_0^I W_0^I\right) - E\right)}{\left(x(1-f)a_0^I + (1-a_0^I)\frac{1}{q_0}\right)W_0^I} =: a_1^{IS}\left(a_0^M\right).$$

This condition implies an affine equation in $\bar{d}/p_1^S(\varphi)$, which gives the following equilibrium price

$$p_1^S(\pi) = \min\left\{\max\left\{\frac{\left(\pi - a_1^{IS}\right)}{\frac{1}{q_1}\left(1-a_1^{IS}\right)}(1-f)\bar{d}, 0\right\}, \bar{d}\pi q_1\right\}.$$

Note that, when there is support, the share investors choose to invest with the manager in period 1 is independent of π.

If sponsors choose to offer support with probability s, assuming the law of large numbers holds, a mass s of managers will offer support and keep the fund open while a mass $(1-s)$ will choose not to offer support and liquidate the fund early. Then the net demand for the risky asset will be

$$\begin{aligned} D(p_1,\pi) &= s\left(\frac{a_1^M(p_1,\pi)\left(A_1^I(p_1,\pi) + W_1^M(p_1, s=1)\right)}{p_1} - \frac{a_0^M\left(a_0^I W_0^I + W_0^M\right)}{p_0}\right) \\ &\quad + (1-s)\left(\frac{a_1^M(p_1,\pi)W_1^M(p_1, s=0)}{p_1} - \frac{a_0^M\left(a_0^I W_0^I + W_0^M\right)}{p_0}\right). \end{aligned}$$

In this case, the equilibrium price if $A_1^I(p_1,\pi)>0$ will be given by

$$\begin{aligned}
0 &= s\left(\left(\pi\bar{d}(1-f)-\frac{p_1}{q_1}\right)\left(1-a_0^I\right)\frac{1}{q_0}-(1-\pi)\bar{d}(1-f)x(1-f)a_0^I\right)W_0^I \\
&\quad+\left(\begin{array}{c}\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right) \\ +E-\frac{p_1}{p_0}a_0^M(1-f)(1-s)a_0^IW_0^I\end{array}\right)\left(\bar{d}(1-f)-\frac{p_1}{q_1}\right) \\
0 &= \bar{d}(1-f)\left(\begin{array}{c}s\left(\pi\left(1-a_0^I\right)\frac{1}{q_0}-(1-\pi)x(1-f)a_0^I\right)W_0^I \\ +\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)+E\end{array}\right) \\
&\quad-\frac{p_1}{q_1}\left(\begin{array}{c}s\left(1-a_0^I\right)\frac{1}{q_0}W_0^I+\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right) \\ +E+\frac{q_1}{p_0}\bar{d}(1-f)\left(a_0^M(1-f)(1-s)a_0^IW_0^I\right)\end{array}\right) \\
&\quad+p_1^2\frac{a_0^M(1-f)(1-s)a_0^IW_0^I}{p_0q_1}
\end{aligned}$$

This quadratic function is negative at $p_1=\bar{d}(1-f)q_1$. Therefore the equilibrium price is given by the smallest root

$$p_1^{SS}(\pi,s)=\frac{-c_1(\pi,s)-\sqrt{c_1(\pi,s)^2-4c_0(\pi,s)c_2(\pi,s)}}{2c_2(\pi,s)}.$$

if $s<1$, where

$$\begin{aligned}
c_0(\pi,s) &= \bar{d}(1-f)\left(\begin{array}{c}s\left(\pi\left(1-a_0^I\right)\frac{1}{q_0}-(1-\pi)x(1-f)a_0^I\right)W_0^I \\ +\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right)+E\end{array}\right) \\
c_1(\pi,s) &= -\frac{1}{q_1}\left(\begin{array}{c}s\left(1-a_0^I\right)\frac{1}{q_0}W_0^I+\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+(f+(1-f)s)a_0^IW_0^I\right) \\ +E+\frac{q_1}{p_0}\bar{d}(1-f)a_0^M(1-f)(1-s)a_0^IW_0^I\end{array}\right) \\
c_2(\pi,s) &= \frac{a_0^M(1-f)(1-s)a_0^IW_0^I}{p_0q_1}
\end{aligned}$$

Otherwise, $A_1^I(p_1,\pi)$ would equal 0. Moreover, since the quadratic function is decreasing and positive at $p_1=0$, $p_1^{SS}>0$.

Finally, if the fund is liquidated the equilibrium price, $p_1^L(\pi)$, is given by

$$p_1^L(\pi)=\min\left\{\frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M+fa_0^IW_0^I\right)+E}{\frac{a_0^M}{p_0}(1-f)a_0^IW_0^I},\bar{d}\pi q_1\right\}$$

and it is determined by the amount of resources managers have that don't come from their holdings of the risky asset, *i.e.*, by the cash in the market.[17]

[17] See Allen and Gale (1994) and Allen and Gale (2005) for more on cash-in-the-market pricing.

Proposition 12 *Under assumption* 4.1.5 *the equilibrium price function* $p_1^*\left(\pi, a_0^M\right)$ *is continuous and non-decreasing in* π *for all* $\pi \in \left[0, \pi^*\left(a_0^M\right)\right) \cup \left(\pi^*\left(a_0^M\right), 1\right]$ *for all* a_0^M.

Proof. To see this note that the expressions for $p_1^{NS}\left(\pi, a_0^M\right)$, $p_1^{SS}\left(\pi, a_0^M\right)$, $p_1^S\left(\pi, a_0^M\right)$ and $p_1^L\left(\pi, a_0^M\right)$ are non-decreasing in π when 4.1.5 holds, that $p_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right) = p_1^{SS}\left(\pi^{**}\left(a_0^M\right), a_0^M\right)$, and that $p_1^{NS}\left(\pi_x, a_0^M\right) = p_1^S\left(\pi_x, a_0^M\right)$. ■

From the definition of $\pi_x\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$, it is easy to see that $p_1^L\left(\pi^{**}\left(a_0^M\right), a_0^M\right) = p_1^{SS}\left(\pi^{**}\left(a_0^M\right), a_0^M\right)$, and that $p_1^{NS}\left(\pi_x, a_0^M\right) = p_1^S\left(\pi_x, a_0^M\right)$. This follows from the continuity of the demand for the risky asset at $\pi_x\left(a_0^M\right)$ and $\pi^{**}\left(a_0^M\right)$. However, the demand for the risky asset may be discontinuous at $\pi^*\left(a_0^M\right)$. For $\pi \geq \pi^*\left(a_0^M\right)$, the demand for the risky asset is equal to the funds' size which include the managers' wealth and the investors' investment with their managers. For $\pi < \pi^*\left(a_0^M\right)$, all funds may be liquidated and the demand for the risky asset is conformed by the managers' wealth only. Then, if all funds are liquidated for $\pi < \pi^*\left(a_0\right)$, *i.e.*, $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$ the demand function is discontinuous at $\pi^*\left(a_0^M\right)$ provided investors choose to invest with the manager when the realized quality of the risky asset is $\pi^*\left(a_0^M\right)$. This, in turn, implies that the equilibrium price may be discontinuous at $\pi^*\left(a_0^M\right)$.

