Buying high and selling low: Stock repurchases and persistent asymmetric information[1]

[Preliminary and Incomplete]

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[1]We thank a seminar audience at the University of Minnesota. Any errors are our own.

Abstract

We investigate the consequences of allowing for repeated capital market transactions in a model with asymmetric information between a firm and its investors. All firms in the model possess a profitable project that they need to raise cash to undertake. However, we show that any equilibrium of the model entails some firms returning cash to investors via share repurchases. Consistent with managerial accounts, some repurchasing firms profit from repurchasing their stock. The ultimate source of these profits is that other firms buy "high" in order to improve the terms of subsequent stock issues. We show the possibility of repurchases lowers social welfare by reducing the fraction of firms that invest, even though repurchasing itself is a costless signal. Our model generates a number of empirical predictions.

An important idea in corporate finance is that firms have more information about their future cash flows than investors. A large body of research has studied the consequences of this asymmetric information for a firm's capital market transactions. However, the vast majority of such papers have restricted firms to a single round of capital market transactions.[1] In this paper, we study the implications of relaxing this assumption for what is arguably the best-known corporate finance model based on asymmetric information, namely Myers and Majluf's (1984) model of equity financing to fund an investment.[2]

Our main finding is that any equilibrium of a dynamic version of Myers and Majluf features share repurchases. In equilibrium, some firms repurchase their stock for strictly less than its fair value, consistent with managerial accounts of these transactions (see Brav et al (2005)). Other firms repurchase stock in order to lower the cost of subsequent equity issuance, consistent with empirical evidence (see Billet and Xue (2007)).

At first sight, the ability of firms to strictly profit from trading on their superior information would appear to violate the no-trade theorem (see, e.g., Milgrom and Stokey (1982)). Existing models of share repurchases generally avoid this problem by introducing an assumption that firms care directly about an interim share price (#CITATIONS). Our model avoids this assumption.[3] Instead, in our model some firms strictly profit from repurchases because other inferior firms also repurchase, and make losses. This second group of firms "buy high" when they repurchase, i.e., buy their stock for more than it is worth.

Why does this second group of inferior firms repurchase at a loss? They do so in order to improve the terms at which they can subsequently issue stock to finance a profitable investment. This is consistent with the empirical findings of Billet and Xue (2007). Nonetheless, and as is standard in models of this type, even the improved issuance terms are still

[1]One well-known exception is Lucas and McDonald (1990). However, in their model a firm's informational advantage only lasts one period.

[2]As we detail below, we focus on the version of this model where firms know more about the value of their existing assets, but have no informational advantage with respect to growth options.

[3]Regarding this assumption, Allen and Michaely (2003) write "For example, why would a management care so much about the stock price next period? Why is its horizon so short that it is willing to 'burn money' (in the form of a payout) just to increase the value of the firm now, especially when the true value will be revealed next period?"

associated with a negative price response at issue (this is the "selling low" of the title).

Repurchases do not carry any deadweight loss in our model; in this, our model is very different from much of the prior literature, which assumes that payouts generate a deadweight loss either via increased taxes or via an increased need for (exogenously) costly external financing (CITATIONS).[4] Nonetheless, the ability of firms to repurchase strictly lowers social welfare (meaning the total amount of profitable investment). In other words, social welfare would be higher if stock repurchases were prohibited. The reason is that firms that issue to finance the profitable investment are forced to first repurchase to signal their quality, and this repurchase generates a loss (which, as discussed above, makes it possible for other firms to strictly profit from repurchases). Note that because repurchases have no deadweight loss, this welfare result is fundamentally different from the commonly-made observation (see, e.g., Arrow (1973)) that social welfare would be higher if a costly signal were prohibited.

Related literature [INCOMPLETE]:

Brav et al (2005) survey managers. A very large fraction of managers agree (Table 3) that "Repurchase decisions convey information about our company to investors." Moreover, a large fraction also agree (Table 6) that the "Market price of our stock (if our stock is a good investment, relative to its true value)" is an important factor. Also consistent with our model, only a very small fraction of managers agree (Table 3) that "We use repurchases to show we can bear costs such as borrowing costly external funds or passing up investment, to make us look better than our competitors." In contrast, the signaling value of repurchases in our model simply stems from a firm's willingness to buy its own shares.

Grullon and Ikenberry (2000) offer a good survey of the literature on repurchasing. They interpret the extant empirical evidence as being more consistent with misvaluation of existing assets, rather than about future cash flows. This is consistent with the focus of our model on asymmetric information about assets-in-place.

The idea that firms repurchase their stock to signal they are good is related to the old idea

[4]Note that Brav et al's (2005) survey of managers finds little support for the idea that repurchases are made to signal that a firm can bear such costs; see below.

that *retaining* equity is a useful signal (Leland and Pyle (1977)). Also related, Example 1 of Brennan and Krause (1987) has a good firm simultaneously repurchasing debt and issuing equity. The debt repurchase allows the firm to signal that it is good.

Our paper is related to the literature on signaling in static payout models. Representative examples include Bhattacharya (1980), Ambarish, John and Williams (1987), Constantinides and Grundy (1989), and Vishwanathan (1995). Perhaps closest to us in this literature is Willams (1988), in which firms use costly dividends as a signal to reduce the cost of a simultaneous equity issuance.

An important assumption in any model of repurchasing based on signaling, including ours, is that a firm's repurchase decision is actually observable. Although regulatory mandates force this to be true in many markets, there has been some debate in the literature about the observability of repurchases in the United States. For example, in an early study of repurchases, Barclay and Smith (1988) find evidence that the announcement of a repurchase program is followed by an increased bid-ask spread, which they interpret as an increase in adverse selection, which they in turn interpret as investors being unsure about whether or not they are trading against the firm. However, in general subsequent research has not supported this original finding (see the discussion in Grullon and Ikenberry (2000)).

A relatively small literature studies dynamic models of trade under asymmetric information. Nöeldeke and van Damme (1990) and Swinkels (1999) study a labor market model where education acts as a signal. Fuchs and Skrzypacz (2013) study trade of a single indivisible asset that is more highly valued by buyers than the seller. They focus on whether more trading opportunities increase or reduce welfare. Kremer and Skrzypacz (2007) and Daley and Green (2011) study a similar model in which information arrives over time. In contrast to these papers, in our model both sales and repurchases are possible; trade is in divisible shares; and the gains from trade arise from the possibility of financing a profitable investment. Finally, a contemporaneous paper by Ordoñez, Perez-Reyna and Yogo (2013) studies a dynamic model of debt issuance.

In a model with moral hazard in place of adverse selection, DeMarzo and Urošević (2006) study the dynamics of a large shareholder selling off his stake in a firm.

Bond and Eraslan (2010) study trade between differentially-informed parties in common-values setting. The no-trade theorem does not apply because the eventual owner of the asset takes a decision that affects the asset's final cash flow. Trade affects the information available to the party making the decision. In the current paper, trade of the asset (i.e., shares) at date 1 instead affects a firm's ability to raise finance at date 2.

1 Example

Firms have cash 2, and the opportunity to invest 8 in a project that yields 10. Hence firms need to raise additional funds of 6 in order to invest. Firms can either repurchase (buy) or issue (sell) shares at each if dates 1 and 2. All uncertainty is resolved at date 3, and firms act to maximize their date 3 share price.

Firm assets-in-place a are distributed over $[0, 13]$. The density of firms over $[0, 4]$ is $\frac{1}{10}$, and the density of firms over $[4, 13]$ is $\frac{1}{15}$.[5]

The following is an equilibrium:

- At date 1, firms with assets-in-place either below 4 or above 7 spend all their cash 2 to repurchase $\frac{2}{9}$ of their shares for a price $P_1 = 9$. The remaining firms do nothing.

- At date 2, firms with assets-in-place below 4 issue $\frac{8}{5\frac{1}{7}} = \frac{14}{9}$ shares at a price $P_2 = 5\frac{1}{7}$ to raise 8, and invest. All other firms do nothing.

We verify this is an equilibrium. First, conditional on firms behaving this way, the repurchase and issues prices are fair, as follows. The date 2 issue price is fair, since it solves

$$P_2 = \frac{E\left[a | a \in [0, 4]\right] + 10}{1 - \frac{2}{9} + \frac{8}{P_2}},$$

[5] For the purposes of verifying that this example satisfies (6) below, observe that the expected value of assets-in-place is $\frac{2}{5} \times 2 + \frac{3}{5} \times 8.5 = 5.9$.

i.e.,

$$P_2 = \frac{2+2}{1-\frac{2}{9}} = \frac{36}{7}.$$

The date 1 repurchase price is fair, since with probability 1/2 the date 2 price will be $P_2 = 5\frac{1}{7}$ and with probability 1/2 it will be $\frac{E[a|a\in[7,13]]}{1-\frac{2}{9}} = 12\frac{6}{7}$, and so the, conditional on date 1 repurchase, the expected date 2 price is 9.

Second, firms respond optimally to the stated repurchase and issue prices. The date 3 share price of a firm with assets-in-place a from repurchasing then issuing is

$$\frac{10+a}{1-\frac{2}{9}+\frac{14}{9}} = \frac{10+a}{\frac{7}{3}},$$

while the date 3 share price from just repurchasing is

$$\frac{a}{\frac{7}{9}},$$

and the date 3 share price from doing nothing at both dates is simply

$$2+a.$$

Out of these three alternatives, firm with assets-place below 4 obtain the highest payoff from repurchasing and then investing; firms with assets-in-place above 7 obtain the highest payoff from just repurchasing; while firms with assets-in-place between 4 and 7 obtain the highest payoff from doing nothing. Moreover, if the off-equilibrium action of skipping the repurchase step and instead directly issuing equity to raise 6 leads investors to assume the firm has no assets-in-place ($a = 0$), the price $\tilde{P}$ associated with this off-equilibrium direct issue would solve $\tilde{P} = \frac{10}{1+\frac{6}{\tilde{P}}}$, i.e., $\tilde{P} = 4$, and so a firm would have to issue $\frac{3}{2}$ shares. Hence the payoff from this off-equilibrium action for a firm with assets-in-place a would be $\frac{10+a}{1+\frac{3}{2}}$, which is indeed less than the conjectured equilibrium payoff of all firms.[6]

[6]We have established that firms act optimally when their choice set is limited to the three equilibrium

Discussion:

Firms with assets-in-place $a > 7$ repurchase shares for strictly less than their true value, $a+2$, and so make strictly positive profits. The reason investors accept the lower price is that these firms pool with worse firms (namely, firms with $a < 4$). But this raises the question of why these worse firms are prepared to repurchase. They do so in order to improve the terms at which they can subsequently issue: if instead they attempt to issue equity directly, they obtain a worse price.

