The Demographics of Innovation and Asset Returns

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Abstract

We study an overlapping-generations economy in which new agents innovate and introduce new products and firms. Innovation is stochastic. The new firms increase overall productivity, but also steal business from pre-existing firms and act as depreciation shocks for the human capital of existing agents. Since new firms belong to newly arriving agents, innovations are a double-edged sword for pre-existing generations: Increased innovation activity increases the total output, but it also reduces the share of the pre-existing agents in the increased output. The latter effect — "the displacement risk" — makes agents reluctant to hold stock in firms whose output is exposed to increased innovation and competition by new firms, while giving a hedging value to firms that can profit from innovation. Therefore the model produces a value effect. At the aggregate level the displacement risk makes agents reluctant to hold stock of existing firms, since their profits are collectively at risk from new entrants. This leads to a higher equity premium. We calibrate the model so that it can match estimated cohort effects for individuals and firms, and evaluate its quantitative implications.

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1 Introduction

In this paper we explore the general-equilibrium asset-pricing implications of innovation. We concentrate on two important facts. First, future innovation is in large part the property of future agents, which makes future firms untradeable at the current date. Second, while new inventions ultimately expand the productive capacity of the economy, they tend to be rivalrous to existing products, squeezing existing firms and eroding the human capital of older workers. We show that these two features yield naturally value and equity premia not explained by the evolution of aggregate consumption. At the same time, interest rates are low and non-volatile.

In order to capture the two features of interest, we modify the standard model to allow for overlapping generations and the "displacement" of the old by the new. Specifically, we present a model where production of a single final good requires labor and several intermediate goods. Newly arriving agents introduce a stochastic amount of new intermediate goods that increase the economy's overall productivity and wages. At the same time, however, the increased competition displaces pre-existing intermediate goods, that is, reduces the profits of existing establishments. In addition, the current workers are not as well adapted to the new technology. This fact diminishes their human capital, making innovation a double edged sword for current agents: On one hand, increased productivity raises wages, output, and consumption at the aggregate level. On the other hand, the share of the wage bill earned by current agents, and ultimately their consumption share, may decline, since the benefits of innovation accrue preponderantly to the newly arriving agents.

It follows that the consumption growth of current agents may behave differently than the aggregate consumption growth. This observation has far reaching implications for asset pricing, because the usual Euler equations need to be satisfied only for the agents who are alive at a given time and not for the unborn agents.

A first consequence of this observation is that if the consumption growth rate of existing agents is lower than aggregate consumption growth, the equilibrium interest rate is lower in our model than in the standard (infinitely-lived) representative-agent model. This implication, which was noted in the seminal paper of Blanchard (1985) and emphasized recently by Gârleanu and Panageas (2007), helps resolve the low risk free rate puzzle.

The equity premium is also larger in our model compared to the standard representative-agent model. This result is due to the opposite impacts of innovation on existing firms' dividends and on the pricing kernel. The first part is clear: innovation increases competition in the product and labor markets, reducing the profits of existing firms. The second part is more nuanced. Innovation increases output and wages, but it also depreciates current agents' human capital, due to assumed vintage effects. As long as the latter effect — the displacement effect — is strong enough, or agents' preferences exhibit sufficiently strong "keeping up with the Joneses" features, the marginal utility of existing agents' consumption increases in response to positive innovation shocks. In that case, the absolute value of the covariance of the returns with the pricing kernel is higher than in the standard model, and therefore so is the risk premium.

Moreover, as long as the stochastic discount factor is positively exposed to the innovation shocks, the model also produces naturally a value premium. To that end, we allow some firms to be endowed with blueprints for the creation of new products in the future. These "growth" firms are valuable hedging instruments and therefore yield lower expected returns. In addition, as their payouts are farther in the future, they have higher P/E ratios at the same time as lower expected returns.

We assess the empirical relevance of our theory in two distinct ways. First, we derive novel implications concerning consumption cohort effects and the value-growth return and test them in the data. Specifically, unlike existing models, ours yields consumption cohort effects that are non-stationary and positively correlated with the excess returns on growth stocks relative to value stocks. The econometric tests confirm these predictions. Second, we calibrate the model and find that it can account quantitatively for joint asset-pricing and consumption-dynamics moments, new-firm creation, and consumption-cohort-effects properties.

The paper relates to several strands of literature. It borrows from the economic-growth literature — in particular, from the seminal paper of Romer (1990) — which concentrates on the processes of innovation, new firm and wealth creation, and rivalry between old and new firms. Given our asset-pricing focus, however, we abstract from such important features as the endogeneity of growth and build a model that emphasizes the potentially important impact of these processes on asset prices. This outcome requires departing form the standard model (e.g., Lucas (1978)), which assumes a single, infinitely-lived agent, who owns both existing and future firms and is therefore unaffected by displacement effects.

A number of papers use an overlapping-generations framework to study asset pricing phenomena. [See, for instance, Abel (2003), Constantinides et al. (2002), DeMarzo et al. (2004, 2008), Gârleanu and Panageas (2007), Gomes and Michaelides (2007), or Storesletten et al. (2007).] None of these papers, however, studies the interaction of the displacement effect with the (lack of) intergenerational risk sharing that is critical for our results.

The present paper also relates to the literature that studies the cross-section of equities in a general-equilibrium framework. We contribute to this active literature, which includes Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004, 2006), Papanikolaou (2007), and Zhang (2005) among many, by providing a new approach to the value-premium puzzle. In particular, we attribute return differences between value and growth firms to differences in their exposure to the displacement risk. Unlike most previous general- or partial-equilibrium approaches, ours does not propose a single-factor model of time-varying conditional betas.[1] Instead, we propose a novel source of risk that is reflected directly in the stochastic discount factor. This makes it quantitatively easier to obtain a large value premium, without having to rely on large variation in conditional betas.

In addition, we contribute simultaneously to the vast literature on the equity-premium puzzle, (e.g., Mehra and Prescott (1985), Campbell and Cochrane (1999)), since the mechanism we propose as an explanation of the value premium — namely, the displacement effect — also generates a high equity premium. We argue that the displacement risk can reconcile the historical moments of stock and bond returns with the fundamentals.

[1] Papanikolaou (2007) is also an exception.

2 Model

2.1 Agents' Preferences and Demographics

We consider a model with discrete and infinite time: $t \in \{\ldots, 0, 1, 2, \ldots\}$. The size of the population is normalized to 1. At each date a mass λ of agents die, and a mass λ of agents are born, so that the population remains constant. An agent who is born at time s has preferences of the form

$$E_s \sum_{t=s}^{s+\tau} \beta^{(t-s)} \frac{\left(c_{t,s}^{\psi} \left(\frac{c_{t,s}}{C_t} \right)^{1-\psi} \right)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where τ is the (geometrically distributed) time of death, $c_{t,s}$ is his consumption at time t, C_t is the aggregate consumption at time t, $0 < \beta < 1$ is a subjective discount factor, $\gamma > 0$ is the agent's relative risk aversion, and ψ is a constant between 0 and 1. Preferences of the form (1) were originally proposed by Abel (1990), and are commonly referred to as "keeping-up-with-the-Joneses" preferences. These preferences capture the idea that agents derive utility from both their own consumption and from their consumption relative to per capita consumption. When $\psi = 1$, these preferences specialize to the standard constant-relative-risk-aversion preferences. In general, for $\psi \in [0, 1]$ agents place a weight ψ on their own consumption (irrespective of what others are consuming) and a weight $1 - \psi$ on their consumption relative to average consumption in the population. Our qualitative results hold independently of the keeping-up-with-the-Joneses feature, which only helps at the calibration stage, by reducing the value of the interest rate.