Proposition 5 Under assumption 4.1.5, the price function $p_1^*\left(\pi, a_0^M\right)$ is non-decreasing π for all $\pi \in [\underline{\pi}, \bar{\pi}]$.

Proof. From proposition 12 the price function is non-decreasing when it is continuous. Therefore, to prove the proposition it is enough to show that

$$\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^-} p_1^*\left(\pi, a_0^M\right) < \lim_{\pi \to \pi^*\left(a_0^{M*}\right)^+} p_1^*\left(\pi, a_0^M\right)$$

when the price function is discontinuous. The price is discontinuous only if $\pi^*\left(a_0^M\right) < \pi^{**}\left(a_0^M\right)$. In this case, $\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^-} p_1^*\left(\pi, a_0^M\right) = p_L\left(\pi^*\left(a_0^{M*}\right), a_0^M\right)$ and $\lim_{\pi \to \pi^*\left(a_0^{M*}\right)^+} p_1^*\left(\pi, a_0^M\right) = p_1^S\left(\pi^*\left(a_0^{M*}\right), a_0^M\right)$. Suppose by contradiction that

$$p_L\left(\pi^*\left(a_0^{M*}\right), a_0^M\right) > p_1^S\left(\pi^*\left(a_0^{M*}\right), a_0^M\right).$$

Then, from the definition of $\pi^*\left(a_0^M\right)$ it follows that

$$\begin{aligned} &\left(x-n\left(p_1^L\left(\pi^*\left(a_0^M\right),a_0^M\right),a_0^M\right)\right)(1-f)A_0^I-fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right),\pi^*\left(a_0^M\right)\right) \\ < &\min\left\{\frac{Bp_1^L\left(\pi^*\left(a_0^M\right),a_0^M\right)}{\bar{d}},n\left(p_1^L\left(\pi^*\left(a_0^M\right),a_0^M\right),a_0^M\right)\left(W_0^M+fA_0^I\right)+E\right\}.\end{aligned}$$

Moreover, using that $\pi^*\left(a_0^M\right)<\pi^{**}\left(a_0^M\right)$ and the definition of $\pi^{**}\left(a_0^M\right)$ one gets

$$\begin{aligned} fA_1^I\left(p_1^L\left(\pi^{**}\left(a_0^M\right),a_0^M\right),\pi^{**}\left(a_0^M\right)\right)-fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right),\pi^*\left(a_0^M\right)\right) &< 0 \\ fA_1^I\left(p_1^S\left(\pi^{**}\left(a_0^M\right),a_0^M\right),\pi^{**}\left(a_0^M\right)\right)-fA_1^I\left(p_1^S\left(\pi^*\left(a_0^M\right),a_0^M\right),\pi^*\left(a_0^M\right)\right) &< 0 \end{aligned}$$

But this implies $\pi^*\left(a_0^M\right)>\pi^{**}\left(a_0^M\right)$ since $A_1^I\left(p_1^S\left(\pi,a_0^M\right),\pi\right)$ is increasing in π which is a contradiction. ■

8.3 Individual threshold characterization

Proposition 13 *There exists a unique threshold $\pi_{x,i}\left(a_{0,i}^M\right)$ such that support is not needed if $\pi\geq\pi_{x,i}\left(a_{0,i}^M\right)$.*

Proof. The proof is straightforward using that the price function $p_1^*(\pi)$ is non-decreasing in π. The left hand side of this expression is always increasing in π.The right hand side is constant in π. Therefore, there is a unique threshold $\pi_{x,i}\left(a_{0,i}^M\right)$ such that for all $\pi\geq\pi_{x,i}\left(a_{0,i}^M\right)$ (**??**) holds. ■

As the previous proposition shows, if the realized probability of success of the risky project is high enough, the manager will not need to offer support to his investor to keep the fund open. In this case, the realized net asset value will be above the liquidation threshold x. Moreover, since the net asset value depends on the manager's portfolio choice at $t=0$, how high a realization of π is needed not to need support will depend on this portfolio choice. The following proposition shows that the higher the risk incurred by the manager in the initial period, the higher the realization of π needed not to need to offer support. A full characterization of the threshold $\pi_{x,i}$ is provided in the appendix and shows that $\boldsymbol{\pi}_{x,i}$ is discontinuous in $a_{0,i}^M$ if the price function is constant for an interval $[\pi_a,\pi_b]$ where $\pi_a<\pi_b$.

Proposition 14 $\pi_{x,i}\left(a_{0,i}^M\right)$ *is increasing in* $a_{0,i}^M$

Proof. The left hand side of (**??**) is constant in $a_{0,i}^M$ whereas the right hand side is increasing in $a_{0,i}^M$ since $x \leq \frac{1}{q_0}$. Therefore, $\pi_{x,i}$ is increasing in $a_{0,i}^M$. ■

If $\pi < \pi_{x,i}$, the manager cannot continue operating the fund unless he supports his investor. A manager that chose to invest $a_{0,i}^M$ in the risky asset in period 0, will choose to offer support if

$$H\left(\pi; a_{0,i}^M\right) \geq 0$$

where

$$\begin{aligned} H\left(\pi; a_{0,i}^M\right) \quad : \quad = & \begin{pmatrix} f a_1^{I*}\left(p_1^*\left(\pi\right),\pi\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\right) a_0^{I*}\left(1-f\right) \end{pmatrix} W_0^I \\ & + \min\left\{\frac{Bp_1^*\left(\pi\right)}{\bar{d}}, n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right) + E\right\}. \end{aligned}$$

This condition is analogous to the one presented to compute the aggregate support threshold π^*. The following two propositions characterize the support decision for an individual manager.