The intermediate interval of firms with $a \in (4,7)$ find issue too dilutive, as in Myers and Majluf, and also find repurchase too expensive.

Firms with $a > 7$ strictly profit from their repurchase transactions, even though these transactions fail to create any value. The ultimate source of these profits is that the investing firms with $a < 4$ end up paying a premium to raise capital. By this, we mean that if firms $a < 4$ could credibly pool and issue directly, the issue price P would satisfy $P = \frac{10+2}{1+\frac{6}{P}}$, i.e., $P = 6$, and so the payoff of each firm $a < 4$ would be $\frac{10+a}{1+\frac{6}{6}}$, which is higher than they get in the above equilibrium.

A related observation is that the equilibrium of the Myers and Majluf setting, where repurchase is impossible, entails investment by firms with assets-in-place between 0 and a cutoff level strictly in excess of 4. In other words, the possibility of repurchase lowers total surplus in the economy (see Section 6). Nonetheless, and as we show below, when repurchase is possible, any equilibrium features some repurchase. Hence welfare would be strictly increased if repurchases were prohibited, even though no deadweight cost is associated with repurchases.

strategies, together with the off-equilibrium strategy of directly issuing $\frac{3}{2}$ shares for a price 4 each. This still leaves open the possibility that a firm could profitably deviate to some strategy other than these four strategies. However, there exist off-equilibrium beliefs such that all other strategies leave firms worse off. The proof of Proposition 4 below includes the description of one such set of off-equilibrium beliefs.

2 Model and preliminary results

Our model is essentially the same as Myers and Majluf (1984). The only substantive difference is that whereas Myers and Majluf consider a firm's interactions with the equity market at just one date, we consider two possible dates. As we will show, this additional feature generates equilibrium share repurchases.

There are four dates, $t = 0, 1, 2, 3$; an all-equity firm, overseen by a manager; and at each of dates 1 and 2, a large number of risk-neutral investors who trade the firm's stock. We normalize the date-0 number of shares to 1.

At date 0, the manager of the firm privately learns the value of the firm's existing assets ("assets-in-place"). Write a for the value of these existing assets, where $a \in [\underline{a}, \overline{a}]$. Let μ be a measure on $[\underline{a}, \overline{a}]$, which determines the distribution of assets-in-place a. We assume a has full support on $[\underline{a}, \overline{a}]$. In addition to assets a, the firm has cash (or other marketable securities) with a value S.

At dates 1 and 2, the firm has an opportunity to undertake a new project. The project requires an initial investment I and generates an expected cash flow $I + b$. For simplicity, we assume that b is common knowledge; in other words, we focus on a version of the Myers and Majluf environment in which asymmetric information is about assets-in-place, not investment opportunities. The project can be undertaken at date 1, or date 2, but not at both dates. Throughout, we assume $I > S$, so that the firm needs to raise external financing to finance the investment I.

Also, at each of dates $t = 1, 2$, the firm can issue new equity and/or repurchase existing equity. Equity issues and repurchases take place as follows. The manager makes a public offer to buy or sell a fixed dollar amount s_t of shares, where $s_t > 0$ corresponds to share repurchases and $s_t < 0$ corresponds to share issues. Investors respond by offering a quantity of shares in exchange. In other words, if $s_t > 0$ each investor offers a number of shares he will surrender in exchange for s_t; and if $s_t < 0$, each investor offers a number of shares he will accept in return for paying the firm $-s_t$.

At date 3, the true value of the firm is realized, including the investment return, and the firm is liquidated.

Write P_3 for the date-3 liquidation share price, and write P_1 and P_2 for the transaction price of the shares at dates $t = 1, 2$. Because the number of investors trading at each of dates 1 and 2 is large, competition among investors implies that the date t share price is

$$P_t = E\left[P_3 | \text{date } t \text{ information, including firm offer } s_t\right]. \tag{1}$$

The manager's objective is to maximize the date 3 share price, namely

$$P_3 = \frac{S - s_1 - s_2 + a + b\mathbf{1}_{\text{investment}}}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}}, \tag{2}$$

where $\mathbf{1}_{\text{investment}}$ is the indicator function associated with whether the firm undertakes the new project, and the denominator reflects the number of shares outstanding at date 3. Note that in the case that only share issues are possible, the manager's objective function coincides with the one specified in Myers and Majluf (1984), which is to maximize the utility of existing shareholders. In our setting, where repurchases are possible, the manager's objective function can be interpreted as maximizing the value of passive shareholders, who neither sell nor purchase the firm's stock at dates 1 and 2. Alternatively, the manager's objective can be motivated by assuming that the manager himself has an equity stake in the firm, and is restricted from trading the firm's shares on his own account.

For use throughout, observe that (1) and (2), together with the fact that the firm invests whenever it has sufficient funds, imply that the date 2 share price conditional on s_1 and s_2 is

$$P_2(s_1, s_2) = \frac{S - s_1 + E\left[a | s_1, s_2\right] + b\mathbf{1}_{S - s_1 - s_2 \geq I}}{1 - \frac{s_1}{P_1}}. \tag{3}$$

Iterating, (1) and (3), together with the law of iterated expectations, imply that the date 1 share price conditional on s_1 is

$$P_1(s_1) = S + E\left[a + b\mathbf{1}_{S-s_1-s_2\geq I}|s_1\right]. \tag{4}$$

From (3) and (4), the equilibrium payoff of firm a from (s_1, s_2) is

$$\frac{S-s_1-s_2+a+b\mathbf{1}_{S-s_1-s_2\geq I}}{\left(1-\frac{s_1}{P_1}\right)\left(1-\frac{s_2}{S-s_1+E[a|s_1,s_2]+b\mathbf{1}_{S-s_1-s_2\geq I}}\right)} = \frac{S-s_1-s_2+a+b\mathbf{1}_{S-s_1-s_2\geq I}}{\left(1-\frac{s_1}{S+E\left[a+b\mathbf{1}_{S-s_1-s_2\geq I}|s_1\right]}\right)\left(1-\frac{s_2}{S-s_1+E[a|s_1,s_2]+b\mathbf{1}_{S-s_1-s_2\geq I}}\right)}. \tag{5}$$

We characterize the perfect Bayesian equilibria of this game. We restrict attention to pure strategy equilibria in which all investors hold the same beliefs off-equilibrium. We focus on equilibria in which all firms play a best response (as opposed to equilibria in which almost all firms play a best response).[7]

Finally, we state here a simple result that we use repeatedly:

Lemma 1 *If in equilibrium firms a' and a'' conduct capital transactions (s_1', s_2') and (s_1'', s_2''), with $S - s_1' - s_2' > S - s_1'' - s_2''$, then $a' < a''$.*

An immediate corollary of Lemma 1 is:

Corollary 1 *In any equilibrium, there exists $a^* \in [\underline{a}, \overline{a}]$ such that all firms $a < a^*$ invest and all firms $a > a^*$ do not invest.*

3 One-period benchmark

Before proceeding to our main analysis, we characterize the equilibrium of the benchmark model in which there is just one trading date, i.e., date 2 is absent. The main conclusion of this section is that the the Myers and Majluf conclusion holds: only the lowest asset firms

[7]Given a perfect Bayesian equilibrium in which almost all firms play a best response, one can easily construct an equilibrium in which all firms play a best response by switching the actions of the measure zero set of firms who originally did not play a best response. Because only a measure zero set of firms are switched, the original set of beliefs remain valid.

issue and invest, and repurchases play no meaningful role. In other words, the addition of the possibility of repurchases to the Myers and Majluf environment is, by itself, inconsequential. Instead, our results further below are driven by the possibility of firms engaging in capital transactions at multiple dates.

The key reason that the firms do not take advantage of repurchases in a one-period model is the no-trade theorem (Milgrom and Stokey (1982)). Even though firms enjoy an informational advantage relative to investors, they are unable to profit from this advantage.

Proposition 1 *In the single stage benchmark game, the set of firms who repurchase and strictly profit relative to doing nothing is of measure* 0*.*

Proposition 1 establishes that, in the one-period benchmark, a firm's ability to repurchase its own stock plays no meaningful role. Accordingly, the equilibria of the one-period benchmark coincide with those of the standard Myers and Majluf (1984) setting, as formally established by the next result:

Proposition 2 *In any equilibrium, there exists* $a^* \in (\underline{a}, \bar{a}]$ *such that almost all firms below* a^* *issue the same amount* s^* *and invest, while almost all firms above* a^* *receive the same payoff as doing nothing (i.e.,* $P_3 = a + S$*).*

Proposition 2 characterizes properties an equilibrium must possess. However, it does not actually establish the existence of an equilibrium. However, this is easily done. In particular, fix any s^* such that $S - s^* \geq I$, and define a^* by

$$a^* = \max\left\{a \in [\underline{a}, \bar{a}] : \frac{S - s^* + a^* + b}{1 - \frac{s^*}{S + E[a|a \in [\underline{a}, a^*] + b]}} \geq S + a^*\right\}.$$

Then there is an equilibrium in which all firms with assets below a^* issue and raise an amount $-s^*$, while firms with assets above a^* do nothing. Off-equilibrium-path beliefs are such that any offer to issue (i.e., $s < 0$ and $s \neq s^*$) is interpreted as coming from the worst type $\underline{a}$, and any offer to repurchase (i.e., $s > 0$) is interpreted as coming from the best type $\bar{a}$.

Observe that if $\frac{I+\overline{a}+b}{1+\frac{I-S}{S+E[a]+b}} \geq S+\overline{a}$, this benchmark model has an equilibrium in which the socially efficient outcome of all firms investing is obtained. In order to focus attention on the case in which asymmetric information causes a social loss, for the remainder of the paper we assume instead that

$$\frac{I+\overline{a}+b}{1+\frac{I-S}{S+E[a]+b}} < S+\overline{a}, \tag{6}$$

so that there is no equilibrium of the benchmark model in which all firms invest. For use below, note that (6) implies

$$\bar{a} > E[a] + b > \underline{a} + b. \tag{7}$$

4 Analysis of the dynamic model

We now turn to the analysis of the full model, in which the firm is able to engage in capital transactions at multiple dates.

Our first result—and one of the main results of the paper—is that any equilibrium of the multi-period model must entail repurchases:

Proposition 3 *In any equilibrium, there exists an upper interval of firms who repurchase; do not invest; and make strictly positive profits both overall (i.e., firm a obtains a payoff strictly in excess of $S+a$), and from the repurchase transaction.*

The economics behind Proposition 3 is as follows. Under assumption (6), the best firms do not invest in equilibrium.[8] Consequently, if they do not repurchase, these firms do not make any profits, and the final payoff of a high-value firm a is simply $S+a$. Consequently, for repurchases to be unattractive in equilibrium for the top firm $\bar{a}$, investors must charge at least $S+\bar{a}$ to surrender their shares; in turn, this requires investors to believe that (off-equilibrium) repurchase offers come from very good firms. But given these beliefs, a low-value firm could profitably deviate from its equilibrium strategy by repurchasing at date 1, thereby triggering beliefs that it is very good, and then issue at a high price at date 2.