A standard argument allows us to integrate over the distribution of the stochastic times of death and re-write preferences of the form (1) as

$$E_s \sum_{t=s}^{\infty} \left[(1-\lambda) \beta \right]^{(t-s)} \frac{\left(c_{t,s}^{\psi} \left(\frac{c_{t,s}}{C_t} \right)^{1-\psi} \right)^{1-\gamma}}{1-\gamma}. \tag{2}$$

2.2 Technology

2.2.1 Final-Good Firms

There is a representative (competitive) final-good producing firm that produces the single final good using two categories of inputs: a) labor and b) a continuum of intermediate goods. Specifically, the production function of a final good producing firm is

$$Y_t = Z_t \left(L_t^F\right)^{1-\alpha} \left[\int_0^{A_t} \omega_{j,t} \left(x_{j,t}\right)^{\alpha} dj\right]. \tag{3}$$

In equation (3) Z_t denotes a stochastic productivity process, L_t^F captures the efficiency units of labor that enter into the production of the final good, A_t is the number of intermediate goods available at time t, and $x_{j,t}$ captures the quantity of intermediate good j that is used in the production of the final good. The constant $\alpha \in [0, 1]$ controls the relative weight of labor and intermediate goods in the production of the final good, while $\omega_{j,t}$ captures the relative importance placed on the various intermediate goods. We specify $\omega_{j,t}$ as

$$\omega_{j,t} = \left(\frac{j}{A_t}\right)^{\chi(1-\alpha)}, \quad \chi \geq 0 \tag{4}$$

For $\chi = 0$, the production function (3) is identical to the one introduced by the seminal Romer (1990) paper in the context of endogenous growth theory. Our version is slightly more general, since the factor weights $\omega_{j,t}$, which are increasing functions of the intermediate good index j, allow the production function to exhibit a "preference" for more recent intermediate goods. Even though our aim here is not to explain the sources of growth in the economy, the production function (3) is useful for our purposes because for several reasons: a) Innovation, i.e., an increase in the variety of intermediate goods (A_t), helps increase aggregate output; b) There is rivalry between existing and newly arriving intermediate goods, since increases in A_t strengthen the competition among intermediate goods producers, and c) Increases in the variety of intermediate, rather than final, goods is technically convenient, since we can keep one unit of the final good as numeraire throughout. An exact illustration of the first two properties is provided in the next section, where we solve the model.

The productivity process Z_t follows a random walk (in logs) with drift μ and volatility σ_ε:

$$\log(Z_{t+1}) = \log(Z_t) + \mu + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2). \tag{5}$$

At each point in time t, the representative final-good firm chooses L_t^F and $x_{j,t}$ (where $j \in [0, A_t]$) so as to maximize its profits

$$\pi_t^F = \max_{L_t^F, x_{j,t}} \left\{ Y_t - \int_0^{A_t} p_{j,t} x_{j,t} dj - w_t L_t^F \right\}, \tag{6}$$

where $p_{j,t}$ is the price of intermediate good j and w_t is the prevailing wage (per efficiency unit of labor).

2.2.2 Intermediate-Goods Firms

The intermediate goods $x_{j,t}$ are produced by monopolistically competitive firms that own non-perishable blueprints to the production of intermediate good $x_{j,t}$. In analogy to Romer (1990), we assume that the production of intermediate good $j \in [0, A_t]$ requires one unit of labor (measured in efficiency units) per unit of intermediate good produced, so that the total number of efficiency units of labor used in the intermediate-goods sector is

$$L_t^I = \int_0^{A_t} x_{j,t} dj. \tag{7}$$

The price $p_{j,t}$ of intermediate good j is determined so as to maximize the profits of the intermediate goods producer taking the demand function of the final good firm $x_j \left(p_{j,t}; p_{j' \neq j}, w_t\right) \equiv \arg\max_{x_{j,t}} \pi_t^F$ as given. To simplify notation, we shall write $x_{j,t}\left(p_{j,t}\right)$ instead of $x_{j,t}\left(p_{j,t}; p_{j' \neq j}, w_t\right)$. The intermediate goods producer sets its price so as to maximize its profits $\pi_t^I(j)$, given by

$$\pi_t^I(j) = \max_{p_{j,t}} \left\{ \left(p_{j,t} - w_t\right) x_{j,t}(p_{j,t}) \right\}. \tag{8}$$

2.3 Arrival of New Intermediate Goods and New Agents

2.3.1 New Products

The number of intermediate goods A_t is not constant. Instead, it expands over time as a result of inventions. Given the asset pricing focus of the paper, we assume that all inventions are purely the result of serendipity, and the number of intermediate goods follows a random walk (in logs):[2]

$$\log(A_{t+1}) = \log(A_t) + u_{t+1}. \tag{9}$$

The increment u_{t+1} is i.i.d. across time for simplicity. To ensure its positivity, we assume that u_{t+1} is Gamma distributed with parameters (k, ν).

The intellectual property rights for the production of the $\Delta A_{t+1} = A_{t+1} - A_t$ intermediate goods belong either to newly arriving agents or to existing firms. Specifically, we assume that a fraction $\kappa \in [0, 1]$ of the value of producing each new intermediate good $j \in [A_t, A_{t+1}]$ belongs to newly arriving agents, while the complementary fraction $1-\kappa$ results as a byproduct of the production process of established firms and hence belongs to existing firms (thus, indirectly, to existing agents, who own these firms).[3]

2.3.2 Value and Growth Firms

Agents who arrive endowed with ideas start a continuum of firms that produce the respective intermediate goods, and introduce them into the stock market.

These new firms come in two varieties, depending on whether they are capable of obtaining blueprints for new intermediate goods in the future or not. The first kind are "value firms,"which cannot obtain any blueprints in future periods. They are only entitled to a fraction $\eta\kappa\Delta A_{t+1}$ of the value of the blueprints introduced at time $t + 1$, where $\eta \in (0, 1]$. The other kind of firms are "growth" firms. They are entitled to a fraction $1 - \eta$ of the value of the $\kappa\Delta A_{t+1}$ new intermediate goods, but they also receive a fraction of the value of new

[2]We choose a random walk specification in order to ensure that aggregate consumption is a random walk. The assumption of a random walk implies that -for a given $u_{t+1}-$ the increase in new products is proportional to the level of pre-existing products. This assumption is routinely used in the literature and is sometimes referred ot as "Standing on the shoulders of giants". See e.g. Jones (1997).

[3]To formalize this division of ownership, we think of intermediate goods as being indexed by (j, k) with $j \in [0, A_t]$ and $k \in [0, 1]$. The final-good production is still given by Equation (3) — in particular, $\omega_{j,k}$ does not depend on k. Of the blueprints for goods (j, k) introduced at time $t + 1$, i.e., $j \in [A_t, A_{t+1}]$, the ones indexed by $k \leq \kappa$ belong to the newly arriving agents; the other ones belong to existing firms.