Proposition 15 *Characterization of* $\pi_{x,i}$:

$$\pi_{x,i}\left(a_{0.i}^M; a_0^M\right) \begin{cases} = \underline{\pi} & \text{for } a_{0,i}^M \leq \widehat{a}_{0,i}^M \\ \left(\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M} + \frac{1}{q_0}\right)\frac{p_0}{\bar{d}q_1} & \text{for } \widehat{a}_{0,i}^M < a_{0,i}^M \leq \underline{a}_{0,i}^M\left(a_0^M\right) \\ = \pi^*\left(a_0^M\right) & \text{for } \underline{a}_{0,i}^M\left(a_0^M\right) < a_{0,i}^M \leq \bar{a}_{0,i}^M\left(a_0^M\right) \\ > \pi^*\left(a_0^M\right) & \text{for } a_{0,i}^M \geq \bar{a}_{0,i}^M\left(a_0^M\right) \end{cases}$$

where

$$\widehat{a}_{0,i}^M = \frac{\left(x - \frac{1}{q_0}\right)}{\left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0}\right)},$$

$$\pi_1\left(a_0^M\right) = \frac{\left(1-a_0^M\right)\frac{1}{q_0}\left(W_0^M + f a_0^I W_0^I\right) + E}{\frac{a_0^M}{p_0}\left(1-f\right)a_0^I W_0^I \bar{d} q_1},$$

$$\underline{a}_{0,i}^M\left(a_0^M\right) = \frac{\left(x - \frac{1}{q_0}\right)a_0^M\left(1-f\right)a_0^I W_0^I}{\left(\left(1-a_0^M\right)\left(W_0^M + f a_0^I W_0^I\right) - a_0^M\left(1-f\right)a_0^I W_0^I\right)\frac{1}{q_0} + E},$$

and

$$\bar{a}_{0,i}^M\left(a_0^M\right) = \frac{\left(x - \frac{1}{q_0}\right)}{\bar{d}\left(1-f\right)\frac{\pi^*\left(a_0^M\right) - a_1^{IS}}{1-a_1^{IS}} - q_0}$$

Proof. $\pi_{x,i}\left(a_{0.i}^{M};a_{\,0}^{\,M}\right)$ is given by the minimum π such that

$$\frac{p_1^*\left(\pi;a_0^M\right)}{p_0}\geq\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M}+\frac{1}{q_0}. \tag{4}$$

If $a_{0,i}^M\leq\widehat{a}_{0,i}^M$, the right hand side of this expression is ≤ 0 which implies that 4 holds for all $\pi\geq\underline{\pi}$ and therefore $\pi_{x,i}\left(a_{0.i}^{M};a_{\,0}^{\,M}\right)=\underline{\pi}$. If $\underline{\pi}<\pi_1\left(a_{\,0}^{\,M}\right)$, for $\pi\in\left[\underline{\pi},\pi_1\left(a_{\,0}^{\,M}\right)\right]$, $p_1^*\left(\pi;a_0^M\right)=\pi\bar{d}q_1$ and $\pi_{x.i}$ is given by

$$\frac{\pi_{x.i}\bar{d}q_1}{p_0}=\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M}+\frac{1}{q_0}.$$

But $\pi_{x.i}\left(a_{0.i}^{M};a_{\,0}^{\,M}\right)\leq\pi_1\left(a_{\,0}^{\,M}\right)$ iff $a_{0,i}^M\leq\underline{a}_{0,i}^M\left(a_0^M\right)$. For $\pi\in\left(\pi_1\left(a_{\,0}^{\,M}\right),\pi^*\left(a_0^M\right)\right)$, $p_1^*\left(\pi\right)=p_L$. Therefore, $\pi_{x.i}\notin\left(\underline{\pi}\left(a_{\,0}^{\,M}\right),\pi^*\left(a_0^M\right)\right)$ since if 4 holds for one $\pi\in\left(\pi_1\left(a_{\,0}^{\,M}\right),\pi^*\left(a_0^M\right)\right)$, it holds for all $\pi_0\in\left[\pi_1\left(a_{\,0}^{\,M}\right),\pi^*\left(a_0^M\right)\right)$. Moreover, $p_L=\pi_1\left(a_{\,0}^{\,M}\right)\bar{d}q_1$. Thus, $\pi_{x.i}\left(a_{0.i}^{M};a_{\,0}^{\,M}\right)\geq\pi^*\left(a_0^M\right)$ for all $a_{0,i}^M>\underline{a}_{0,i}^M\left(a_0^M\right)$. Finally, for $\underline{a}_{0,i}^M\left(a_0^M\right)<a_{0,i}^M\leq\bar{a}_{0,i}^M\left(a_0^M\right)$

$$\frac{p_1^*\left(\pi^*;a_0^M\right)}{p_0}\geq\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M}+\frac{1}{q_0}$$

and

$$\frac{\lim_{\pi\to\pi^*}p_1^*\left(\pi;a_0^M\right)}{p_0}<\frac{\left(x-\frac{1}{q_0}\right)}{a_{0.i}^M}+\frac{1}{q_0}$$

which implies $\pi_{x,i}\left(a_{0.i}^{M};a_{\,0}^{\,M}\right)=\pi^*\left(a_0^M\right)$. ■

Lemma $H\left(\pi;a_{0,i}^M\right)$ is increasing in π whenever it is continuous.

Proof. If the equilibrium price function is continuous in π, $H\left(\pi;a_{0,i}^M\right)$ is always increasing in π. To see this note that $a_1^{I*}\left(p_1^*\left(\pi\right),\pi\right)$ is always increasing in π if the equilibrium price is continuous in π. ■

Proposition 6 Given the equilibrium price, there exists a unique $\pi_i^*\left(a_{0,i}^M\right)$ such that for all $\pi_i^*\left(a_{0,i}^M\right)<\pi<\pi_{x,i}\left(a_{0,i}^M\right)$ there is an equilibrium in which all individual managers strictly prefer to offer support.

Proof. An individual manager will offer support in all states π such that $H\left(\pi;a_{0,i}^M\right)\geq 0$. If the equilibrium price is continuous in π or if the equilibrium price is discontinuous at $\pi^*\left(a_0^M\right)$ but $\lim_{\pi\to\pi^*}H\left(\pi;a_{0,i}^M\right)<H\left(\pi^*;a_{0,i}^M\right)$, the proof follows from monotonicity of $H\left(\pi;a_{0,i}^M\right)$ in π, using lemma 8.3. If $\lim_{\pi\to\pi^*}H\left(\pi;a_{0,i}^M\right)>H\left(\pi^*;a_{0,i}^M\right)$ and $\lim_{\pi\to\pi^*}H\left(\pi;a_{0,i}^M\right)>H\left(\pi^*;a_{0,i}^M\right)>$

0 or $0 > \lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M\right) > H\left(\pi^*; a_{0,i}^M\right)$, $H\left(\pi; a_{0,i}^M\right)$ crosses 0 only once and the statement of the proposition holds.