[8]Formally, this is established in Corollary A-3 in the appendix.

Because investors break even in expectation, an immediate corollary of Proposition 3 is:

Corollary 2 *In any equilibrium, there exist firms that repurchase and make strict losses on the repurchase transaction.*

This is the "buying high" of the paper's title.

A second important implication of Proposition 3 is that the equilibrium of the one-period benchmark economy is not an equilibrium of the full model. At first sight, this might seem surprising: one might imagine that one could take the equilibrium of the one-period economy and then assign off-equilibrium beliefs to make other actions, and in particular repurchases, unattractive. However, the dynamic nature of the model makes this impossible. The reason is that, as just illustrated, to deter repurchases, off-equilibrium beliefs must assign a large weight to a repurchasing-firm being a high type; but given these beliefs, a deviating firm can issue at attractive terms at date 2. In brief, it is impossible to assign off-equilibrium beliefs that deter *both* date 1 repurchase and date 2 issue.

In light of this discussion, it is important to establish that an equilibrium actually exists. Our environment features a continuum of types and the possibility of signaling at two dates.[9] Consequently, we are unaware of any result in the existing literature that guarantees existence. Instead, we prove existence by explicitly constructing an equilibrium. The equilibrium constructed is either similar to the above example, or else features all firms repurchasing at date 1, with a strict subset then issuing equity to fund investment at date 2.

Proposition 4 *An equilibrium exists.*

5 Stock price reactions

A large empirical literature has examined stock price reactions to repurchase and issuance announcements; see, e.g., Allen and Michaely (2003) for a survey. As documented by this lit-

[9] The model also features a continuum of actions. However, it is the first two features that make establishing equilibrium existence difficult.

erature, repurchase announcements are associated with price increases, and issue announcements are associated with price declines.

Our model provides a natural explanation of both these announcement effects. Issue announcements generate negative price responses because lower-value firms issue. This is the "selling low" of the paper's title, and is very much in line with the existing literature (again, see Allen and Michaely (2003)).

Repurchase announcements generate positive price reactions. The reason is that some of the firms repurchasing are high-value firms. This is an effect present in several existing models in the literature. With respect to this previous literature, the innovation of our paper is to obtain this effect without exogenously assuming that firms care about the interim stock price. Specifically, the reason high-value firms repurchase in our model is that they pool with low-value firms, and so are able to repurchase at an attractive price.

The reason low-value firms repurchase—and do so at a price that is high for them—is that by doing so they reduce the price of subsequent equity issues. This is one of the primary empirical implications of our model. Billet and Xue (2007) find evidence for this effect. They compare the issuance price reactions of firms that previously repurchased stock with the issuance price reactions of firms that did not previously repurchase. The price decline of the former group is smaller, consistent with our model.

The following result formalizes these predictions of our model:

Proposition 5 *Let* $s_1 \geq 0$ *be a date* 1 *repurchase decision used by a positive measure of firms. Then:*

(A, price drops at issue) A positive-measure subset of these firms issue an amount s_2 *such that* $S - s_1 - s_2 \geq I$ *at date* 2*, at a price* $P_2 \leq P_1$*. Moreover, the date-*2 *price of non-issuing firms exceeds* P_1*. Both relations are strict whenever* $\Pr(s_2|s_1) < 1$.

(B, repurchase increases subsequent issue price) Suppose that a positive measure of firms issue $s_1' < 0$ *at date* 1*. Then there exists* s_2' *such that* $s_2' \leq 0$*,* $S - s_1' - s_2' \geq I$*,* $\Pr(s_2'|s_1') = 1$*, and* $P_2(s_1', s_2') = P_1(s_1') \leq P_2(s_1, s_2)$*. Likewise, if* $(0, s_2')$ *with* $s_2' < 0$ *is played by a positive*

measure of firms, then $P_2(0, s_2') \leq P_2(s_1, s_2)$*. Both price relations are strict if* $s_1 > 0$ *and* $\Pr(s_2|s_1) < 1$.

(C, price increases at repurchase) If, in addition, a positive measure of firms take no action at date 1*, then* $P_1(s_1) \geq P_1(0)$*, with the inequality strict under the same conditions as in Part (B).*

Our model also generates cross-sectional predictions between, on the one hand, the *size* of repurchases and issues, and on the other hand, the price response associated with these transactions. These predictions emerge in equilibria of the model in which multiple repurchase and issue levels coexist (in contrast to the example, which features just one repurchase level and one issue level).[10]

As one would expect, larger repurchases are associated with higher repurchase prices, since they are conducted by firms that are, on average, better. Similarly, larger issues are associated with lower issue prices. Both predictions are consistent with empirical evidence: see, for example, Ikenberry, Lakonishok and Vermaelen (1995) for evidence on repurchases, and Asquith and Mullins (1986) for evidence on issues.

Proposition 6 *(A, repurchases) Consider an equilibrium in which* s' *and* $s'' > s'$ *are repurchase levels, with associated prices* P' *and* P''*, and such that there exist firms* a' *and* a'' *where firm* a' *(respectively,* a''*) repurchases* s' *(respectively,* s''*) and does not conduct any other capital transaction at any other date. Then (i)* $P'' \geq P'$*, (ii)* $s''/P'' > s'/P'$*, and (iii)* $a'' > a'$*. In particular, repurchase size is positively correlated with repurchase price.*

(B, issues) Let (s_1', s_2') *and* (s_1'', s_2'') *be equilibrium strategies such that* $S - s_1'' - s_2'' > S - s_1' - s_2'$*. Then* $P_2(s_1', s_2') > P_2(s_1'', s_2'')$*. In particular, if* $s_2' < 0$ *and* $s_2'' < 0$*, then greater cumulative issue is associated with lower date* 2 *issue prices.*[11]

[10] One can show, via numerical simulation, that such equilibria exist.

[11] It is also possible to establish that $s_1' > s_1''$, i.e., greater cumulative issue is associated with smaller initial repurchases. A proof is available upon request.

6 Welfare

As we have established, any equilibrium of our economy entails some firms repurchasing. As we noted earlier, an immediate implication of this result is that the equilibrium of the one-period benchmark—in other words, the Myers and Majluf equilibrium—is not an equilibrium of the full model. In contrast, if repurchases are prohibited, it is very straightforward to show that all equilibria of the one-period benchmark remain equilibria of the full model.

Here, we ask how the possibility of repurchases affects social welfare. The appropriate welfare measure is simply the fraction of firms that invest. We compare welfare in our model with welfare in the same model when repurchases are prohibited. We obtain the following strong result:

Proposition 7 *Consider any equilibrium featuring a finite number of actions.* [12] *If repurchases are prohibited, there exists an equilibrium with strictly high welfare.*[13]

The example illustrates the basic economics of this result. In the equilibrium of the example, some high-value firms strictly profit from repurchasing their stock for less than its true value. Because investors break even in expectation, the ultimate source of these profits is low-value firms who initially pool with high-value firms and repurchase, in order to reduce the cost of subsequent issues. Low-value firms lose money on the repurchase leg of this transaction. If repurchases are prohibited, low-value firms no longer have to endure this loss-making leg. This allows them to issue at better terms, which in turn means that a greater fraction of firms find issuance (and investment) preferable to non-issuance.

At least since Arrow (1973), it has been understood that the possibility of economic agents signaling their type by undertaking a socially costly action may result in lower welfare relative to a situation in which signaling is prohibited or otherwise impossible.[14] In our

[12] This restriction is made for simplicity, to avoid mathematical complication. The result covers equilibria with an arbitrarily large (but finite) number of equilibrium actions.

[13] In particular, if the one-period benchmark has a unique equilibrium in the class of equilibria with $S - s_1 + I$, then welfare in this equilibrium exceeds welfare in any equilibrium of the full model.

[14] For a recent result along these lines, see Hoppe, Moldovanu and Sela (2009).

setting, however, repurchases carry no deadweight cost, yet welfare is still reduced.

7 Conclusion

We investigate the consequences of allowing for repeated capital market transactions in a model with asymmetric information between a firm and its investors. All firms in the model possess a profitable project that they need to raise cash to undertake. However, we show that any equilibrium of the model entails some firms returning cash to investors via share repurchases. Consistent with managerial accounts, some repurchasing firms profit from repurchasing their stock. The ultimate source of these profits is that other firms buy "high'' in order to improve the terms of subsequent stock issues. We show the possibility of repurchases lowers social welfare by reducing the fraction of firms that invest, even though repurchasing itself is a costless signal. Our model generates a number of empirical predictions.

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Appendix

Proof of Lemma 1: Suppose to the contrary that $a' \geq a''$. Since firms a' and a'' follow different strategies, $a' > a''$. Let P_1' and P_2' (respectively, P_1'' and P_2'') be the prices associated with s_1' and s_2' (respectively, s_1'' and s_2''). Also, let $\mathbf{1}'$ and $\mathbf{1}''$ be the investment decisions of firms a' and a''.

From the equilibrium conditions,

$$\frac{a'' + S - s_1'' - s_2'' + b\mathbf{1}''}{1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''}} \geq \frac{a'' + S - s_1' - s_2' + b\mathbf{1}'}{1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}}. \tag{A-1}$$

By supposition, and given optimal investment decisions, the numerator of the LHS is strictly smaller than the numerator of RHS. Hence the denominator of the LHS must also be strictly smaller, i.e.,

$$1-\frac{s_1''}{P_1''}-\frac{s_2''}{P_2''}<1-\frac{s_1'}{P_1'}-\frac{s_2'}{P_2'}. \tag{A-2}$$

Also from the equilibrium conditions,

$$\frac{a'+S-s_1'-s_2'+b\mathbf{1}'}{1-\frac{s_1'}{P_1'}-\frac{s_2'}{P_2'}}\geq\frac{a'+S-s_1''-s_2''+b\mathbf{1}''}{1-\frac{s_1''}{P_1''}-\frac{s_2''}{P_2''}}.$$

From (A-2),

$$\frac{a'-a''}{1-\frac{s_1'}{P_1'}-\frac{s_2'}{P_2'}}<\frac{a'-a''}{1-\frac{s_1''}{P_1''}-\frac{s_2''}{P_2''}},$$

which implies

$$\frac{a''+S-s_1'-s_2'+b\mathbf{1}'}{1-\frac{s_1'}{P_1'}-\frac{s_2'}{P_2'}}>\frac{a''+S-s_1''-s_2''+b\mathbf{1}''}{1-\frac{s_1''}{P_1''}-\frac{s_2''}{P_2''}},$$

contradicting the equilibrium condition (A-1) and completing the proof.