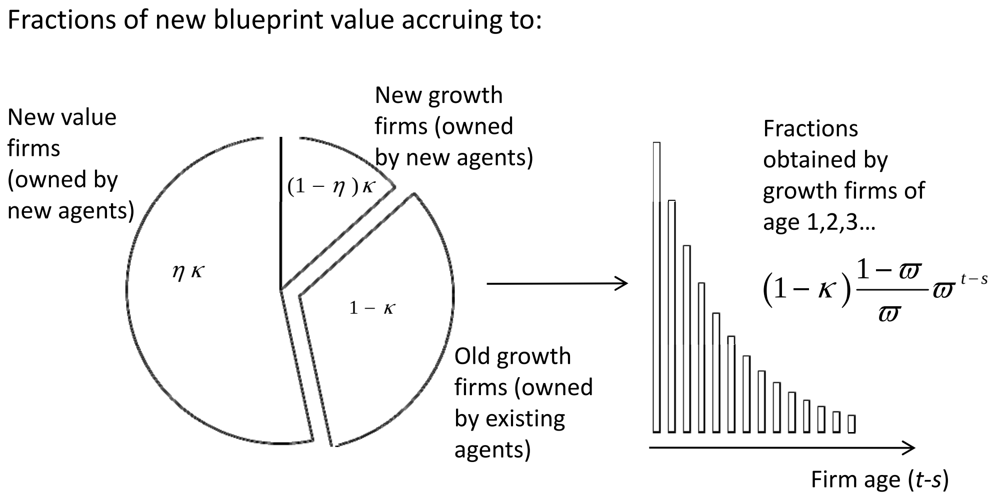

Figure 1: Illustration of the allocation of new blueprint value

blueprints in future periods. Specifically, in period n, growth firms born at $s \in (-\infty, n-1]$ obtain a fraction

$$(1-\kappa)\left(\frac{1-\varpi}{\varpi}\right)\varpi^{n-s} \tag{10}$$

of the value of the ΔA_n new blueprints. To simplify matters, we assume that there are no intra-cohort differences between growth firms and any two growth firms of the same cohort obtain the same value of blueprints in any given period. The geometric decay in the fraction of new blueprints that accrues to a given growth firm as a function of its age ensures that asymptotically the market capitalization of any growth firm goes to zero as a fraction of aggregate market capitalization.

2.3.3 Workers

New business owners make up a fraction $\phi \in (0,1)$ of newly arriving agents. The rest of the newly arriving agents are workers. Workers arrive in life with an endowment of hours $\overline{h}$. As in Blanchard (1985), the endowment of hours changes geometrically with age at the rate δ, so that a worker who was born at time $s < t$ has an endowment of hours equal to $h_{t,s} =$

$\overline{h}\,(1-\delta)^{t-s}$ at time t. For simplicity, agents have no utility of leisure and supply their hours inelastically.

In the real world younger workers are likely to be more productive in the presence of increased technological complexity than older workers. One potential reason is that their education gives them the appropriate skills for understanding the technological frontier. By contrast, older workers are likely to be challenged by technological advancements. In Appendix B we present a simple vintage model of the labor market that introduces imperfect substitution across labor supplied by agents born at different times. To expedite the presentation of the main results, in this section we assume that labor is a homogenous good and that workers' endowment of efficiency units depreciates in a way that replicates the outcome of the more elaborate model in Section 6.

Specifically, we assume that a worker's total supply of efficiency units of labor is given by $h_{t,s}q_{t,s}$ with

$$\log(q_{t+1,s}) = \log(q_{t,s}) - \rho u_{t+1}, \tag{11}$$

and $\rho \geq 0$. This specification captures the idea that advancements of the technological frontier act as depreciation shocks to the productivity of old agents. In addition to being intuitively appealing, this feature generates cohort effects in consumption that are consistent with, and can be estimated directly from, the data. We undertake this exercise in Section 5.1.

To ensure a constant number of efficiency units in the economy, we normalize the initial endowment of efficiency units to

$$q_{s,s} = 1 - (1-\lambda)\,(1-\delta)\,e^{-\rho u_{t+1}}, \tag{12}$$

and assume that $(1-\lambda)\,(1-\delta) \leq 1$. We also normalize the initial endowment of hours to $\overline{h} = \frac{1}{\lambda}$. This implies that the number of per-worker efficiency units, $L_t/\,(1-\phi)$, is always equal to 1, and hence $h_{t,s}q_{t,s}$ can be interpreted as the fraction of total wages that accrues to workers born at time s.[4]

[4]Note that our assumptions imply

$$\frac{L_t}{(1-\phi)} = \lambda \sum_{s=-\infty\ldots t} \left[(1-\lambda)\,(1-\delta)\right]^{t-s} q_{t,s},$$

2.4 Asset Markets

Once born, agents face dynamically complete markets. This means that at each point in time agents are able to trade in zero net supply Arrow-Debreu securities on the realization of next period's shocks ε_{t+1} and u_{t+1}. This assumption implies the existence of a stochastic discount factor ξ_t, so that the time-s value of a claim paying a stream of dividends D_t with $\lim_{t\to\infty} E_s[\xi_t D_t] = 0$ is given by $E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) D_t$.

Finally, agents have access to annuity markets as in Blanchard (1985). (We refer the reader to that paper for details). The joint assumptions of dynamically complete markets *for existing agents* and frictionless annuity markets simplifies the analysis considerably, since in a dynamically complete market with annuities an agent's feasible consumption choices are constrained by a *single* intertemporal budget constraint. For a worker, that intertemporal budget constraint is given by

$$E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) c^w_{t,s} = E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) w_t q_{t,s} h_{t,s}, \tag{13}$$

where $c^w_{t,s}$ denotes the time-t consumption of a representative worker who was born at time s. The left hand side of (13) represents the present value of a worker's consumption, while the right hand side represents the present value of her income. Similarly, letting $c^e_{t,s}$ denote the time-t consumption of a representative inventor who was born at time s, her intertemporal budget constraint is

$$E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) c^e_{t,s} = \frac{1}{\lambda\phi} V_{s,s}, \tag{14}$$

where $V_{s,s}$ is the time-s total market capitalization of new firms created at time s. The left-hand side of equation (14) is the present value of a representative inventor's consumption,

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so that iterating forward once to obtain $\frac{L_{t+1}}{(1-\phi)}$ and using (11) and (12) give

$$\begin{aligned} \frac{L_{t+1}}{(1-\phi)} &= \lambda \sum_{s=-\infty\ldots t+1} (1-\lambda)(1-\delta)^{t+1-s} q_{t+1,s} \\ &= (1-\lambda)(1-\delta)\left(\frac{L_t}{1-\phi}\right) e^{-\rho u_{t+1}} + \lambda \overline{h} q_{t+1,t+1}. \end{aligned}$$

Setting $q_{t+1,t+1}$ as in (12) implies $\frac{L_{t+1}}{(1-\phi)} = \frac{L_t}{(1-\phi)} = 1$.

while the right-hand side is the value of all new firms divided by the mass of new business owners $(\lambda\phi)$. To determine the total market value of firms (newly) created at time s, let $\Pi_{j,s}$ be the fair value of the ability of firm j to produce a new intermediate good as given by the net present value of the associated profits:

$$\Pi_{j,s} = \left[E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) \pi_{j,t}^{I} \right]. \tag{15}$$

The total market capitalization of all new firms can consequently be written as

$$V_{s,s} = \kappa \int_{A_{s-1}}^{A_s} \Pi_{j,s} dj + \left(\frac{1-\varpi}{\varpi} \right) E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s} \right) (1-\kappa) \varpi^{t-s} \int_{A_{t-1}}^{A_t} \Pi_{j,t} dj. \tag{16}$$

The first term in equation (16) is the value of the blueprints for the production of new intermediate goods that are introduced by new firms (both "growth" and " value" firms) at time s. Similarly the latter term captures the value of "growth opportunities," i.e., the value of blueprints to be received by growth firms in future periods.