Suppose that $\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M\right) > 0 > H\left(\pi^*; a_{0,i}^M\right)$. Then, the set of realizations of π for which the manager offers support is given by $\left[\underline{\pi}_i^*\left(a_{0,i}^M\right), \pi^*\right] \cup \left[\bar{\pi}_i^*\left(a_{0,i}^M\right), 1\right]$ where

$$\begin{aligned} 0 &= \begin{pmatrix} f a_1^{I*}\left(p_1^L\left(\underline{\pi}_i^*\right), \underline{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^L\left(\underline{\pi}_i^*\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right) \end{pmatrix} W_0^I \\ &\quad + \min\left\{\frac{Bp_1^L\left(\underline{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^L\left(\underline{\pi}_i^*\right), a_{0,i}^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right)\right\}, \end{aligned}$$

or $\underline{\pi}_i^*\left(a_{0,i}^M\right) = 0$ and

$$\begin{aligned} 0 &= \begin{pmatrix} f a_1^{I*}\left(p_1^S\left(\bar{\pi}_i^*\right), \bar{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^S\left(\bar{\pi}_i^*\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right) \end{pmatrix} W_0^I \\ &\quad + \min\left\{\frac{Bp_1^S\left(\bar{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^S\left(\bar{\pi}_i^*\right), a_{0,i}^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right)\right\}. \end{aligned}$$

Note that $\underline{\pi}_i^*\left(a_{0,i}^M\right) \leq \pi^*\left(a_0^M\right) \leq \bar{\pi}_i^*\left(a_{0,i}^M\right)$, and therefore, the LHS of the expressions above is decreasing in $a_{0,i}^M$, since $p_1^L\left(\underline{\pi}_i^*\right) < p_1^S\left(\bar{\pi}_i^*\left(a_{0,i}^M\right)\right) \leq \frac{1}{q_0}$ (if $\bar{\pi}_i^*\left(a_{0,i}^M\right) > \pi_x\left(a_{0,i}^M\right)$, LHS>0). Moreover, from the definition of $\pi^*\left(a_0^M\right)$ and using that $H\left(\pi; a_{0,i}^M\right)$ is increasing in π in $\left(\pi^*\left(a_0^M\right), 1\right]$,

$$\begin{aligned} 0 &< \begin{pmatrix} f a_1^{I*}\left(p_1^*\left(\bar{\pi}_i^*\right), \bar{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^*\left(\bar{\pi}_i^*\right), a_0^M\right)\right)a_0^{I*}\left(1-f\right) \end{pmatrix} W_0^I \\ &\quad + \min\left\{\frac{Bp_1^*\left(\bar{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^*\left(\bar{\pi}_i^*\right), a_0^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right)\right\} \end{aligned}$$

which implies that $a_{0,i}^M \geq a_0^M$. From the definition of $\pi^*\left(a_0^M\right)$,

$$\begin{aligned} 0 &> \begin{pmatrix} f a_1^{I*}\left(p_1^L\left(\pi^*\left(a_0^M\right)\right), \pi^*\left(a_0^M\right)\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^L\left(\pi^*\left(a_0^M\right)\right), a_0^M\right)\right)a_0^{I*}\left(1-f\right) \end{pmatrix} W_0^I \\ &\quad + \min\left\{\frac{Bp_1^L\left(\pi^*\left(a_0^M\right)\right)}{\bar{d}}, n\left(p_1^L\left(\pi^*\left(a_0^M\right)\right), a_0^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right)\right\} \end{aligned}$$

and since the LHS is decreasing in a_0^M, this implies

$$\begin{aligned} 0 &> \begin{pmatrix} f a_1^{I*}\left(p_1^L\left(\underline{\pi}_i^*\right), \underline{\pi}_i^*\right)\left(x\left(1-f\right)a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^L\left(\underline{\pi}_i^*\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right) \end{pmatrix} W_0^I \\ &\quad + \min\left\{\frac{Bp_1^L\left(\underline{\pi}_i^*\right)}{\bar{d}}, n\left(p_1^L\left(\underline{\pi}_i^*\right), a_{0,i}^M\right)\left(W_0^M + f a_0^{I*} W_0^I\right)\right\} \end{aligned}$$

which is a contradiction.

Therefore, $H\left(\pi, a_{0,i}^{M}\right)$ crosses 0 at most once and the proposition holds. ■

Proposition ?? $\pi_i^*\left(a_{0.i}^{M}; a_{\ 0}^{M}\right)$ is increasing in $a_{0,i}^{M}$.

Proof. Follows from the definition of H and the fact that $p_1^*(\pi) < \frac{1}{q_0} < x$ for all $\pi \leq \pi_{x,i}\left(a_{0,i}^{M}\right)$. ■

8.4 Investor's problem

The $t=2$ wealth of an investor with manager i is given by

$$
\begin{aligned}
& W_2^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right) \\
&= \begin{cases} \left(a_1^{I*}\left(p_1(\pi), \pi\right)\left((1-f)\, d - \frac{1}{q_1}\right) + \frac{1}{q_1}\right) W_1^I & \text{if} \quad \pi \geq \pi_i^* \\ \\ \frac{1}{q_1} W_1^I \left(\bar{s}(\pi)\, a_1^{I*}\left(p_1(\pi), \pi\right)\left((1-f)\, d - \frac{1}{q_1}\right) + \frac{1}{q_1}\right) & \text{if} \quad \pi < \pi_i^* \end{cases},
\end{aligned}
$$

where $W_1 = W_1\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right)$

$$
\begin{aligned}
& W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right) \\
&= \begin{cases} \left(x(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right) W_0^I & \text{if } \pi_{x,i} > \pi \geq \pi_i^* \\ \\ \left(\bar{s}(\pi)\, x + (1-\bar{s}(\pi))\, n\left(p_1(\pi), a_{0,i}^M\right)\right)(1-f)\, A_0^I + \left(1-a_0^I\right)\frac{W_0^I}{q_0} & \text{otherwise} \end{cases}.
\end{aligned}
$$

Because of log utility, the investor's problem can be rewritten as

$$
\begin{aligned}
& \max_{a_0^I \in [0,1]} \mathbb{E}_\pi s^*(\pi) \log W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right) \\
& + \mathbb{E}_\pi\left(1-s^*(\pi)\right) \log W_1^I\left(a_0^I; a_{0i}^M, \pi_i^{**}, \pi_i^*, \pi_{x,i}, \bar{s}(\pi), \pi\right).
\end{aligned}
$$