Proof of Corollary 1: Suppose to the contrary that the claim does not hold, i.e., there exists an equilibrium in which there are firms a' and $a''>a'$ where a'' invests and a' does not invest. Since investment decisions are optimal, the capital transactions of firms a' and a'', say (s_1', s_2') and (s_1'', s_2''), must satisfy $S-s_1'-s_2'<I\leq S-s_1''-s_2''$. This contradicts Lemma 1, completing the proof.

Proof of Proposition 1: Suppose otherwise. Let $s_1(a)$ be the strategy of firm a, and $A^{rep}=\{a: s_1(a)>0\}$ be the set of firms who repurchase in equilibrium. By supposition, $\mu(A^{rep})>0$. On the one hand, a firm prefers repurchasing to doing nothing if and only if $\frac{a+S-s_1}{1-\frac{s_1}{P_1(s_1)}}\geq a+S$, or equivalently, $P_1(s_1)\leq a+S$. Since by supposition a strictly positive mass of repurchasing firms have a strict preference for repurchasing,

$$E\left[P_1(s_1(a))-(a+S)\,|a\in A^{rep}\right]<0.$$

One the other hand, investors only sell if $P_1(s_1)\geq E\left[\frac{a+S-s_1}{1-\frac{s_1}{P_1(s_1)}}|s_1\right]$, or equivalently, $P_1(s_1)\geq$

$E[a|s_1] + S$. By the law of iterated expectations, this implies

$$E[P_1(s_1(a)) - (a+S)|a \in A^{rep}] \geq 0.$$

The contradiction completes the proof.

Proof of Proposition 2: Fix an equilibrium. From Proposition 1, there cannot be a positive mass of firms who repurchase and obtain $P_3 > a + S$. By a parallel proof, there cannot be a positive mass of firms who issue, do not invest, and obtain $P_3 > a + S$. By (4), any issue s that is enough for investment is associated with the price

$$P_1(s) = S + E[a|s] + b. \tag{A-3}$$

Given these observations, standard arguments then imply that there exists some $\varepsilon > 0$ such that almost all firms in $[\underline{a}, \underline{a} + \varepsilon]$ issue and invest: if an equilibrium does not have this property, then these firms certainly have the incentive to deviate and issue and invest, since this is profitable under any investor beliefs. So by Corollary 1, there exists $a^* > \underline{a}$ such that all firms in $[\underline{a}, a^*)$ issue and invest.

Finally, suppose that contrary to the claimed result that different firms in $[\underline{a}, a^*)$ issue different amounts. Given Lemma 1, it follows that there exists $\breve{a} \in (\underline{a}, a^*)$ such that any firm in $[\underline{a}, \breve{a})$ issues strictly more than any firm in $(\breve{a}, a^*)$. Hence there must exist firms $a' \in [\underline{a}, \breve{a})$ and $a'' \in (\breve{a}, a^*)$ such that

$$P_1(s(a')) \leq S + a' + b < S + a'' + b \leq P_1(s(a'')).$$

Since $-s(a') > -s(a'')$, this combines with the equilibrium condition for firm a' to deliver the following contradiction, which completes the proof:

$$\frac{S - s(a'') + a' + b}{1 - \frac{s(a'')}{P_1(s(a''))}} \leq \frac{S - s(a') + a' + b}{1 - \frac{s(a')}{P_1(s(a'))}} \leq \frac{S - s(a'') + a' + b}{1 - \frac{s(a'')}{P_1(s(a'))}} < \frac{S - s(a'') + a' + b}{1 - \frac{s(a'')}{P_1(s(a''))}}.$$

Lemma A-1 *There is no equilibrium in which almost all firms invest.*

Proof of Lemma A-1: Suppose to the contrary that there is an equilibrium in which almost all firms invest. By assumption (6), it follows that there is a firm a' that invests and such that $a' > E[a]$ and

$$S + a' > \frac{I + a' + b}{1 + \frac{I-S}{S+E[a]+b}}.$$

Let (s_1, s_2) be the strategy of firm a', and let (P_1, P_2) be the associated prices. So the equilibrium condition for firm a' implies

$$\frac{S - s_1 - s_2 + a' + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} \geq S + a' > \frac{I + a' + b}{1 + \frac{I-S}{S+E[a]+b}} \geq \frac{S - s_1 - s_2 + a' + b}{1 - \frac{s_1+s_2}{S+E[a]+b}},$$

where the final inequality makes use of $-s_1 - s_2 \geq I - S$ (since firm a' invests) and $a' > E[a]$. Since any firm has the option of following strategy (s_1, s_2), it follows that the equilibrium payoff of an arbitrary firm a is at least

$$\frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} > \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1+s_2}{S+E[a]+b}}.$$

Consequently, the unconditional expected firm payoff is *strictly* greater than

$$E\left[\frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1+s_2}{S+E[a]+b}}\right] = S + E[a] + b.$$

But this violates investor rationality (formally, it violates (4)), giving a contradiction and completing the proof.

Corollary A-3 *In any equilibrium, there is a non-empty interval* $[\bar{a} - \delta, \bar{a}]$ *of firms that do not invest.*

Proof of Corollary A-3: Immediate from Corollary 1 and Lemma A-1.

Proof of Proposition 3:

Claim: There is a non-empty interval $[\bar{a}-\delta,\bar{a}]$ of firms that make strictly positive profits, i.e., obtain a payoff strictly in excess of $S+a$.

Proof of Claim: Suppose to the contrary that this is not the case. i.e., that one can find a firm a arbitrarily close to $\bar{a}$ that has a payoff of $S+a$.

Consider any repurchase offer $s_1 > 0$. If $P_1(s_1) < S+\bar{a}$, then by supposition one can find a firm that could strictly increase its payoff by repurchasing s_1, a contradiction. Hence $P_1(s_1) \geq S+\bar{a}$. So from (4), the beliefs associated with s_1 must be such that $E\left[a+b\mathbf{1}_{S-s_1-s_2\geq I}|s_1\right] \geq \bar{a}$. So by Corollary A-3, there exists s_2 with $S-s_1-s_2 \geq I$ such that $E\left[a+b|s_1,s_2\right] \geq \bar{a}$. So by (3), firm a's payoff from playing (s_1,s_2) is weakly greater than

$$\frac{S-s_1-s_2+a+b}{\left(1-\frac{s_1}{P_1(s_1)}\right)\left(1-\frac{s_2}{S-s_1+\bar{a}}\right)}.$$

By the equilibrium condition, the unconditional expected equilibrium payoff of a firm is at least

$$\frac{S-s_1-s_2+E\left[a\right]+b}{\left(1-\frac{s_1}{P_1(s_1)}\right)\left(1-\frac{s_2}{S-s_1+\bar{a}}\right)} \geq \frac{I+E\left[a\right]+b}{\left(1-\frac{s_1}{P_1(s_1)}\right)\left(\frac{I+\bar{a}}{S-s_1+\bar{a}}\right)}, \tag{A-4}$$

where the inequality follows from (7) and $S-s_1-s_2 \geq I$.

Since $P_1(s_1)$ is bounded below by $S+\underline{a}$, the term $\frac{s_1}{P_1(s_1)}$ approaches 0 as s_1 approaches 0. Consequently, the limiting value of the RHS of (A-4) is

$$\left(I+E\left[a\right]+b\right)\frac{S+\bar{a}}{I+\bar{a}}. \tag{A-5}$$

Because the above argument holds for any initial choice of $s_1 > 0$, the unconditional expected equilibrium payoff of a firm is at least (A-5). Moreover, by (7), expression (A-5) is itself strictly greater than $S+E\left[a\right]+b$. But this violates investor rationality (formally, it violates (4)), giving a contradiction.

Completing the proof: By Corollary A-3 and the Claim, there exists $\delta' > 0$ such that all firms in $[\bar{a}-\delta',\bar{a}]$ make strictly positive profits and do not invest. Let $\varepsilon > 0$ be the minimum profits made by a firm in this interval. (Note that the minimum is well-defined because a firm's equilibrium payoff is continuous in a: if this is not the case, there is a profitable deviation for some a.) Then choose $\delta \in (0, \delta')$ sufficiently small such that, for all $a \in [\bar{a} - \delta, \bar{a}]$, $a + \varepsilon > \bar{a}$,$a + b > \bar{a}$, and $(S + a)\frac{\bar{a}}{a} < S + a + \varepsilon$. To complete the proof, we show all firms in $[\bar{a} - \delta, \bar{a}]$ repurchase, and make strictly positive profits from the repurchase transaction.

Suppose to the contrary that there exists some firm $a \in [\bar{a} - \delta, \bar{a}]$ that either does not repurchase, or else makes weakly negative profits from the repurchase: formally, either $s_1(a) \leq 0$, or $s_1(a) > 0$ with $P_1(s_1(a)) \geq S + a$; and either $s_2(a) \leq 0$, or $s_2(a) > 0$ with $P_2(s_1(a), s_2(a)) \geq \frac{S - s_1(a) + a}{1 - \frac{s_1(a)}{P_1(s_1(a))}}$.

We first show that firm a's payoff is bounded above by

$$\frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}}. \tag{A-6}$$

If $s_2(a) > 0$ this is immediate. Otherwise, (5) and the fact that by (3) (and using $a \leq \bar{a}$ and the firm does not invest) $P_2(s_1(a), s_2(a)) \leq \frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}}$ together imply that the firm's payoff is bounded above by

$$\frac{S - s_1(a) - s_2(a) + a}{\left(1 - \frac{s_1(a)}{P_1(s_1(a))}\right)\left(1 - \frac{s_2(a)}{S - s_1(a) + \bar{a}}\right)} = \frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}} \frac{S - s_1(a) - s_2(a) + a}{S - s_1(a) - s_2(a) + \bar{a}},$$

which is below expression (A-6).

If $s_1(a) > 0$, expression (A-6) is in turn bounded above by

$$\frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{S + a}} = (S + a)\frac{S - s_1(a) + \bar{a}}{S - s_1(a) + a} \leq (S + a)\frac{\bar{a}}{a}.$$

But this is less than $S + a + \varepsilon$, a contradiction.