2.5 Equilibrium

The definition of equilibrium is standard. To simplify notation, we let ϕ^e and ϕ^w denote the fraction of entrepreneurs and workers (respectively) in the population so that

$$\phi^i = \begin{cases} \phi \text{ if } i = e \\ 1-\phi \text{ if } i = w \end{cases}. \tag{17}$$

An equilibrium is defined as follows

Definition 1 *An equilibrium is defined as a tuple of adapted stochastic processes* $\{x_{j,t},\ L_t^F,\ c_{t,s}^w,\ c_{t,s}^e,\ \xi_t,\ p_{j,t},\ w_t\}$ *where* $j \in [0, A_t]$ *and* $t \geq s$ *such that*

1. *(Consumer optimality): Given* ξ_t*, the process* $c_{t,s}^w$ *(respectively,* $c_{t,s}^e$*) solves the optimization problem (2) subject to the constraint (13) (respectively, constraint (14)).*

2. *(Profit maximization) The prices* $p_{j,t}$ *solve the optimization problem (8) and* $L_t^F, x_{j,t}$ *solve the optimization problem (6) given* $p_{j,t}, w_t$*.*

3. (Market clearing). Labor and goods markets clear

$$L_t^F + L_t^I = (1-\phi) \tag{18}$$

$$\lambda \sum_{s=-\infty}^{t} \sum_{i\in\{w,e\}} (1-\lambda)^{t-s} \phi^i c_{t,s}^i = Y_t. \tag{19}$$

Conditions 1 and 2 are the usual optimality conditions. Condition 3 requires that total labor demand $L_t^F + L_t^I$ equals total labor supply $1-\phi$. Finally, the last condition requires that aggregate consumption be equal to aggregate output.

3 Solution

3.1 Equilibrium Output, Profit, and Wages

We start with the intermediate goods and the labor markets. As a first step, we derive the demand curve of the final goods firm for the intermediate input j at time t. Maximizing (6) with respect to $x_{j,t}$, we obtain

$$x_{j,t} = L_t^F \left[\frac{p_{j,t}}{\omega_{j,t} Z_t \alpha}\right]^{\frac{1}{\alpha-1}}. \tag{20}$$

Substituting this expression into (8) and maximizing over $p_{j,t}$ leads to

$$p_{j,t} = \frac{w_t}{\alpha}, \tag{21}$$

while combining (20) and (21) yields

$$x_{j,t} = L_t^F \left[\frac{w_t}{\omega_{j,t} Z_t \alpha^2}\right]^{\frac{1}{\alpha-1}}. \tag{22}$$

Turning to labor markets, maximizing (6) with respect to L_t^F gives the first order condition

$$w_t L_t^F = (1-\alpha) Y_t. \tag{23}$$

Substituting (22) into (3) and then into (23) and simplifying yield

$$w_t = \left(\alpha^2\right)^\alpha \left(\frac{1-\alpha}{1+\chi}\right)^{1-\alpha} Z_t A_t^{1-\alpha}. \tag{24}$$

Substituting equation (24) into (22) and then into (18) allows us to obtain the following expressions for $x_{j,t}$ and L_t^F

$$x_{j,t} = \frac{1+\chi}{A_t}\left(\frac{j}{A_t}\right)^\chi \frac{(1-\phi)\,\alpha^2}{\alpha^2+1-\alpha} \tag{25}$$

$$L_t^F = \frac{1-\alpha}{\alpha^2+1-\alpha}\left(1-\phi\right). \tag{26}$$

Combining (25) and (26) inside (3) leads to the following simple expression for aggregate output:

$$Y_t = \frac{\left(\alpha^2\right)^\alpha \left(\frac{1-\alpha}{1+\chi}\right)^{1-\alpha}}{\alpha^2+1-\alpha}\left(1-\phi\right) Z_t A_t^{1-\alpha}. \tag{27}$$

An interesting feature of equation (27) is that the number of intermediate inputs (A_t) is raised to the power $1-\alpha$. This means that aggregate output is increasing as the number of intermediate inputs increases. However, the extent of the increase depends on the degree of subsitutability between different varieties of intermediate goods. For instance, as α approaches 1 existing and new intermediate goods become perfect substitutes, and so the only effect of increases in variety is that newly arriving technologies "steal business" from existing ones, without changing the overall productive capacity of the economy.

Before proceeding, it is useful to compute the income shares of labor and the profits of the firms in the economy. Combining (24) and (27) implies that total payments to labor $w_t\left(1-\phi\right)$ are simply equal to a fraction $\left(\alpha^2+1-\alpha\right)$ of output Y_t.Because of constant returns to scale in the production of final goods the profits of the final goods firm are given by $\pi_t^F = 0$. The profits to an intermediate goods producer are obtained by combining (25) and (21) with (8) and (24) to obtain

$$\pi_{j,t}^I = \left(1+\chi\right)\left(\frac{j}{A_t}\right)^\chi \frac{Y_t}{A_t}\alpha\left(1-\alpha\right). \tag{28}$$

An interesting observation about (28) is that $\pi_{j,t}^I$ is not cointegrated with Y_t, since over time A_t increases and hence asymptotically $\pi_{j,t}^I/A_t \to 0$ almost surely. The lack of cointegration between the dividends of an *individual firm* and aggregate output is to be expected because of the constant arrival of competitors. Nevertheless, aggregate profits are a constant fraction of total output, since $\int_0^{A_t} \pi_{j,t}^I dj = \alpha (1-\alpha) Y_t$. This follows in a straightforward way, since in a general-equilibrium framework the total income shares must add up to aggregate output: $w_t (1-\phi) + \pi_t^F + \int_0^{A_t} \pi_{j,t}^I dj = Y_t$.

3.2 The Stochastic Discount Factor

To determine the stochastic discount factor ξ_t, we recall that, since agents face dynamically complete markets *after their birth,* a consumer's lifetime consumption profile can be obtained by maximizing (2) subject to a single intertemporal budget constraint (constraint [13] if the agent is a worker and constraint [14] if the agent is an inventor). Attaching a Lagrange multiplier to the intertemporal budget constraint, maximizing with respect to $c_{t,s}^i$, and relating the consumption at time t to the consumption at time s for a consumer whose birth date is s gives

$$c_{t,s}^i = c_{s,s}^i \left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}} \quad \text{for } i \in \{e, w\}. \tag{29}$$

From this equation, the aggregate consumption at any point in time is

$$C_t = \lambda \sum_{s=-\infty}^{t} \sum_{i \in \{w,e\}} (1-\lambda)^{t-s} \phi^i c_{s,s}^i \left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s} \right)^{-\frac{1}{\gamma}}, \tag{30}$$

where ϕ^i was defined in (17). Iterating forward once to obtain C_{t+1} and then using (30) gives

$$C_{t+1} = (1-\lambda) C_t \left(\beta^{-1} \frac{C_{t+1}^{(1-\psi)(1-\gamma)}}{C_t^{(1-\psi)(1-\gamma)}} \frac{\xi_{t+1}}{\xi_t} \right)^{-\frac{1}{\gamma}} + \lambda \sum_{i \in \{w,e\}} \phi^i c_{t+1,t+1}^i. \tag{31}$$

Dividing both sides of (31) by C_t, solving for $\frac{\xi_{t+1}}{\xi_t}$ and noting that $C_t = Y_t$ in equilibrium leads to

$$\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-1+\psi(1-\gamma)} \left[\frac{1}{1-\lambda} \left(1 - \lambda \sum_{i \in \{w,e\}} \phi^i \frac{c_{t+1,t+1}^i}{Y_{t+1}} \right) \right]^{-\gamma}. \tag{32}$$

To obtain an intuitive understanding of equation (32) it is easiest to focus on the case $\psi = 1$, so that agents have standard CRRA preferences. In this case, the stochastic discount factor consists of two parts. The first part is $\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}$, which is the standard expression for the stochastic discount factor in an (infinitely-lived) representative-agent economy. The second part — the term contained inside square brackets in equation (32) — gives the proportion of output at time $t{+}1$ that accrues to agents already alive at time t. To see this, note that only a proportion $1-\lambda$ of existing agents survive between t and $t+1$, and that the newly arriving generation claims a proportion $1-\lambda\sum_{i\in\{w,e\}}\phi^i\frac{c^i_{t+1,t+1}}{Y_{t+1}}$ of aggregate output. The combination of the two parts yields the consumption growth between t and $t+1$ of the surviving agents.