The first order condition for an interior solution is

$$
\begin{aligned}
0 &= \int_{\pi_x\left(a_0^M\right)}^{\bar{\pi}} \frac{n\left(p_1(\pi), a_0^M\right)(1-f) - \frac{1}{q_0}}{\left(n\left(p_1(\pi), a_0^M\right)(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}\right)} dG(\pi) \\
& + \int_{\pi^*\left(a_0^M\right)}^{\pi_x\left(a_0^M\right)} \frac{x(1-f) - \frac{1}{q_0}}{x(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} dG(\varphi) \\
& + \int_0^{\pi^*\left(a_0^M\right)} \frac{n\left(p_1(\pi), a_0^M\right)(1-f) - \frac{1}{q_0}}{n\left(p_1(\pi), a_0^M\right)(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} dG(\pi)
\end{aligned}
$$

The SOC is negative.

$$
\begin{aligned}
0 \quad > \quad & -\int_{\pi_x\left(a_0^M\right)}^{\bar{\pi}} \left(\frac{n\left(p_1\left(\pi\right), a_0^M\right)(1-f) - \frac{1}{q_0}}{n\left(p_1\left(\pi\right), a_0^M\right)(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} \right)^2 dG\left(\pi\right) \\
& -\int_{\pi^*\left(a_0^M\right)}^{\pi_x\left(a_0^M\right)} \left(\frac{x\left(1-f\right) - \frac{1}{q_0}}{x\left(1-f\right) a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} \right)^2 dG\left(\pi\right) \\
& -\int_{0}^{\pi^*\left(a_0^M\right)} \left(\frac{\lambda n\left(p_1\left(\pi\right), a_0^M\right)(1-f) - \frac{1}{q_0}}{\lambda n\left(p_1\left(\pi\right), a_0^M\right)(1-f)\, a_0^I + \left(1-a_0^I\right)\frac{1}{q_0}} \right)^2 dG\left(\pi\right)
\end{aligned}
$$

8.5 Manager's Objective Function

Manager i's welath at $t=2$ is given by $\frac{\pi \bar{d}}{p_1} W_1^M$ where $W_1^M = W_1^M\left(a_{0,i}^M; \pi_{x,i}\left(a_{0,i}^M\right), \pi_i^*\left(a_{0,i}^M\right), a_0^I, \pi\right)$. If no support is needed for the fund to continue at $t=1$,

$$
W_1^M = n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E + f A_1^{I*}\left(\pi\right).
$$

If the manager offers support to his investor at $t=1$,

$$
\begin{aligned}
W_1^M \quad = \quad & n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E + f A_1^{I*}\left(\pi\right) \\
& -\left(x - n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\right) a_0^{I*}\left(1-f\right) W_0^I.
\end{aligned}
$$

Finally, if the fund is liquidated

$$
W_1^M = n\left(p_1^*\left(\pi\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) + E.
$$

8.5.1 Continuity

Since $\pi_{x,i}\left(a_{0,i}^M\right)$ can have a discontinuity at $\underline{a}_{0,i}^M\left(a_0^M\right)$, the objective function can be discontinuous at $\underline{a}_{0,i}^M\left(a_0^M\right)$ too.

If $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) = \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)$,

$$
\begin{aligned}
& Obj\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) - \lim_{a \to \underline{a}_{0,i}^M\left(a_0^M\right)^+} Obj\left(a\right) \\
= \quad & \int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\pi^*} \left(\frac{\pi \bar{d}}{p_1^*\left(\pi\right)} f a_1^{I*}\left(\pi\right) \left(a_0^{I*}\left(1-f\right)\left(a_{0,i}^M \left(\frac{p_1^*\left(\pi\right)}{p_0} - \frac{1}{q_0} \right) + \frac{1}{q_0} \right) + \left(1-a_0^{I*}\right)\frac{1}{q_0} \right) W_0^I + B\pi \right) dG\left(\pi\right)
\end{aligned}
$$

and the objective function jumps down and a max always exists.

When $B = 0$, $p_1^*\left(\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right) = \bar{d}\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)q_1$ which implies that

$$a_1^I\left(p_1^*\left(\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right), \pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)\right) = 0$$

and, hence, that $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) = \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)$.

If $\pi_{x,i}\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) > \pi_i^*\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right)$

$$\begin{aligned}
&Obj\left(\underline{a}_{0,i}^M\left(a_0^M\right)\right) - \lim_{a\to\underline{a}_{0,i}^M\left(a_0^M\right)^+} Obj\left(a\right) \\
= &\int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\pi^*} \frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(fa_1^{I*}\left(\pi\right)-1\right)\left(\left(a_{0,i}^M\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)+\frac{1}{q_0}\right)-x\right)a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right) \leq 0
\end{aligned}$$

and the objective function jumps up at $\underline{a}_{0,i}^M\left(a_0^M\right)$ and I cannot guarantee the existence of a maximum. In a symmetric equilibrium, if $\pi_{x,i}\left(a_{0,i}^M\right)$ is discontinuous, $\underline{a}_{0,i}^M\left(a_0^M\right) < a_0^M$.

8.5.2 Differentiability

The first derivative of the manager's objective function with respect to $a_{0,i}^M$ when the objective function is differentiable is given by

$$\begin{aligned}
&\int_{\pi_i^{*E}\left(a_{0,i}^M\right)} \frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)\left(W_0^M + fa_0^I W_0^I\right)dG\left(\pi\right) \\
&+\int_{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi\bar{d}}{p_1^*\left(\pi\right)} fa_1^{I*}\left(\pi\right)\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right) \\
&+\int_{\pi_i^*\left(a_{0,i}^M\right)}^{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi\bar{d}}{p_1^*\left(\pi\right)}\left(\frac{p_1^*\left(\pi\right)}{p_0}-\frac{1}{q_0}\right)a_0^{I*}\left(1-f\right)W_0^I dG\left(\pi\right) \\
&-\frac{\partial\pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\left(\begin{array}{c}
n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + fa_0^I W_0^I\right) \\
+fa_1^{I*}\left(\pi_i^*\left(a_{0,i}^M\right)\right)\left(a_0^{I*}\left(1-f\right)x + \left(1-a_0^{I*}\right)\frac{1}{q_0}\right)W_0^I \\
-\left(x - n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\right)a_0^{I*}\left(1-f\right)W_0^I \\
+\frac{B}{\bar{d}}p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)
\end{array}\right)dG
\end{aligned}$$