Consequently, it must be the case that $s_1(a) \leq 0$. Observe that if $P_1(s_1(a)) \leq S + a + \varepsilon$, from (A-6) and the fact that $a + \varepsilon > \bar{a}$, firm a's payoff is bounded above by

$$(S + a + \varepsilon) \frac{S - s_1(a) + \bar{a}}{S - s_1(a) + a + \varepsilon} < S + a + \varepsilon,$$

which again is a contradiction. Hence $P_1(s_1(a)) > S + a + \varepsilon > S + \bar{a}$. It then follows from (4) that there must exist s_2 such that $S - s_1(a) - s_2 \geq I$ and

$$E\left[S + a + b\mathbf{1}_{S - s_1 - s_2 \geq I} | s_1 = s_1(a), s_2\right] \geq P_1(s_1(a)). \tag{A-7}$$

On the one hand, the equilibrium payoff of firm $a \in A$ is—using (A-6), together with $P_1(s_1(a)) > S + \bar{a}$—bounded above by

$$P_1(s_1(a)) \frac{S - s_1(a) + \bar{a}}{P_1(s_1(a)) - s_1(a)} \leq P_1(s_1(a)) \frac{I + \bar{a}}{P_1(s_1(a)) + I - S}.$$

On the other hand, the payoff to firm a to instead deviating and using strategy $(s_1(a), s_2)$, where s_2 is as above, is bounded below by

$$\frac{S - s_1(a) - s_2 + a + b}{1 - \frac{s_1(a) + s_2}{P_1(s_1(a))}} \geq \min\left\{P_1(s_1(a)), P_1(s_1(a)) \frac{I + a + b}{P_1(s_1(a)) + I - S}\right\}.$$

Since this is strictly greater than the upper bound on firm a's equilibrium payoff, firm a has the incentive to deviate. This contradicts the equilibrium condition, and completes the proof.

Proof of Proposition 4:

Preliminaries:

Given any date 1 repurchase level $s_1 > 0$, define $a^\star(s_1)$ to be the smallest solution of

$$\frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a | a \leq a^\star] + b}} (I + a^\star + b) - (S - s_1 + a^\star) = 0. \tag{A-8}$$

We first show that $a^\star(s_1)$ is well-defined, decreasing in s_1, and strictly exceeds $\underline{a}$. The proof is as follows. The LHS of (A-8) is strictly positive at $a^\star = \underline{a}$. The LHS of (A-8) is strictly decreasing in s_1 for any $a^\star > \underline{a}$. Consequently, (6) implies that the LHS of (A-8) is strictly negative at $a^\star = \bar{a}$. Existence of $a^\star(s_1)$ follows by continuity. The other two properties are immediate.

Observe that at $s_1 = 0$ and $a_1 = \underline{a}$,

$$\frac{1}{1-\frac{s_1}{S+\bar{a}}}\frac{1}{1+\frac{I-S+s_1}{S-s_1+E[a|a\leq a_1]+b}}(I+a_1+b) > S+a_1. \tag{A-9}$$

By continuity, choose $\bar{a}_1 > \underline{a}$ and $\bar{s}_1 > 0$ such that inequality (A-9) holds for all $(a_1, s_1) \in [\underline{a}, \bar{a}_1] \times [0, \bar{s}_1]$.

Fix $s_1 \in (0, \min\left\{\bar{s}_1, \frac{S}{2}\right\}]$ sufficiently small such that

$$\max\left\{\frac{I-S+s_1}{S-s_1+E\left[a|a<a^\star(\frac{S}{2})\right]+b}, \frac{I-S+s_1}{S-s_1+E\left[a|a<\bar{a}_1\right]+b}\right\} \leq \frac{I-S}{S+\underline{a}+b}. \tag{A-10}$$

Given s_1, we explicitly construct an equilibrium. To do so, we first consider the artificial game in which the only allowable strategies are: do-nothing; repurchase s_1; repurchase s_1 and then issue $I+s_1-S$ at date 2; and directly issue $I-S$ at date 1. After constructing an equilibrium of this artificial game, we then show that the same strategies constitute an equilibrium of the full game, by exhibiting off-equilibrium beliefs for all other strategies. Moreover, in the equilibrium we construct for the artificial game, direct issue of $I-S$ is never played, and the off-equilibrium beliefs attached to this strategy are that the firm is the lowest type $\underline{a}$, and so the associated share price for direct issue is $S+\underline{a}+b$.

The equilibrium construction depends on the relative magnitude of $S + a^\star(s_1)$ and $S + E[a] + b\Pr(a \leq a^\star(s_1))$:

Case 1: $S + a^\star(s_1) \geq S + E[a] + b\Pr(a \leq a^\star(s_1))$.

In this case, we show there is an equilibrium in which at date 1 all firms repurchase s_1; and at date 2 firms $a \leq a^\star(s_1)$ issue $I - S + s_1$ and invest, while other firms do nothing at date 2. The date 1 repurchase price P_1 and date 2 issue price P_2 in such an equilibrium are

$$\begin{aligned} P_1 &= S + E[a] + b \Pr(a \leq a^\star(s_1)) \\ P_2 &= \frac{S - s_1 + E[a|a \leq a^\star(s_1)] + b}{1 - \frac{s_1}{P_1}}. \end{aligned}$$

Hence the payoff for a firm a from repurchase-issue is

$$\frac{1}{1 - \frac{s_1}{P_1} + \frac{I - S + s_1}{P_2}}(I + a + b) = \frac{1}{1 - \frac{s_1}{P_1}} \frac{I + a + b}{1 + \frac{I - S + s_1}{S - s_1 + E[a|a \leq a^\star(s_1)] + b}}. \tag{A-11}$$

By (A-10) and the fact that $a^\star(\cdot)$ is decreasing, the payoff (A-11) is at least

$$\frac{1}{1 - \frac{s_1}{P_1}} \frac{I + a + b}{1 + \frac{I - S}{S + \underline{a} + b}} > \frac{I + a + b}{1 + \frac{I - S}{S + \underline{a} + b}}.$$

The RHS of this inequality is the payoff to issuing directly given out-of-equilibrium beliefs in which direct issue is associated with the worst firm $\underline{a}$. Hence all firms prefer the equilibrium repurchase-issue strategy to the off-equilibrium direct issue strategy.

Firms $a \geq a^\star(s_1)$ prefer repurchase-do-nothing to do-nothing. To see this, simply note that the payoff for a firm a from repurchase-do-nothing is $\frac{S - s_1 + a}{1 - \frac{s_1}{P_1}}$, which exceeds the payoff from do-nothing, i.e., $S + a$, if and only if $P_1 \leq S + a$. Since we are in Case 1, this condition is satisfied for all firms $a \geq a^\star(s_1)$.

Firms $a \geq a^\star(s_1)$ prefer repurchase-do-nothing to repurchase-issue by the definition of $a^\star(s_1)$.

Likewise, firms $a \leq a^\star(s_1)$ prefer repurchase-issue to repurchase-do-nothing by the definition of $a^\star(s_1)$.

Finally, firms $a \leq a^\star(s_1)$ prefer repurchase-issue to do-nothing because this is true for firm $a^\star(s_1)$; and is also true for firm $\underline{a}$, since this firm prefers direct issue to do-nothing. Since all payoffs are linear in firm type, it then follow that all firms between $\underline{a}$ and $a^\star(s_1)$ likewise prefer repurchase-issue to do-nothing.

Case 2: $S + a^{\star}(s_1) < S + E(a) + b \Pr(a \leq a^{\star}(s_1))$.

In this case, we show there exists a_1 and a_2 such that the following is an equilibrium: At date 1 firms $[\underline{a}, a_1] \cup [a_2, \bar{a}]$ repurchase s_1, while other firms do nothing; and at date 2 firms $[\underline{a}, a_1]$ issue $I - S + s_1$ and invest, while other firms do nothing at date 2. The date 1 repurchase price P_1 and date 2 issue price P_2 in such an equilibrium are

$$\begin{aligned} P_1 &= S + E[a|a \leq a_1 \ \ or\ a \geq a_2] + b \frac{\Pr(a \leq a_1)}{\Pr(a \leq a_1 \ \ or\ a \geq a_2)} \\ P_2 &= \frac{S - s_1 + E[a|a \leq a_1] + b}{1 - \frac{s_1}{P_1}}. \end{aligned}$$

We show that the following pair of equations has a solution with $a_1, a_2 \in [\underline{a}, \bar{a}]$ and $a_1 < a_2$:

$$\frac{1}{1 - \frac{s_1}{P_1}} \frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a|a \leq a_1] + b}} (I + a_1 + b) = S + a_1 \quad \text{(A-12)}$$

$$P_1 = S + a_2 \quad \text{(A-13)}$$

Condition (A-12) states that firm a_1 is indifferent between repurchase-issue and do-nothing, while condition (A-13) states that firm a_2 is indifferent between repurchase-do-nothing and do-nothing. Note first that this pair of equations (together with the definition of P_1) has a solution if and only if the following pair has a solution:

$$\begin{aligned} \frac{1}{1 - \frac{s_1}{S + a_2}} \frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a|a \leq a_1] + b}} (I + a_1 + b) &= S + a_1 && \text{(A-14)} \\ E[a|a \leq a_1 \ \ or\ a \geq a_2] + b \frac{\Pr(a \leq a_1)}{\Pr(a \leq a_1 \ \ or\ a \geq a_2)} - a_2 &= 0 && \text{(A-15)} \end{aligned}$$

Claim: There exists $\hat{a} \in [\bar{a}_1, \bar{a}]$ such that for $a_1 \in [\hat{a}, a^{\star}(s_1)]$, equation (A-14) has a unique solution, which we denote $a_2(a_1)$. Moreover, $a_2(a_1)$ is continuous in a_1, with $a_2(\hat{a}) = \bar{a}$ and $a_2(a^{\star}(s_1)) = a^{\star}(s_1)$, and $a_2(a_1) > a_1$.

We prove the Claim below. Given the Claim, equation (A-15) with $a_2 = a_2(a_1)$ has a solution, as follows. At $a_1 = \hat{a}$, the LHS of (A-15) is $E[a|a \leq \hat{a}] + b - \bar{a}$, which strictly negative since (6) implies that $b < \overline{a} - E[a]$. At $a_1 = a^\star(s_1)$, the LHS of (A-15) is $E[a] + b\Pr(a \leq a^\star(s_1)) - a^\star(s_1)$, which is strictly positive since we are in Case 2. Hence a solution exists by continuity. The solution is the desired equilibrium.