Equation (32) states an intuitive point: Since only agents alive at time t are relevant for asset pricing, it is exclusively their consumption growth that determines the stochastic discount factor, not the aggregate consumption growth. We elaborate on this point further in the next section.

To conclude the computation of equilibrium, we need to obtain an expression for the term inside square brackets on the right-hand side of (32). This can be done by using the intertemporal budget constraints (14) and (13). Proposition 1 in the Appendix shows that

$$1-\lambda\sum_{i\in\{w,e\}}\phi^i\frac{c^i_{t+1,t+1}}{Y_{t+1}} = \widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b),$$

with

$$\begin{aligned}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b) \;\equiv\; & 1-\theta^e\alpha\left(1-\alpha\right)\left(\kappa\left(1-e^{-(1+\chi)u_{t+1}}\right)+\left(\frac{1-\varpi}{\varpi}\right)\theta^b\right) \\ & -\theta^w\left(\alpha^2+1-\alpha\right)\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho u_{t+1}}\right),\end{aligned} \tag{33}$$

and $\theta^e,\theta^b,\theta^w$ are three appropriate constants obtained from the solution of a system of three nonlinear equations in three unknowns. Given the interpretation of $\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)$ as the fraction of consumption that accrues to new agents, we shall refer to it as the “displacement factor”.

4 Qualitative Properties

As we already discussed, equation (32) captures the intuition that only the consumption of existing agents matters for the stochastic discount factor. A first implication of this fact is that the commonly used consumption CAPM (using aggregate consumption data) may fail in explaining asset returns. The easiest way to illustrate this point is to consider a limiting case of our model in which the consumption CAPM (using aggregate data) would yield that all risk premia are zero.

Specifically, suppose that $\sigma = 0$, $\rho > 0$ and α approaches 1. Equation (27) implies that the volatility of output approaches zero. In a standard model, therefore, equity premia would approach zero, as well. However, this is not the case in our model, as the following Lemma illustrates.

Lemma 1 *Assume that $\sigma = 0$, $\rho > 0$, $\kappa = 1$, and*

$$1 > \beta (1-\lambda)^{\gamma} e^{\mu\psi(1-\gamma)} \tag{34}$$

$$1 \leq \beta (1-\delta)^{-\gamma} E[e^{\rho\gamma u_{t+1}}]. \tag{35}$$

Then, letting R_t be the return of any stock, an equilibrium exists and

$$\lim_{\alpha\to 1} Var\left(\Delta Y_{t+1}\right) = 0 \tag{36}$$

$$\lim_{\alpha\to 1} \frac{\partial\left(\xi_{t+1}/\xi_t\right)}{\partial u_{t+1}} > 0 \tag{37}$$

$$\lim_{\alpha\to 1} \left\{E(R_t) - \left(1 + r^f\right)\right\} > 0. \tag{38}$$

Conditions (34)–(35) are technical conditions, both necessary and sufficient for the existence of an equilibrium. They are satisfied for a wide range of parameter combinations and distribution functions.

The intuition behind Lemma 1 is straightforward. Even though the volatility of aggregate consumption becomes negligible as α approaches 1, the volatility of existing agents' consumption doesn't. As α approaches 1, the competition between firms is fierce, as all intermediate inputs start becoming perfect substitutes. This implies that new innovations result almost exclusively in redistribution from old to young firms and from old to young agents (since $\rho > 0$). Since it is only the consumption growth of existing agents that matters for asset prices, innovation shocks (u_t) present "aggregate" shocks for existing agents, even though they are not "aggregate" shocks from the perspective of the entire economy. Since the profits of existing firms are exposed to these innovation shocks (u_t), existing companies' stock exposes agents to these shocks and commands a risk premium.

Even though the limiting case $\alpha = 1$ is clearly an extreme special case of the model,[5] the results in Lemma 1 help explain why asset pricing based on the standard (aggregate) consumption CAPM relationship can understate the risks associated with investing in stocks from the perspective of existing agents.

The distinction between consumption of current agents and aggregate consumption has also important implications for the value premium. To start, assume henceforth that $\kappa < 1$, so that "growth firms" receive a fraction of the value of new blueprints over time. Letting Π_s be given as in (15), define the price to earnings ratio q for a typical value firm as $q \equiv \frac{\Pi_s}{\pi_s}$. Since the increments to the (log) stochastic discount factor and the increments to (log) profits of value firms are i.i.d., q is a constant.[6] Then, for any set of parameter values such that there exists a solution to (52)–(54) we have the following Lemma.

Lemma 2 *The (end of) period-t value of the representative "growth firm" created at time s is given by*

$$P_{t,s} = \alpha (1-\alpha) Y_t \left[(q-1) N_{t,s} + (1-\varpi) \varpi^{t-s} \frac{\theta^b}{\varpi} q \right], \tag{39}$$

[5] A caveat behind Lemma 1 is that in the limit $\alpha = 1$ the profits of intermediate goods firms disappear. Hence, even though the rate of return on a stock is well defined in the limit (because rates of returns are not affected by the level of dividends and prices), the limiting case $\alpha = 1$ is of limited practical relevance. However, it has theoretical interest, because it illustrates in a simple way the asset pricing implications of the wedge between aggregate consumption and existing agents' consumption.

[6] We show this formally as part of the proof of Proposition 1.

where

$$N_{t,s} = (1-\eta)\kappa\left(\frac{A_s}{A_t}\right)^{1+\chi}\left(1-e^{-(1+\chi)u_s}\right) + \sum_{n=s+1}^{t}(1-\varpi)(1-\kappa)\,\varpi^{n-(s+1)}\left(\frac{A_n}{A_t}\right)^{1+\chi}\left(1-e^{-(1+\chi)u_n}\right).$$

The first term inside the square brackets in equation (39) is the value of all the blueprints that the growth firm has received since its creation, while the second term is the value of remaining growth opportunities. Based on (39), the (gross) rate of return of the representative "growth firm" R^g_{t+1} at time $t+1$ is given by the dividends from pre-existing blueprints $\alpha(1-\alpha)Y_{t+1}N_t$, the dividends from the new blueprints $\alpha(1-\alpha)Y_{t+1}(N_{t+1}-N_t)$, and the end of period price $P_{t+1,s}$, all divided by the beginning of the period price $P_{t,s}$:

$$R^g_{t+1,s} \equiv \frac{\alpha(1-\alpha)Y_{t+1}N_{t+1}+P_{t+1,s}}{P_{t,s}}.$$

It will be useful to decompose this expression into two components. Specifically, define the gross rate of return of a "pure" value firm R^I_{t+1} and the gross rate of return of a "pure" growth firm R^o_{t+1} as[7]

$$R^I_{t+1} = \left(\frac{q}{q-1}\right)\left(\frac{\pi_{t+1}}{\pi_t}\right), \tag{40}$$

$$R^o_{t+1} = \varpi\left(\frac{Y_{t+1}}{Y_t}\right)\frac{(1-\kappa)\left(1-e^{-(1+\chi)u_{t+1}}\right)+\theta^b}{\theta^b}. \tag{41}$$

Then, the rate of return on any growth firm can be expressed as

$$R^g_{t+1,s} = \left(1-w^o_{t,s}\right)R^I_{t+1} + w^o_{t,s}R^o_{t+1}, \tag{42}$$

where $w^o_{t,s}$ is the relative weight of growth options in the value of the firm, and is obtained from Lemma 2 as

$$w^o_{t,s} = \frac{(1-\varpi)\,\varpi^{t-s}\frac{\theta^b}{\varpi}q}{(q-1)N_t+(1-\varpi)\,\varpi^{t-s}\frac{\theta^b}{\varpi}q}.$$

[7] A pure value firm is one that receives no new blueprints. A pure growth firm is one that has no blueprints currently, but will receive a geometrically decaying number of such blueprints in each future year.