Using the definition of π_i^* and the fact that if (condition) holds with strict inequality then $\frac{\partial\pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M} = 0$ (when it exists), the last term in the first derivative is equal to

$$\frac{\partial\pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}\min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + fa_0^I W_0^I\right) + E - \frac{B}{\bar{d}}p_1^*\left(\pi_i^{*E}\left(a_{0,i}^M\right)\right)\right\}dG.$$

Therefore, the first derivative of the objective function with respect to $a_{0,i}^{M}$ is

$$\int_{\underline{\pi}}^{\bar{\pi}} \frac{\pi \bar{d}}{p_1^*(\pi)} \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) \left(W_0^M + f a_0^I W_0^I \right) dG(\varphi)$$
$$+ \int_{\pi_{x,i}\left(a_{0,i}^M\right)}^{\bar{\pi}} \frac{\pi \bar{d}}{p_1^*(\pi)} f a_1^{I*}(\pi) \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) a_0^{I*} (1-f) W_0^I dG(\pi)$$
$$+ \int_{\pi_i^*\left(a_{0,i}^M\right)}^{\pi_{x,i}\left(a_{0,i}^M\right)} \frac{\pi \bar{d}}{p_1^*(\pi)} \left(\frac{p_1^*(\pi)}{p_0} - \frac{1}{q_0} \right) a_0^{I*} (1-f) W_0^I dG(\pi)$$
$$+ \frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^M} \frac{\pi_i^*\left(a_{0,i}^M\right) \bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \min \left\{ \begin{array}{c} 0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + f a_0^I W_0^I\right) \\ +E - \frac{B}{\bar{d}} p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right) \end{array} \right\} dG.$$

When $B = 0$, this first derivative is continuous when the objective function is differentiable.

When $B > 0$, the first derivative is not defined at the discontinuity points of $\pi_i^*\left(a_{0,i}^M\right)$, at $\underline{a}_{0,i}^M\left(a_0^M\right)$, $\hat{a}_{0,i}^M\left(a_0^M\right)$, and $\bar{a}_{0,i}^M\left(a_0^M\right)$.

$$\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M} \begin{cases} = 0 & \text{if } a_{0,i}^M < \underline{a}_{0,i}^M\left(a_0^M\right) \\ > 0 & \text{if } a_{0,i}^M \in \left(\underline{a}_{0,i}^M\left(a_0^M\right), \hat{a}_{0,i}^M\left(a_0^M\right)\right) \\ = 0 & \text{if } a_{0,i}^M \in \left(\hat{a}_{0,i}^M\left(a_0^M\right), \bar{a}_{0,i}^M\left(a_0^M\right)\right) \\ > 0 & \text{if } a_{0,i}^M > \bar{a}\left(a_0^M\right) \end{cases}$$

Therefore

$$\frac{\partial Obj}{\partial a_{0,i}^M}(\bar{a}) - \lim_{a \to \left(a_{0,i}^M\right)} \frac{\partial Obj}{\partial a_{0,i}^M}(a) \begin{cases} > 0 & \text{if } \bar{a} = \underline{a}_{0,i}^M\left(a_0^M\right) \\ < 0 & \text{if } \bar{a} = \hat{a}_{0,i}^M\left(a_0^M\right) \\ > 0 & \text{if } \bar{a} = \bar{a}_{0,i}^M\left(a_0^M\right) \\ = 0 & \text{else} \end{cases}.$$

Therefore, $\underline{a}_{0,i}^M\left(a_0^M\right)$ and $\bar{a}_{0,i}^M\left(a_0^M\right)$ are candidates for a maximum.

When the first derivative is differentiable, the second derivative of the objective function is given by $\frac{\partial^2 Obj}{\partial a_{0,i}^{M2}} =$

$$\left(1 - fa_1^{I*}\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)\right) \frac{\pi_{x,i}\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)} \left(\frac{p_1^*\left(\pi_{x,i}\left(a_{0,i}^M\right)\right)}{p_0} - \frac{1}{q_0}\right) a_0^{I*}\left(1-f\right) W_0^I dG \frac{\partial \pi_{x,i}\left(a_{0,i}^M\right)}{a_{0,i}^M}$$
$$+ \frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \left(\frac{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)}{p_0} - \frac{1}{q_0}\right) a_0^{I*}\left(1-f\right) W_0^I dG\left(\pi\right) \frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{a_{0,i}^M}$$
$$+ \frac{\partial^2 \pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^{M2}} \frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + fa_0^I W_0^I\right) + E - \frac{B}{\bar{d}} p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)\right\} dG$$
$$+ \left(\frac{\partial \pi_i^*\left(a_{0,i}^M\right)}{\partial a_{0,i}^M}\right)^2 \frac{\partial}{\partial a_{0,i}^M} \left(\frac{\pi_i^*\left(a_{0,i}^M\right)\bar{d}}{p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)} \min\left\{0, n\left(p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right), a_{0,i}^M\right)\left(W_0^M + fa_0^I W_0^I\right) + E - \frac{B}{\bar{d}} p_1^*\left(\pi_i^*\left(a_{0,i}^M\right)\right)\right\}\right.$$

When $B = 0$,

$$\frac{\partial^2 Obj}{\partial a_{0,i}^{M2}}\left(a_0^M\right) > 0$$

therefore, there cannot be a symmetric equilibrium in which a_0^M is interior.

8.6 Numerical Analysis

To continue illustrating the forces at work in the model, I compute the equilibria for several examples and provide some intuition for how these equilibria change with the cost faced by mangers upon early liquidation of the fund, and with the amount of capital in the funds.

Figure 9 depicts symmetric equilibria for different values of B. For all parameterizations shown in the figure, the risk behavior of the managers remains unchanged: at $t = 0$, managers invest all the funds' wealth in the risky asset. Fees are increasing in B: a higher B implies a higher loss for managers if their fund is liquidated early. Since managers are taking risky positions in equilibrium, they higher the loss B, the higher the compensation needed for managers to be willing to open a fund. Moreover, the higher the loss due to the early liquidation of a fund, the higher the incentive of managers to offer support, and the lower the probability of breaking the buck. Finally, the change in the supply of liquidity will be driven by the change in the amount intermediated in the economy. Investors face higher fees and receive higher insurance as B increases. These two forces have countervailing effects on their willingness to invest with managers and can drive liquidity supply up or down. In the examples shown here, the effect of the higher fees prevails and liquidity supply decreases with B.