Most of the equilibrium conditions follow from (A-12) and (A-13). The remaining condition to check is that no firm prefers to deviate and issue directly. This follows as in Case 1, together with the fact that $a_1 > \hat{a} \geq \bar{a}_1$ and (A-10) imply

$$\frac{I - S + s_1}{S - s_1 + E[a|a < a_1] + b} \leq \frac{I - S + s_1}{S - s_1 + E[a|a < \bar{a}_1] + b} \leq \frac{I - S}{S + \underline{a} + b}.$$

Proof of Claim:

The LHS of (A-14) is strictly decreasing in a_2, so if a solution exists it is continuous. By the definition of $a^\star(s_1)$, the LHS of (A-8) is positive for all $a_1 \in [\underline{a}, a^\star(s_1)]$, and strictly so except for at $a_1 = a^\star(s_1)$. Consequently, the LHS of (A-14) evaluated at $a_2 = a_1$ is greater than $\frac{S - s_1 + a_1}{1 - \frac{s_1}{S + a_1}} = S + a_1$, and strictly so except for at $a_1 = a^\star(s_1)$. So at $a_1 = a^\star(s_1)$ we have $a_2(a_1) = a_1$, while for $a_1 < a^\star(s_1)$ any solution to (A-14) must strictly exceed a_1.

Evaluated at $a_1 = \bar{a}_1$ and $a_2 = \bar{a}$, the LHS strictly exceeds the RHS of (A-14) by (A-9). Evaluated at $a_1 = a^\star(s_1)$ and $a_2 = \bar{a}$, the LHS of (A-14) is

$$\frac{S - s_1 + a^\star(s_1)}{1 - \frac{s_1}{S + \bar{a}}} < S + a^\star(s_1) = S + a_1.$$

So by continuity, there exists $\hat{a} \in (\bar{a}_1, a^\star(s_1))$ such that, for all $a_1 \in [\hat{a}, a^\star(s_1)]$, the LHS of (A-14) evaluated at $a_2 = \bar{a}$ is below the RHS, with equality at $a_1 = \hat{a}$.

Consequently, for $a_1 \in [\hat{a}, a^\star(s_1)]$ equation (A-14) has a unique solution in a_2. The solution lies in the interval $[a_1, \bar{a}]$, equals a_1 when $a_1 = a^\star(s_1)$ and equals $\bar{a}$ when $a_1 = \hat{a}$. This completes the proof of the Claim, and hence the treatment of this case.

Completing the proof: Off-equilibrium beliefs

We now allow for arbitrary strategies, which we denote $(\tilde{s}_1, \tilde{s}_2)$, and show that the equilibrium constructed above remains an equilibrium for at least the following specification of off-equilibrium beliefs: At date 2, repurchases $\tilde{s}_2 > 0$ are associated with the best firm $\bar{a}$ and issues $\tilde{s}_2 < 0$ are associated with the worst firm $\underline{a}$. At date 1, repurchases $\tilde{s}_1 > 0$ are associated with the best firm $\bar{a}$ with probability $1-\varepsilon$ and the worst firm with probability ε; while issues $\tilde{s}_1 < 0$ are associated with the best firm $\bar{a}$ with probability ε and the worst firm $\underline{a}$ with probability $1-\varepsilon$. Note that these date 1 beliefs mean the specification of date 2 beliefs is consistent.

Write $\tilde{P}_1$ and $\tilde{P}_2$ for the associated off-equilibrium prices. Given the stated off-equilibrium beliefs, there exists some $\kappa > 0$ such that

$$\tilde{P}_1 \begin{array}{ccc} \geq & S+\bar{a}-\varepsilon\kappa & \text{if} \quad \tilde{s}_1 > 0 \\ \leq & S+\underline{a}+b+\varepsilon\kappa & \text{if} \quad \tilde{s}_1 < 0 \end{array}. \tag{A-16}$$

Moreover,

$$\tilde{P}_2 = \begin{cases} \frac{S-\tilde{s}_1+\bar{a}+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{1-\frac{\tilde{s}_1}{\tilde{P}_1}} & \text{if} \quad \tilde{s}_2 > 0 \\ \frac{S-\tilde{s}_1+\underline{a}+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{1-\frac{\tilde{s}_1}{\tilde{P}_1}} & \text{if} \quad \tilde{s}_2 < 0 \end{cases}. \tag{A-17}$$

From above, note that the artificial-equilibrium payoff of any firm $a \in [\underline{a}, \bar{a}]$ strictly exceeds the payoff from direct issue, namely $\frac{I+a+b}{1+\frac{I-S}{S+\underline{a}+b}}$. Moreover, for firms a sufficiently close to $\bar{a}$, the artificial-equilibrium payoff also strictly exceeds the payoff from doing nothing, namely $S+a$. (Of course, this relation holds weakly for *all* firms.) Hence it is possible to choose $\varepsilon > 0$ such that, for all firms $a \in [\underline{a}, \bar{a}]$,

$$\max\left\{\frac{I+a+b}{1+\frac{I-S-\varepsilon\kappa}{S+\underline{a}+b+\varepsilon\kappa}}, a\frac{S+\bar{a}-\varepsilon\kappa}{\bar{a}-\varepsilon\kappa}\right\} < \text{artificial-equilibrium payoff of firm } a. \tag{A-18}$$

Moreover, and using $b > 0$ and inequality (7), choose $\varepsilon > 0$ sufficiently small such that, in addition to inequality (A-18), the following pair of inequalities holds:

$$\frac{a}{\underline{a}+b} \leq \frac{I+a+b}{I+\underline{a}+b} \quad \text{if} \quad a \in [\underline{a}+b, \underline{a}+b+\varepsilon\kappa], \tag{A-19}$$

$$\underline{a}+b+\varepsilon\kappa \leq \bar{a}-\varepsilon\kappa. \tag{A-20}$$

Firm a's payoff from an arbitrary off-equilibrium strategy $(\tilde{s}_1, \tilde{s}_2)$ is

$$\frac{S-\tilde{s}_1-\tilde{s}_2+a+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{1-\frac{\tilde{s}_1}{\tilde{P}_1}-\frac{\tilde{s}_2}{\tilde{P}_2}}.$$

First, observe that

$$-\frac{\tilde{s}_2}{\tilde{P}_2} \geq -\frac{\tilde{s}_2}{S-\tilde{s}_1+\underline{a}+b}\left(1-\frac{\tilde{s}_1}{\tilde{P}_1}\right).$$

This follows directly from (A-17) if $\tilde{s}_2 < 0$, and from (A-17) together with (7) if $\tilde{s}_2 > 0$. Second, observe that

$$-\frac{\tilde{s}_1}{\tilde{P}_1} \geq -\frac{\tilde{s}_1}{S+\underline{a}+b+\varepsilon\kappa}.$$

This follows directly from (A-16) if $\tilde{s}_1 < 0$, and from (A-16) together with (A-20) if $\tilde{s}_1 > 0$.

Consequently, firm a's payoff is bounded above by

$$\frac{S-\tilde{s}_1-\tilde{s}_2+a+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{\left(1-\frac{\tilde{s}_1}{S+\underline{a}+b+\varepsilon\kappa}\right)\left(1-\frac{\tilde{s}_2}{S-\tilde{s}_1+\underline{a}+b}\right)} = \frac{S-\tilde{s}_1-\tilde{s}_2+a+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{S-\tilde{s}_1-\tilde{s}_2+\underline{a}+b}\,\frac{S-\tilde{s}_1+\underline{a}+b}{S-\tilde{s}_1+\underline{a}+b+\varepsilon\kappa}\left(S+\underline{a}+b+\varepsilon\kappa\right). \tag{A-21}$$

To complete the proof, by (A-18) it is sufficient to show that expression (A-21) is bounded above by either the LHS of (A-18), or by $S+a$. There are four cases:

If $S-\tilde{s}_1-\tilde{s}_2 \geq I$ it is immediate that (A-21) is bounded above by $\frac{I+a+b}{I+\underline{a}+b}\left(S+\underline{a}+b+\varepsilon\kappa\right)$, which is the first term in the LHS of (A-18).

If $S-\tilde{s}_1-\tilde{s}_2 < I$ and $a \leq \underline{a}+b$ then (A-21) is bounded above by $(S+\underline{a}+b+\varepsilon\kappa)$.

If $S-\tilde{s}_1-\tilde{s}_2 < I$ and $a \in [\underline{a}+b, \underline{a}+b+\varepsilon\kappa]$ then (A-21) is bounded above by $\frac{a}{\underline{a}+b}\left(S+\underline{a}+b+\varepsilon\kappa\right)$, and the result then follows from (A-19).

Finally, consider the case $S - \tilde{s}_1 - \tilde{s}_2 < I$ and $a > \underline{a} + b + \varepsilon\kappa$. Note first that since $S - \tilde{s}_1 - \tilde{s}_2 < I$, the off-equilibrium beliefs imply that the firm weakly loses money on its date 2 transactions, so that its payoff is bounded above by

$$\frac{S - \tilde{s}_1 + a}{1 - \frac{\tilde{s}_1}{\tilde{P}_1}} = \tilde{P}_1 \frac{S - \tilde{s}_1 + a}{\tilde{P}_1 - \tilde{s}_1}.$$

If $\tilde{s}_1 > 0$, this expression is bounded above by $\max\left\{S + a, \frac{a\tilde{P}_1}{\tilde{P}_1 - S}\right\}$, which by (A-16) is bounded above by $\max\left\{S + a, a\frac{S+\bar{a}-\varepsilon\kappa}{\bar{a}-\varepsilon\kappa}\right\}$. If instead $\tilde{s}_1 < 0$ this expression is bounded above by $\max\left\{S + a, \tilde{P}_1\right\}$, which by (A-16) is bounded above by $\max\left\{S + a, S + \underline{a} + b + \varepsilon\kappa\right\} = S + a$. This completes the proof.

Lemma A-2 *If an equilibrium features capital transactions (s_1', s_2') and (s_1'', s_2'') with $S - s_1' - s_2' = S - s_1'' - s_2''$, then the associated transaction prices P_1', P_2', P_1'', P_2'' are such that*

$$1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''} = 1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}. \tag{A-22}$$

Proof of Lemma A-2: Let a' and a'' be firms that play (s_1', s_2') and (s_1'', s_2'') respectively. The equilibrium conditions for firm a' include

$$\frac{S - s_1' - s_2' + a' + b\mathbf{1}_{S-s_1'-s_2' \geq I}}{1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}} \geq \frac{S - s_1'' - s_2'' + a' + b\mathbf{1}_{S-s_1''-s_2'' \geq I}}{1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''}},$$

which simplifies to $1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''} \geq 1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}$. The symmetric equilibrium condition for a firm a'' playing (s_1'', s_2'') then implies (A-22). QED

Proof of Proposition 5:

Part (A): Firms that repurchase s_1 at date 1 are, at date 2, in exactly the situation characterized by Proposition 2. Consequently, at date 2 a positive-measure subset of these firms must issue an amount s_2 such that investment is possible, i.e., $S - s_1 - s_2 \geq I$ at date 2. If almost all firms that repurchase s_1 also issue s_2, then $P_1 = P_2$, and the proof is complete.