Expressions (40) and (41) imply that R^I_{t+1} and R^o_{t+1} are exposed differently to innovation shocks u_t. Specifically, equations (28) and (27) imply that $\frac{\partial R^I_{t+1}}{\partial u_{t+1}} < 0$, while $\frac{\partial R^o_{t+1}}{\partial u_{t+1}} > 0$. Hence, if the marginal utility of existing agents increases in response to u_t shocks (mathematically, $\frac{\partial(\xi_{t+1}/\xi_t)}{\partial u_{t+1}} > 0$), these agents will require a higher rate of return to hold value stocks than growth stocks. The reason is that the growth opportunities embedded in growth stocks act as a hedge against u_t shocks and hence drive the return of growth stocks down. Equation (37) of Lemma 1 asserts that this will be the case whenever α is close enough to 1.[8]

We conclude this section with a few remarks on the relationship between the equilibrium stochastic discount factor in our model and some popular asset pricing models, such as the CAPM and the consumption-CAPM.

A closer inspection of equations (33), (32) and (27) reveals that both the increments of the (log) stochastic discount factor and the increments to (log) aggregate consumption are i.i.d.. Hence, equation (29) implies that a consumer's (log) lifetime consumption process is a random walk with drift, and the ratio of an agent's wealth to her current consumption $\left(W^i_{t,s}/c^i_{t,s}\right)$ is constant for all t and s[9]. Therefore equation (29) can be re-written as

$$W^i_{t,s} = W^i_{s,s}\left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s{}^{(1-\psi)(1-\gamma)}}\beta^{-(t-s)}\frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}} \quad \text{for } i \in \{e, w\}. \tag{43}$$

Letting $\overline{W}_t$ denote aggregate wealth in the economy, and repeating the same steps as in equations (30)-(32) implies

$$\frac{\xi_{t+1}}{\xi_t} = \beta\frac{C_{t+1}^{(1-\psi)(\gamma-1)}}{C_t{}^{(1-\psi)(\gamma-1)}}\left\{\frac{1}{1-\lambda}\frac{\overline{W}_{t+1}\left[1-\lambda\left(\phi^w\frac{W^w_{t+1,t+1}}{\overline{W}_{t+1}}+\phi^e\frac{W^e_{t+1,t+1}}{\overline{W}_{t+1}}\right)\right]}{\overline{W}_t}\right\}^{-\gamma}. \tag{44}$$

[8]We note that equation (37) continues to hold when $\sigma > 0$, after a few minor modifications of the technical conditions of the Lemma.

[9]To see this, note that at any point in an agent's life her consumption and her total wealth $W^i_{t,s}$ (the sum of financial wealth and human capital) are related through the net present value relationship

$$W^i_{t,s} = c_{t,s}E_t\sum_{u=t}^{\infty}(1-\lambda)^{u-t}\left(\frac{c_{u,s}}{c_{t,s}}\right)\left(\frac{\xi_u}{\xi_t}\right).$$

Because both $\frac{c_{u,s}}{c_{t,s}}$ and $\frac{\xi_u}{\xi_t}$ are random walks (with drift) in logarithms, the expectation on the right hand side of the above equation is a constant and the result follows.

Equations (32) and (44) are equivalent ways of expressing the stochastic discount factor. However, they produce different insights as to why some standard asset pricing models fail within our model. As we have already discussed, equation (32) implies that the consumption CAPM (implemented empirically by using aggregate consumption data) will not hold.

Equation (44) shows why the conventional CAPM also does not hold in our model. At first pass, equation (44) implies that *if* $\psi = 1$, then variations in the stochastic discount factor are identifiable with changes in the *total* wealth of pre-existing agents. (To see this, note that the term inside square brackets in equation (44) simply subtracts the wealth that accrues to newly arriving agents from the total wealth in the economy at time $t+1$). One might therefore conjecture that since all stocks are held by pre-existing agents, and newly arriving agents have no financial wealth, it might be possible to use the CAPM to identify changes in the wealth of pre-existing agents. The problem with this argument is that the total wealth of pre-existing agents is comprised of both human and financial wealth. As long as the two are not perfectly correlated (or if $\psi \neq 1$), the CAPM will not hold.

Since the usual approaches to quantifying the variability of the stochastic discount factor are inadequate, in the next section we employ an alternative approach to quantify the variation of the stochastic discount factor. This approach estimates directly changes in the consumption of pre-existing agents by using microeconomic data and cohort analysis.

5 Quantitative Evaluation

5.1 Cohort Effects and Asset Returns

The model has robust novel implications for the joint behavior of consumption cohort effects and cross-sectional asset-pricing returns. In this section we derive these implications and test them in the data. In the next section we use the moments that we estimate here to calibrate the model.

Taking logarithms on both sides of (29), we obtain the following decomposition of an agent's consumption into a cohort effect (a_s), a time effect (b_t) and an individual specific effect $\widetilde{\varepsilon}^i$ for $i \in \{e, w\}$:

$$\log c^i_{t,s} = a_s + b_t + \widetilde{\varepsilon}^i, \tag{45}$$

where

$$a_s = \sum_{j\in\{e,w\}} \phi^j \log c^j_{s,s} + \frac{1}{\gamma}\log\left(C_s^{(1-\psi)(1-\gamma)}\beta^{-s}\xi_s\right), \tag{46}$$

$$b_t = -\frac{1}{\gamma}\xi_t\beta^{-t}C_t^{(1-\psi)(1-\gamma)}, \tag{47}$$

$$\widetilde{\varepsilon}^i = \log c^i_{s,s} - \sum_{j\in\{e,w\}} \phi^j \log c^j_{s,s} \tag{48}$$

Equation (45) provides a first testable implication of the model: In a model with unrestricted transfers between altruistically linked generations, every agent's consumption is a constant proportion of the aggregate consumption regardless of her birth date.[10] Consequently, cohort effects are zero in such a model. By contrast, in our model log consumption exhibits non-zero cohort effects. The following lemma derives the dynamic behavior of these cohort effects according to the model.

Lemma 3 *Let z_s be defined as*

$$z_s \equiv (1-\phi)\log\left(\frac{c^w_{s,s}}{Y_s}\right) + \phi\log\left\{\frac{c^e_{s,s}}{Y_s}\right\}$$

Then for any $T \geq 1$,

$$a_{s+T} - a_s = -\sum_{i=1}^{T}\log\left(\frac{\widetilde{\upsilon}\left(u_{s+i}\right)}{1-\lambda}\right) + z_{s+T} - z_s. \tag{49}$$

Equation (49) implies that cohort effects should be non-stationary according to our model, since the first term on the right hand side of (49) is a random walk with drift. Moreover, as we show in the Appendix[11], z_s is i.i.d. and captures the fraction of income accruing to newly arriving agents at time s. Another interesting implication of (49) is that the increments

[10] Such a model is isomorphic to the standard infinitely-lived representative-agent model.