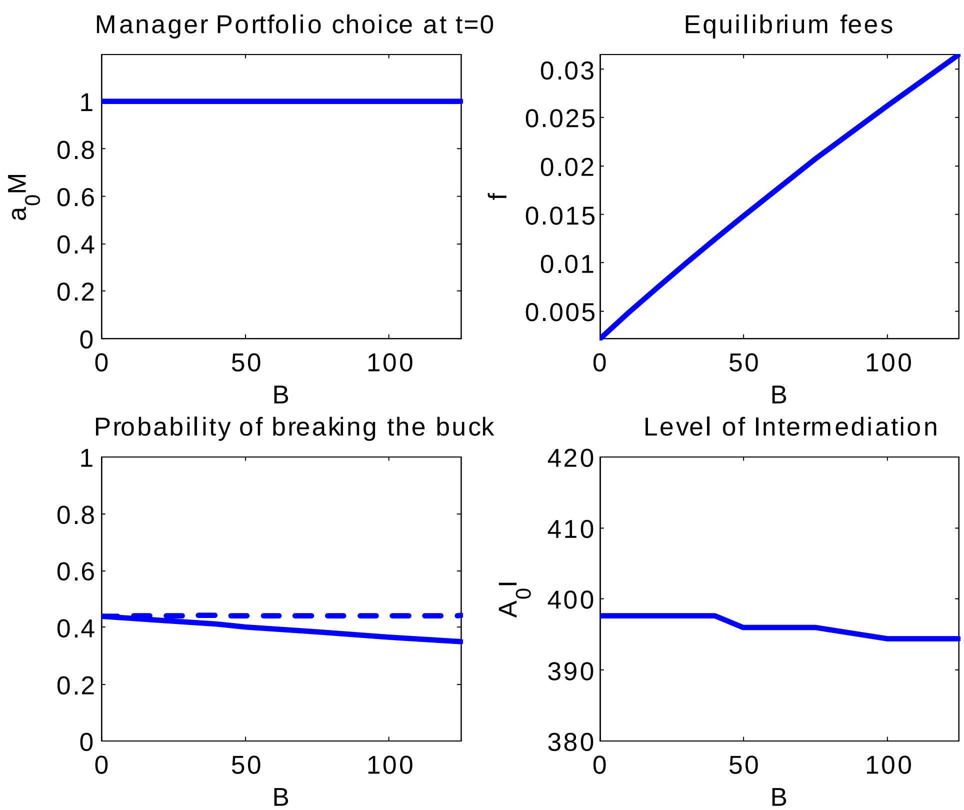

Figure 9: Equilibrium when B changes.

Figure 10 shows symmetric equilibria for different values of W_0^M when $B > 0$. As in the previous examples, the risk behavior of the managers is the same for all parameterizations shown in the figure and managers take as much risk as they can at $t = 0$. Moreover, the equilibrium fees and the probability of breaking the buck are increasing in W_0^M. A higher W_0^M changes the amount of resources in the economy and it increases the demand for the risky asset at $t = 0$. This drives the price of the risky asset at $t = 0$ up and the return of one unit invested in the risky asset and, hence, the NAV down. The decrease in the NAV for all realizations of π implies that the manager will need to offer more support in order to keep the fund going at $t = 0$. If the incentives to offer support remain unchanged, this will translate into a higher probability of breaking the buck. Moreover, these effects decrease the return of investing with the manager at $t = 0$. Therefore, fees must increase to keep the managers indifferent between opening a fund and managing only their own funds. In this example, the increase in the fees is not enough to increase the managers' incentives to offer support enough to offset the effect of a lower NAV over the probability of breaking the buck. Thus, investors face higher fees and less insurance which translates into lower intermediation levels and lower liquidity.

8.7 Results

Proposition ??If $B = 0$, $a_0^{M*} = 1$ in a symmetric equilibrium.

Proof. Using the characterization of the objective function for the manager, one can see this function is differentiable and convex at $a_{0,i}^M = a_0^M$ when $B = 0$. Therefore, in a symmetric equilibrium a_0^M cannot be an interior solution. Assuming that the supply function is such that there is always some risky asset bough at $t = 0$, a_0^M has to be 1 in a symmetric equilibrium. ■

Proposition 8 The amount of assets managed for investors at $t = 1$, A_1^I, is increasing in the net asset value, $n\left(p_1, a_0^M\right)$.

Proof. From the definition of A_1^I, we have

$$A_1^I = \begin{cases} a_1^I\left(p_1, \pi\right) W_1 & \text{if the fund continues} \\ 0 & \text{if the fund is liquidated} \end{cases}$$

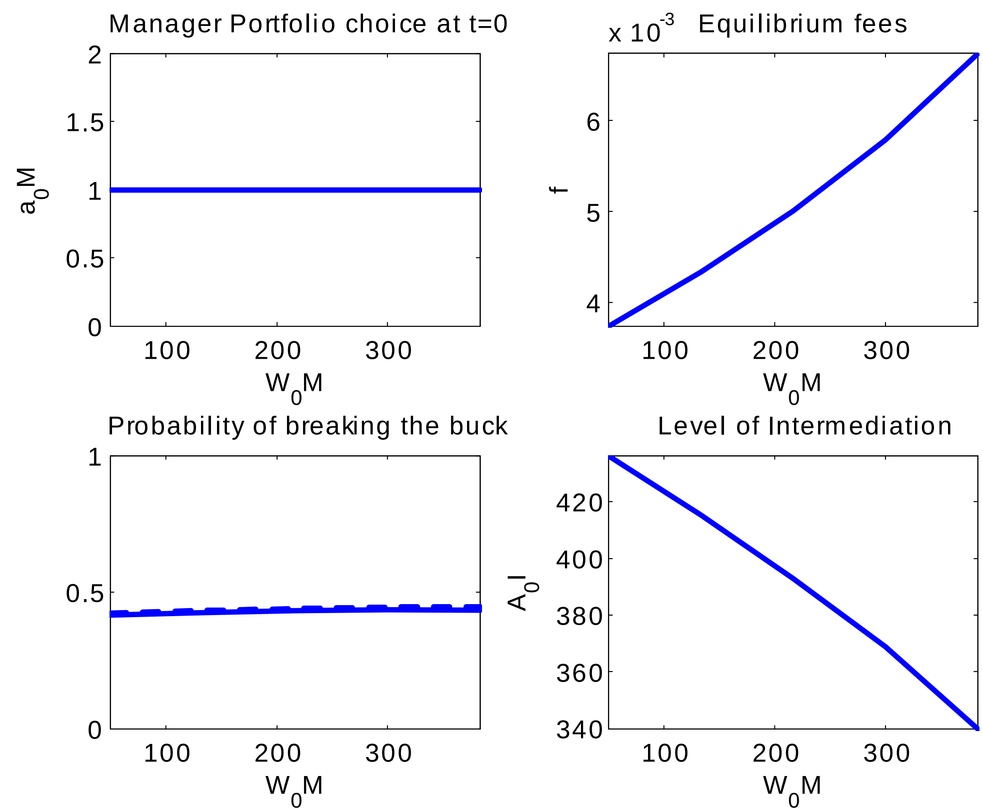

Figure 10: Equilibrium when W_0^M changes.