Otherwise, let $A_1^{s_1}$ denote the set of firms that repurchase s_1 at date 1. From Proposition 2, there exists a^* such that almost all firms in $A_1^{s_1} \cap [a^*, \bar{a}]$ choose not to issue s_2 at date 2. The equilibrium condition for any firm $a \in A_1^{s_1} \cap [a^*, \bar{a}]$ in this non-issuing set is

$$\frac{S - s_1 + a}{1 - \frac{s_1}{P_1}} \geq \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}}.$$

Hence

$$E\left[\frac{S - s_1 + a}{1 - \frac{s_1}{P_1}} | a \in A_1^{s_1} \cap [a^*, \bar{a}]\right] > E\left[\frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} | a \in A_1^{s_1} \cap [\underline{a}, a^*]\right],$$

so that the date 2 share price of non-issuing firms strictly exceeds the date 2 share price of issuing firms, i.e., P_2. Since the date 1 share price equals the conditional expectation of the date 2 share price, it follows that $P_2 < P_1$.

Part (B): First, suppose a positive measure of firms issue $s_1' < 0$. If $S - s_1' \geq I$, then by the argument of Proposition 2, almost all firms play $s_2' = 0$. If instead $S - s_1' < I$, then by the argument of Proposition 2, there exists s_2' such that $S - s_1' - s_2' \geq I$ and such that a positive measure of firms play (s_1', s_2'), and almost all the remainder play $(s_1', 0)$. Moreover, $\Pr(s_2'|s_1') = 1$, as follows. Suppose to the contrary that $\Pr(s_2'|s_1') < 1$. The equilibrium condition for a firm a that plays $(s_1', 0)$ is

$$\frac{S - s_1' + a}{1 - \frac{s_1'}{P_1(s_1')}} \geq \frac{S - s_1' - s_2' + a + b}{\left(1 - \frac{s_1'}{P_1(s_1')}\right)\left(1 - \frac{s_2'}{E\left[S - s_1 + a + b | s_1', s_2'\right]}\right)},$$

which simplifies (using $s_2' < 0$) to

$$\frac{S - s_1' + a}{E\left[S - s_1' + a + b | s_1', s_2'\right]} \geq 1 - \frac{b}{s_2'}.$$

Hence any firm a that plays $(s_1', 0)$ must satisfy $a > E\left[a + b | s_1', s_2'\right]$. By Lemma 1, firms that play $(s_1', 0)$ are better than firms that play (s_1', s_2'). Hence $P_1(s_1') < S + \sup\{a : a \text{ plays } s_1'\}$;

and almost all firms sufficiently close to $\sup\{a : a \text{ plays } s_1'\}$ play $(s_1', 0)$, and would obtain a higher payoff by doing nothing, a contradiction. This establishes that $P_2(s_1', s_2') = P_1(s_1')$.

We next establish the price comparison with firms that issue after previously repurchasing, i.e., $P_2(s_1, s_2)$. Given the first step, we handle the two cases in the proposition together: let (s_1', s_2') be a strategy with $S - s_1' - s_2' \geq I$ and $s_1', s_2' \leq 0$. At any date with strictly positive issue, the price is $P_2(s_1', s_2') = E[S + a + b | s_1', s_2']$. We first show that

$$S - s_1' - s_2' \geq S - s_1 - s_2. \tag{A-23}$$

The proof is by contradiction: suppose instead that $S - s_1' - s_2' < S - s_1 - s_2$. So by Lemma 1, $E[a|s_1', s_2'] > E[a|s_1, s_2]$. By Part (A), $P_1(s_1) \geq P_2(s_1, s_2) = \frac{S - s_1 + E[a|s_1,s_2] + b}{1 - \frac{s_1}{P_1(s_1)}}$, and so $P_1(s_1) \geq S + E[a|s_1, s_2] + b$. Hence

$$\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(\frac{1}{S - s_1 + E[a|s_1, s_2] + b}\right) \geq \frac{1}{S + E[a|s_1, s_2] + b}.$$

So the payoff to firm a from (s_1, s_2) is

$$\frac{S - s_1 - s_2 + a + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(1 - \frac{s_2}{S - s_1 + E[a|s_1,s_2] + b}\right)} \leq \frac{S - s_1 - s_2 + a + b}{\frac{S - s_1 - s_2 + E[a|s_1,s_2] + b}{S + E[a|s_1,s_2] + b}}.$$

Fix a firm playing (s_1, s_2) with $a > E[a|s_1, s_2]$. By the supposition $S - s_1' - s_2' < S - s_1 - s_2$, the payoff from (s_1, s_2) for firm a is strictly less than

$$\frac{S - s_1' - s_2' + a + b}{\frac{S - s_1' - s_2' + E[a|s_1,s_2] + b}{S + E[a|s_1,s_2] + b}},$$

which since $E[a|s_1', s_2'] > E[a|s_1, s_2]$ is in turn strictly less than

$$\frac{S - s_1' - s_2' + a + b}{\frac{S - s_1' - s_2' + E[a|s_1',s_2'] + b}{S + E[a|s_1',s_2'] + b}} = \frac{S - s_1' - s_2' + a + b}{1 - \frac{s_1' + s_2'}{S + E[a|s_1',s_2'] + b}}.$$

But this contradicts the equilibrium condition, since the RHS is firm a's payoff from deviating and playing (s_1', s_2'), and establishes inequality (A-23).

To complete the proof of Part (B), we consider in turn the cases in which (A-23) holds with equality, and in which it holds strictly. First, if (A-23) holds with equality, Lemma A-2 implies

$$-\frac{s_1}{P_1(s_1)} - \frac{s_2}{P_2(s_1, s_2)} = -\frac{s_1' + s_2'}{P_2(s_1', s_2')}.$$

From Part (A), $P_2(s_1, s_2) \leq P_1(s_1)$, and since $s_1 \geq 0$, this implies

$$-\frac{s_1 + s_2}{P_2(s_1, s_2)} \leq -\frac{s_1' + s_2'}{P_2(s_1', s_2')},$$

which since (A-23) holds with equality, implies $P_2(s_1, s_2) \geq P_2(s_1', s_2')$.

Second, if instead (A-23) holds strictly, taking the expectation over the equilibrium condition for all firms a playing (s_1, s_2), together with the implication of Lemma 1 that $E[a|s_1, s_2] > E[a|s_1', s_2']$, yields

$$\frac{S - s_1 - s_2 + E[a|s_1, s_2] + b}{1 - \frac{s_1}{P_1(s_1)} - \frac{s_2}{P_2(s_1, s_2)}} \geq \frac{S - s_1' - s_2' + E[a|s_1, s_2] + b}{1 - \frac{s_1'}{P_1(s_1')} - \frac{s_2'}{P_2(s_1', s_2')}} > \frac{S - s_1' - s_2' + E[a|s_1', s_2'] + b}{1 - \frac{s_1'}{P_1(s_1')} - \frac{s_2'}{P_2(s_1', s_2')}}.$$

Since the first and last terms in this inequality are simply $P_2(s_1, s_2)$ and $P_2(s_1', s_2')$ respectively, this establishes $P_2(s_1, s_2) > P_2(s_1', s_2')$.

Part (C): By the argument of Proposition 2, either almost all firms that play 0 at date 1 also play 0 at date 2; or there exists s_2' such that $S - s_2' \geq I$ and almost all firms play either 0 or s_2' at date 2. The date 1 price for do-nothing firms satisfies

$$P_1(0) = E[S + a|(0,0)] \Pr(0|0) + P_2(0, s_2') \Pr(s_2'|0).$$

From Parts (A) and (B), we know $P_2(0, s_2') \leq P_2(s_1, s_2) \leq P_1(s_1)$. From the equilibrium condition, and firm a that plays $(0,0)$ satisfies $S + a \leq P_1(s_1)$, since otherwise firm a would be strictly better off playing $(s_1, 0)$. The result then follows, completing the proof.

Proof of Proposition 6: *Part (A):* By the equilibrium condition for firm a'',

$$\frac{S - s'' + a''}{1 - \frac{s''}{P''}} \geq \frac{S - s' + a''}{1 - \frac{s'}{P'}}. \tag{A-24}$$

Since $s'' > s'$, it is immediate that $s''/P'' > s'/P'$, establishing (ii). By the equilibrium condition for firm a',

$$\frac{S - s' + a'}{1 - \frac{s'}{P'}} \geq \frac{S - s'' + a'}{1 - \frac{s''}{P''}}. \tag{A-25}$$

Multiplying (A-25) by -1 and combining with (A-24) yields

$$\frac{a'' - a'}{1 - \frac{s''}{P''}} \geq \frac{a'' - a'}{1 - \frac{s'}{P'}}.$$

If $a' > a''$ then this inequality contradicts (ii); hence $a'' \geq a'$, which (since $a'' \neq a'$) establishes (iii).

Firm a' also has the choice of doing nothing, and so the equilibrium condition implies $S + a' \geq P'$, i.e., firm a' pays weakly less than its stock is worth. Consequently,

$$\frac{S - s'' + a'}{1 - \frac{s''}{P'}} \geq \frac{S - s' + a'}{1 - \frac{s'}{P'}},$$

i.e., if firm a' were able to repurchase more stock at the constant price P', it would weakly prefer to do so. Combined with (A-25), it then follows that $P'' \geq P'$, establishing (i), and completing the proof of Part (A).

Part (B): The proof is exactly the same as the final paragraph of the proof of Part (B) of Proposition 5.

Proof of Proposition 7: By hypothesis, there are only a finite number of strategies played in equilibrium. Throughout the proof, we ignore any firm that plays a strategy that is played by only a measure zero set of firms. Partition the remaining firms so that if two firms share the same $s_1 + s_2$ and make the same investment decision, then they lie in the same partition element. Let $A^1, \ldots, A^M$ be the partition elements in which firms invest. Let A^0 be the set of non-investing firms. Without loss, order the sets $A^1, \ldots, A^M$ so that $i > j$ is equivalent to $S - s_1 - s_2$ being smaller for firms in A^i than A^j. By Lemma 1, it follows that A^i are intervals, with $A^i > A^j$ if $i > j$. By Corollary 1, $\inf A^1 = \underline{a}$. Define $s^i = s_1 + s_2$ for all firms in A^i, and by Lemma A-2, and define $N^i = 1 - \frac{s_1(a)}{P_1(s_1(a))} - \frac{s_2(a)}{P_2(s_1(a), s_2(a))}$ for all firms $a \in A^i$.