[11] We show this as part of the proof of Proposition 1.

to the random walk component of the consumption cohort effects "reveal" the unobserved displacement factor $\widetilde{v}(u_s)$. Specifically, taking (de-meaned) first differences of the cohort effects Δa_{s+i} and forming an estimate of their "long run" variance by a heteroscedasticity- and autocorrelation-consistent variance estimator (such as Newey-West) gives a consistent estimate of the variance of $\log\left(\frac{\widetilde{v}(u_{s+i})}{1-\lambda}\right)$. This provides useful guidance in calibrating the model, since we can obtain a measurement of the variance of the displacement factor.

Before proceeding, we note an additional implication of (49). According to the model, the innovation shocks u_t affect both the displacement factor $\widetilde{v}(u_t)$ and the return spread of a growth-value stock. Equations (40) and (41), together with (28) and (27), imply that R^o_{t+1}/R^I_{t+1} is an increasing function of u_{t+1}. Hence for any firm (or portfolio of firms), whose value contains a fraction w^o_t of growth options, we obtain that $R^g_{t+1}/R^I_{t+1} = 1 + w^o_t\left(\frac{R^o_{t+1}}{R^I_{t+1}} - 1\right) = f(u_{t+1})$, for an increasing function f. Finally, letting $l(x)$ be defined as $l(x) \equiv -\log\left(\widetilde{v}\left(f^{-1}(x)\right)\right)$, equation (49) can be rewritten as

$$a_{s+T} - a_s = \sum_{i=1}^{T} l\left(\log \frac{R^g_{s+i}}{R^I_{s+i}}\right) + T\log(1-\lambda) + z_{s+T} - z_s. \tag{50}$$

Since the function $\widetilde{v}(u_t)$ is decreasing in u_t and the function $f^{-1}(x)$ is increasing in x, $l(x)$ is an increasing function. Consequently, the "long-run" covariance of Δa_s and $\log\left(R^g_s/R^I_s\right)$ should be non-negative.

To estimate consumption cohort effects we use CEX data from 1981-2003 as provided on the NBER website[12]. For each household we obtain a measurement of consumption over four quarters, the year corresponding to the first observation quarter and the age of the head of household at that time. We define the cohort of households who were "born" in a given year as all those households whose head was twenty years old in that year. Given these measurements it is possible to estimate equation (45).

Equation (45) corresponds exactly to the model's prediction of how consumption should behave. A practical issue with Equation (45) is that according to the model consumption

[12]The CEX data on the NBER website were compiled by Ed Harris and John Sabelhaus. See http://www.nber.org/ces_cbo/Cexfam.pdf for a description of the data.

only features time and cohort effects, but no age effects. Realistically, one would expect the presence of age effects either because of borrowing constraints early in life, or because of changes in the consumption patterns of households as children age, etc. In order to be conservative with the attribution of consumption variation to cohort effects, we estimated (45) allowing for age effects. We also include a control for (log) household size.[13]

An issue that arises when we include age effects in (45) is that linear trends in age, cohort, and time effects cannot be identified separately.[14] Fortunately, the exact identification of the linear trends is immaterial for our purposes. As has been shown in the literature, it is possible to estimate differences in differences of cohort effects $(a_{s+1} - a_s - (a_s - a_{s-1}))$ without any normalizing assumption and even after including a full set of age dummies.[15] [16] An implication of this statement is that there exists a function of time a_s^* such that, for any normalizing assumptions, the estimated cohort effects $\widehat{a}_s$ are given by $\widehat{a}_s = \beta_0 + \beta_1 s + a_s^*$. (The coefficients β_0 and β_1 are not identified, i.e., their magnitude depends on the normalizing assumptions.)

According to our model, a_s^* is non-zero, but instead behaves like a random walk. Hence the first hypothesis we test is that $a_s^* = 0$. The three columns of Table 1 report the results of estimating equation (45) including 1) no age effects, 2) parametric age effects, and 3) a full set of age dummies. The model with parametric age effects is fitted by assuming that age effects are given by some function $h(age)$ which we parameterize with a cubic spline

[13] We also adjusted for family size by dividing by the average family equivalence scales reported in Fernandez-Villaverde and Krueger (2007) and the results remained unchanged.

[14] Sometimes the literature addresses this problem by following Deaton and Paxson (1994) and making the normalizing assumption that the time effects add up to zero and are orthogonal to a time trend. In our model the time effects b_t follow a random walk and hence such an assumption is not appropriate.

[15] See McKenzie (2006) for a proof.

[16] The easiest way to see this, is to allow for age effects in equation (45), consider the resulting equation $\log c_{t,s}^i = a_s + b_t + \gamma_{t-s} + \widetilde{\varepsilon}^i$, and observe that

$$E \log c_{t+1,s+1}^i - E \log c_{t+1,s}^i - \left(E \log c_{t,s}^i - E \log c_{t,s-1}^i\right) = a_{s+1} - a_s - (a_s - a_{s-1}).$$

Replacing expectations with the respective cross sectional averages shows how the differences in differences of cohort effects $(a_{s+1} - a_s - (a_s - a_{s-1}))$ can be estimated in the data.

	No age effects	Parametric Age Effects	Age Dummies
Wald Test $a^* = 0$	31.3	4.21	4.25
p-value	0.000	0.000	0.000
Observations	52245	52245	52245
R-squared	0.373	0.382	0.384

TABLE 1: Results from regression of log consumption expenditure on quarter dummies, a control for log(fam.size) and various specifications of cohort and age effects. Cohort effects are included via cohort dummies. In the first specification the regression does not contain age effects, while the second specification allows age effects parameterized via a cubic spline. The third specification allows for a full set of age dummies. The Wald test refers to the test that $a_{s+1} - a_s - (a_s - a_{s-1}) = 0$ for all s. Standard errors are computed using a robust covariance matrix clustered by cohort and quarter. The CEX data are from 1980-2003 and include observations on cohorts as far back as 1911.

having knots at ages 33, 45, and 61. The first row reports the results from a Wald test that the estimated differences $\widehat{a}_{s+1} - \widehat{a}_s - (\widehat{a}_s - \widehat{a}_{s-1}) = 0$ for all s. The second row reports the associated $p-$values. As can be seen, the data strongly reject the hypothesis that cohort effects are either non-existent or given by a deterministic linear trend. For our purposes, this implies that we can reject the prediction of a standard representative-agent model, namely that cohort effects are uniformly zero.

The first two rows of Table 2 report results on the magnitude of the variation of these cohort effects for different ways of accounting for age effects. The first row contains information on the standard deviation of the first differences of cohort effects $(\widehat{a}_{s+1} - \widehat{a}_s)$. The second row fits an ARIMA (1,1,1) to $\widehat{a}_s$ and then uses the methods of Beveridge and Nelson (1981) to obtain the standard deviation of the permanent component of $\widehat{a}_s$. As two alternative ways to obtain an estimate of this standard deviation, we report in the third row Newey-West estimates of the long-run variance of the first differences of $\widehat{a}_s$ using 10-year autocovariance lags. In the fourth row we report the standard deviation of cohort effects summed over

	Param. Age Effects	Age Dummies
Std. Dev. (Cohort−Lagged Cohort)	0.030	0.030
Std. Dev. (Perm. Component (Newey West))	0.020	0.020
Std. Dev. (Perm. Component (Beveridge Nelson))	0.021	0.023
Std. Dev. (Perm. Component $\sqrt{10}$(10-year Aveg))	0.019	0.018
$\frac{\text{Cov(coh.diffs; FF)}}{\text{Var(coh.diffs)}}$ (Newey West)	3.39	3.92
$\frac{\text{Cov(10-year coh.diffs; 10-year FF)}}{\text{Var(10-year coh.diffs)}}$	2.93	3.43
Observations	68	68

Table 2: Various moments of the permanent components of the estimated cohort effects.