where a_1^I is independent of the return of the investor's shares bought at $t = 0$. Moreover, the investor's wealth is (weakly) increasing in the net asset value since

$$W_1 = \max\left\{x, n\left(p_1, a_0^M\right)\right\}(1-f)\, a_0^I W_0^I + \frac{1}{q_0}\left(1 - a_0^I\right) W_0^I.$$

■

Lemma 1 *When $B = 0$, the manager's support decision is a threshold decision for all $B_i > 0$ and all $W_{0,i}^M$ at the equilibrium price (given the equilibrium decisions for other managers).*

Proof. To show that the proposition holds it is enough to show that it cannot be the case that

$$\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M, B_i, W_{0,i}^M\right) \geq 0 > H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right)$$

Let

$$\begin{aligned} H\left(\pi; a_{0,i}^M, B_i, W_{0,i}^M\right) \quad : \quad &= \begin{pmatrix} f a_1^{I*}\left(p_1^*(\pi), \pi\right)\left(x(1-f)\, a_0^I + \left(1 - a_0^I\right)\frac{1}{q_0}\right) \\ -\left(x - n\left(p_1^*(\pi), a_{0,i}^M\right)\right) a_0^{I*}(1-f) \end{pmatrix} W_0^I \\ &\quad + \min\left\{\frac{B p_1^*(\pi)}{\bar{d}}, n\left(p_1^*(\pi), a_{0,i}^M\right)\left(W_{0,i}^M + f a_0^{I*} W_0^I\right) + E\right\}. \end{aligned}$$

■

I know from the definition of the aggregate support decision threshold that

$$\begin{aligned} H\left(\pi^*; 1, 0, 0\right) &= 0 \\ f a_1^{I*}\left(p_1^S(\pi^*), \bar{\pi}_i^*\right)\left(x(1-f)\, a_0^I + \left(1 - a_0^I\right)\frac{1}{q_0}\right) W_0^I - \left(x - \frac{p_1^S(\pi^*)}{p_0}\right) a_0^{I*}(1-f)\, W_0^I &= 0 \end{aligned}$$

and that

$$\begin{aligned} \lim_{\pi \to \pi^*} H\left(\pi; 1, 0, 0\right) &= f a_1^{I*}\left(p_1^L(\pi^*), \bar{\pi}_i^*\right)\left(x(1-f)\, a_0^I + \left(1 - a_0^I\right)\frac{1}{q_0}\right) W_0^I \\ -\left(x - \frac{p_1^L(\pi^*)}{p_0}\right) a_0^{I*}(1-f)\, W_0^I &\leq 0 \end{aligned}$$

$H\left(\pi; a_{0,i}^M, B_i\right)$ is decreasing in $a_{0,i}^M$ and increasing in B_i and $W_{0,i}^M$. Therefore, it cannot be the case that for some triplet $\left(a_{0,i}^M, B_i, W_{0,i}^M\right)$

$$\lim_{\pi \to \pi^*} H\left(\pi; a_{0,i}^M, B_i; W_{0,i}^M\right) \geq 0 > H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right)$$

since $H\left(\pi^*; a_{0,i}^M, B_i, W_{0,i}^M\right) \geq 0$ always.

Proposition 9 Suppose $B = 0$ for all managers but for manager j. Then, in equilibrium, the risk taken by manager j at $t = 0$ will be decreasing in B_j and he will choose $a_{0,j}^M \in \left\{a_0^{MS}, 1\right\}$ where $a_0^{MS} = \max a_{0,i}^M$ s.t. $\pi_{x,i}\left(a_{0,i}^M\right) = \underline{\pi}$.

Proof. Using lemma 1 and the characterization of the manager's objective function when $B = 0$, one can see that the two candidates for maxima are $V^M\left(W_0^M; a_0^{MS}\right)$ and $V^M\left(W_0^M; 1\right)$, where a_0^{MS} is the highest $a_{0,i}^M$ such that $\pi_{x,i}\left(a_{0,i}^M\right) = \underline{\pi}$. Then,

$$a_{0,j}^{M*} = \begin{cases} a_0^{MS} & \text{if } V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right) > 0 \\ 1 & \text{if } V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right) < 0 \end{cases}.$$

When $B_j = 0$, $V^M\left(W_0^M; a_0^{MS}\right) < V^M\left(W_0^M; 1\right)$ (follows from Proposition **??**). Moreover,

$$\lim_{B\to\infty}\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right) = \infty.$$

Let

$$\Theta = \mathbf{1}\left\{\frac{Bp_1^*\left(\pi^*\left(1\right)\right)}{\bar{d}} > \frac{p_1^*\left(\pi_i^*\left(1\right)\right)}{p_0}\left(W_{0,i}^M + fa_0^{I*}W_0^I\right) + E\right\}$$

Using the characterization of the objective function in the previous section,

$$\begin{aligned} & \frac{\partial\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right)}{\partial B_j} \\ \propto\ & \begin{cases} \int_{\underline{\pi}}^{\pi_i^*(1)} B_i\pi d\pi + \frac{\partial\pi_i^*}{\partial B_i}\left(1\right)\left(\frac{\pi_i^*(1)\bar{d}}{p_1^*(\pi)}\left(\frac{p_1^*\left(\pi_i^*(1)\right)}{p_0}\left(W_{0,i}^M + fa_0^{I*}W_0^I\right) + E\right) - B_i\pi_i^*\left(1\right)\right) & \text{if } \Theta = 1 \\ \int_{\underline{\pi}}^{\pi_i^*(1)} B_i\pi d\pi & \text{if } \Theta = 0 \end{cases} \\ 0 <\ & \frac{\partial\left(V^M\left(W_0^M; a_0^{MS}\right) - V^M\left(W_0^M; 1\right)\right)}{\partial B_j} \end{aligned}$$

since $\frac{\partial\pi_i^*}{\partial B_i}\left(1\right) < 0$. ■