If $s_1 \leq 0$ for some firm in A^i, an easy adaption of the arguments of Propositions 1 and 2 implies that all firms that use this action at date 1 take the same date 2 action, s_2. Moreover, by the definition of A^i, all such firms invest. So for these firms, the date 1 and 2 transaction prices coincide, and by (4), both equal $E\left[S + a + b | s_1(a), s_2(a)\right]$. Hence in this case $N^i = 1 - \frac{s^i}{E[S+a+b|s_1(a), s_2(a)]}$.

If instead $s_1 > 0$ for some firm in $a \in A^i$, then by Proposition 5, the date 1 and 2 transaction prices satisfy $P_1 \geq P_2$, and so using $s_2(a) < 0$, $N^i \geq 1 - \frac{s^i}{P_2(s_1(a), s_2(a))}$. Since $P_2(s_1(a), s_2(a)) = E\left[\frac{S - s^i + a + b}{N^i} | s_1(a), s_2(a)\right]$, we know

$$P_2(s_1(a), s_2(a)) \leq E\left[\frac{S - s^i + a + b}{1 - \frac{s^i}{P_2(s_1(a), s_2(a))}} | s_1(a), s_2(a)\right].$$

and hence

$$P_2(s_1(a), s_2(a)) \leq E\left[S + a + b | s_1(a), s_2(a)\right],$$

and so

$$N^i \geq 1 - \frac{s^i}{E\left[S + a + b | s_1(a), s_2(a)\right]}.$$

Moreover, by Proposition 5, the inequality is strict whenever $\Pr(\text{invest} | s_1(a)) < 1$.

The above observations imply

$$N^i \geq 1 - \frac{s^i}{E\left[S + a + b | a \in A^i\right]}, \tag{A-26}$$

with the inequality strict whenever $\Pr(\text{invest} | s_1 \in s_1(A^i)) < 1$.

We next show that $\Pr(\text{invest} | s_1 \in s_1(A^i)) < 1$ for at least some i. Suppose to the contrary that this is not the case. Then $\Pr(\text{not invest}|s_1 \in s_1(A^0)) = 1$. So $E[P_3|s_1 \in s_1(A^0)] = E[P_3|a \in A^0] = S + E[a|a \in A^0]$. But a straightforward adaption of the proof of Proposition 3 implies that there exists an upper interval of firms who obtain a payoff strictly in excess of $S + a$, and by Corollary A-3, this upper interval has a non-null intersection with A^0. But then $E[P_3|a \in A^0] > S + E[a|a \in A^0]$, a contradiction.

Boundary firms $a^{i*} \equiv \sup(A^i)$ must be indifferent across two adjacent issue paths s^{i-1} and s^i, i.e., for all $i < M$,

$$\frac{1}{N^i}(a^{i*} + S + b - s^i) = \frac{1}{N^{i+1}}(a^{i*} + S + b - s^{i+1}). \tag{A-27}$$

The heart of the proof is to establish that inequality (A-26), with the inequality strict for at least some i, implies

$$N^M > 1 - \frac{s^M}{E[a + S + b|a \in [\underline{a}, a^{M*}]]}. \tag{A-28}$$

We establish (A-28) by showing inductively that for any $i = 1, \ldots, M$,

$$N^i \geq 1 - \frac{s^i}{E[a + S + b|a \in [\underline{a}, a^{i*}]]}. \tag{A-29}$$

The initial case $i = 1$ is immediate from (A-26) and the earlier observation that $\inf A_1 = \underline{a}$. For the inductive step, suppose (A-29) holds at $i = K - 1 < M$. We show that (A-29) also holds at $i = K$.

Observe first that inequality (A-29) at $i = K - 1$ is equivalent to

$$\frac{S + a^{(K-1)*} + b - s^{K-1}}{N^{K-1}} \leq \frac{S + a^{(K-1)*} + b - s^{K-1}}{1 - \frac{s^{K-1}}{E[a+S+b|a \in [\underline{a}, a^{(K-1)*}]]}}.$$

Since $a^{(K-1)*} \geq E[a|a \in [\underline{a}, a^{(K-1)*}]]$, the RHS of this inequality is increasing in s^{K-1}, i.e., if the share price is $E[a + S + b|a \in [\underline{a}, a^{(K-1)*}]]$, the best firm $a^{(K-1)*}$ in pool $[\underline{a}, a^{(K-1)*}]$ would be better off raising fewer funds than $S - s^{K-1}$. We know $S - s^K < S - s^{K-1}$, and so

$$\frac{S + a^{(K-1)*} + b - s^{K-1}}{N^{K-1}} < \frac{S + a^{(K-1)*} + b - s^K}{1 - \frac{s^K}{E[a+S+b|a\in[\underline{a},a^{(K-1)*}]]}}.$$

Combined with the indifference condition (A-27) at $i = K - 1$, it follows that

$$N^K \geq 1 - \frac{s^K}{E[a + S + b|a \in [\underline{a}, a^{(K-1)*}]]}.$$

Combined with (A-26), it then follows that

$$N^K \geq 1 - \frac{s^K}{E[a + S + b|a \in A^K \cup [\underline{a}, a^{(K-1)*}]]} = 1 - \frac{s^K}{E[a + S + b|a \in [\underline{a}, a^{K*}]]},$$

which establishes the inductive step. Moreover, this inequality must hold strictly for at least one step.

To complete the proof, note that in equilibrium, for all $a \in A^M$,

$$\frac{S - s^M + a + b}{N^M} \geq S + a.$$

So by (A-28)

$$\frac{S - s^M + a^{M*} + b}{1 - \frac{s^M}{E[S+a+b|a\in[\underline{a},a^{M*}]]}} > S + a^{M*}.$$

Consequently, by continuity together with (6), there exists $\tilde{a}^* > a^{M*}$ such that

$$\frac{S - s^M + \tilde{a}^* + b}{1 - \frac{s^M}{E[S+a+b|a\in[\underline{a},\tilde{a}^*]]}} = S + \tilde{a}^*.$$

It is straightforward to show that there is an equilibrium of the one-period benchmark in which firms in $[\underline{a}, \tilde{a}^*]$ issue shares at a price $E\left[S + a + b|a \in [\underline{a}, \tilde{a}^*]\right]$ to raise funds $-s^M$ and invest, while firms $a \in (\tilde{a}^*, \bar{a}]$ do nothing. This completes the proof.

Omitted results

Lemma A-3 *Let $s_1' > s_1'' \geq 0$ be date 1 repurchase sizes both used by a positive measure of firms. Then there exist s_2' and s_2'' such that $S - s_1' - s_2' \geq I$ and $S - s_1'' - s_2'' \geq I$, and such that a positive measure of firms play each of (s_1', s_2') and (s_1'', s_2''). If in addition a positive measure of firms plays $(s_1'', 0)$, then $E[a|s_1', s_2'] > E[a|s_1'', s_2'']$.*

Proof of Lemma A-3: The first statement follows by standard argument from a one-period setting. We focus here on the second statement. Suppose that, contrary to the claimed result, $E[a|s_1', s_2'] \leq E[a|s_1'', s_2'']$. Combined with the equilibrium condition for any firm a' that plays (s_1', s_2'), we have (using (5))

$$\frac{S - s_1' - s_2' + a' + b}{\left(1 - \frac{s_1'}{P_1(s_1')}\right)\left(1 - \frac{s_2'}{S - s_1' + E[a|s_1', s_2'] + b}\right)} \geq \frac{S - s_1'' - s_2'' + a' + b}{\left(1 - \frac{s_1''}{P_1(s_1'')}\right)\left(1 - \frac{s_2''}{S - s_1'' + E[a|s_1'', s_2''] + b}\right)}$$
$$\geq \frac{S - s_1'' - s_2'' + a' + b}{\left(1 - \frac{s_1''}{P_1(s_1'')}\right)\left(1 - \frac{s_2''}{S - s_1'' + E[a|s_1', s_2'] + b}\right)}.$$

Taking the expectation of this inequality over all firms playing (s_1', s_2') gives

$$\frac{S - s_1' + E[a|s_1', s_2'] + b}{1 - \frac{s_1'}{P_1(s_1')}} \geq \frac{S - s_1'' + E[a|s_1', s_2'] + b}{1 - \frac{s_1''}{P_1(s_1'')}}.$$

Let a'' be any firm that repurchases s_1'' then does nothing. By the equilibrium condition,

$$\frac{S - s_1'' + a''}{1 - \frac{s_1''}{P_1(s_1'')}} \geq \frac{S - s_1' + a''}{1 - \frac{s_1'}{P_1(s_1')}}.$$

These last two inequalities imply

$$\frac{S - s_1'' + a''}{S - s_1' + a''} \geq \frac{S - s_1'' + E[a|s_1', s_2'] + b}{S - s_1' + E[a|s_1', s_2'] + b}.$$

Since $s_1' > s_1''$, this is equivalent to

$$E\left[a|s_1', s_2'\right] + b \geq a''.$$

Since a'' repurchases s_1'' then does nothing, by the equilibrium condition $S + a'' \geq P_1\left(s_1''\right)$. Together with the supposition $E\left[a|s_1'', s_2''\right] \geq E\left[a|s_1', s_2'\right]$, these observations imply

$$P_2\left(s_1'', s_2''\right) = \frac{S - s_1'' + E\left[a|s_1'', s_2''\right] + b}{1 - \frac{s_1''}{P_1\left(s_1''\right)}} \geq \frac{S - s_1'' + a''}{1 - \frac{s_1''}{P_1\left(s_1''\right)}} \geq \frac{P_1\left(s_1''\right) - s_1''}{1 - \frac{s_1''}{P_1\left(s_1''\right)}} = P_1\left(s_1''\right).$$

But this contradicts Part (A) of Proposition 5, completing the proof.

Proposition 8 *Let (s_1', s_2') and (s_1'', s_2'') be strategies both played by a positive measure of firms, with $s_1' > 0$, $s_1'' > 0$ and $S - s_1'' - s_2'' > S - s_1' - s_2' \geq I$. If in addition a positive measure of firms play $(s_1', 0)$, then $s_1' > s_1''$.*

Proof of Proposition 8: Suppose to the contrary that $s_1'' \geq s_1'$. First, consider the case $s_1'' > s_1'$. By Lemma 1, $E\left[a|s_1', s_2'\right] > E\left[a|s_1'', s_2''\right]$, which contradicts Lemma A-3. Second, consider the case $s_1'' = s_1'$. This implies $s_2'' = s_2'$ by the arguments of Proposition 2, contradicting $S - s_1'' - s_2'' > S - s_1' - s_2'$. This completes the proof.