10-year intervals, computing the associated standard deviation, and dividing by $\sqrt{10}$. All methods yield roughly similar results. We use these estimates in the next section, when we calibrate the model.

To obtain a visual impression of these results, the solid line in Figure 2 depicts the estimated cohort effects.[17] As can be seen from the figure, these cohort effects are persistent, in line with the results reported in the first two rows of Table 2. The dashed line depicts $\sum_{i=1}^{T} \log\left(R^g_{t+i}/R^I_{t+i}\right)$, where we have used the negative of the logarithmic gross return associated with the HML factor of Fama and French (1992) as a measure of $\log\left(R^g_{t+i}/R^I_{t+i}\right)$ and have removed a linear trend. According to Equation (50), the consumption cohort effects should be co-trending with the sum of an appropriate non-linear increasing function $l(\log\left(R^g_{t+1}/R^I_{t+1}\right))$. Assuming that $l(\cdot)$ is reasonably well approximated by an affine first order Taylor expansion, cohort effects and cumulative (log) returns on a growth-value portfolio should be co-trending, as the picture suggests.

For the purpose of the calibration exercise that follows, a useful quantity is the covariance between the innovations to the permanent components of a_s and to the permanent

[17]We report the cohort effects from 1927 onward, since data on the Fama French factors are available from 1927 onward. We also report results up to 1995 because of the sparsity of data on cohorts post 1995. However, cohort effects prior to 1927 and post 1995 are included in the estimation and all testing exercises.

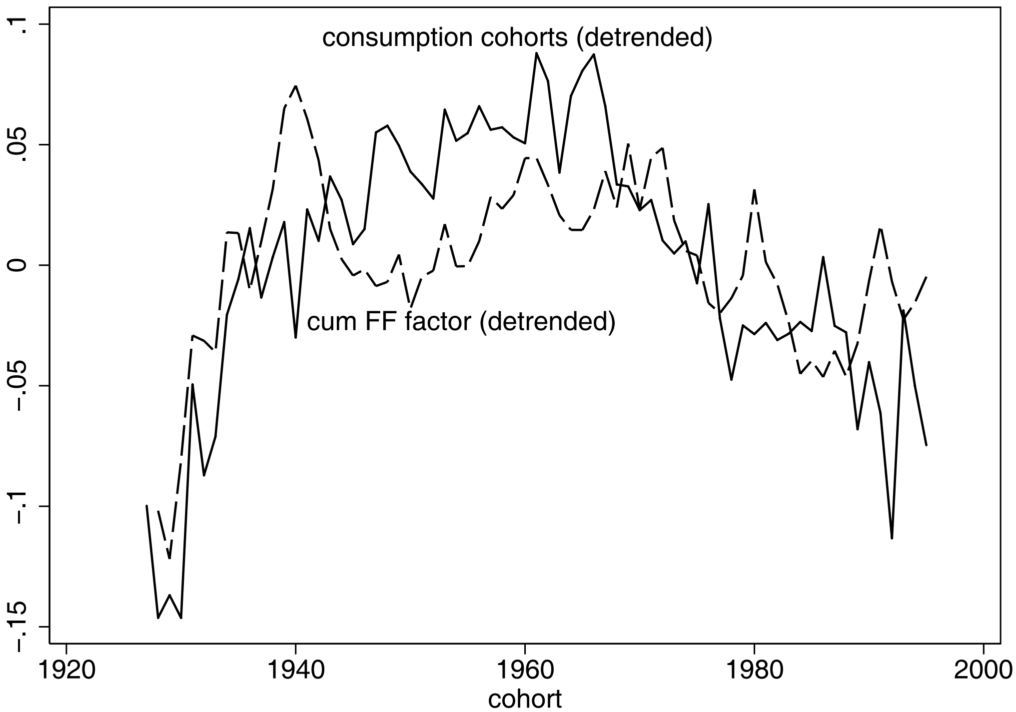

FIGURE 2: Consumption cohort effects and cumulative returns on a growth-value portfolio (negative of the HML factor) after removing a constant time trend from both series and multiplying the latter series by a scalar to fit in one scale. A full set of age dummies was used in estimating the consumption cohort effects.

components of $\sum_{i=1}^{s} \log\left(R_{t+i}^{g}/R_{t+i}^{I}\right)$. According to (50), the covariance between the innovations to these permanent components (to which we will refer as the long-run covariance) provides a measure of $cov(\log\left(R_t^g/R_t^I\right), \log\widetilde{v}\left(u_t\right))$, i.e. a measure of how the displacement factor covaries with the returns to a growth-value portfolio. We obtain an estimate of this covariance by using a multivariate Newey-West (10 lags) estimate of the covariance between the negative of the logarithmic gross return associated with the HML factor and the first differences of the estimated cohort effects. The fifth row of Table 2 reports this covariance, normalized by the long-run variance of the consumption cohort effects as obtained in the second row of the table. As a robustness check, we also report in the final row of the table the results from computing the covariance of 10-year consumption cohort differences and 10-year cumulative returns on the growth-value portfolio, normalized by the variance of 10-year consumption-cohort differences.

Table 3 reports results on a stronger implication of the model, namely that the permanent component of cohort effects can help explain return differentials of stocks in different book to market deciles. Specifically, the second line of this table reports the average return differential between the (log) gross return of a portfolio of stocks with book to market values in various deciles vs. the respective return of a portfolio in the 10th decile. The third row of table 3 reports the analog of the 5th row of table 2 for these return differentials, namely the covariance between these return differentials and the first differences of the permanent component of (log) consumption cohort effects (normalized by the variance of the permanent component of these cohort effects). Consistent with the empirically well established value premium, stocks with lower book to market values (growth stocks) exhibit a lower return than stocks with higher book to market values (value stocks). Interpreting the (permanent component of) consumption cohort effects as a measure of the displacement factor, the model implies that stocks in lower deciles should act as hedges compared to the stocks in the top decile. That is, the covariance between the (permanent component of) consumption cohort effects and the $i^{\text{th}} - 10$ return differential should be negative, and higher in absolute value, the smaller is i. This is the general pattern in the last row of the table.

Before proceeding with the calibration, we note that so far we have focused on the empirical implications of the model for consumption and returns. This is of first-order importance for asset pricing because these two quantities determine expected returns, which are the focus of this paper. However, the model makes additional predictions, which allow us to inspect further whether the model's mechanisms can be detected in the data. An obvious candidate, which also plays a role in the calibration of the model, is whether agents' income is affected by intergenerational risk. Applying a similar argument to the one that led to Equation (45), Equations (11) and (12) imply the presence of cohort effects in income data that should be correlated with the cumulative return of a growth-value portfolio. Figure 3 is

1 -10	2-10	3-10	4-10	5-10	6-10	7-10	8-10	9-10
-0.033	-0.020	-0.023	-0.028	-0.015	-0.011	-0.016	0.005	0.009
6.293	7.162	5.609	4.995	5.125	4.184	3.090	3.474	2.636

TABLE 3: Return differentials between portfolios sorted on book to market and their respective "long run" covariances with cohort effects. The first line refers to the return differential under consideration. For instance 3-10 refers to the return differential between the (log) gross return of a portfolio of stocks with book to market values in the 3rd decile vs. the respective return of a portfolio in the 10th decile. The second line gives the mean of this return differential. The third line reports the covariance between this return differential and first differences in the permanent component of (log) consumption cohort effects normalized by the variance of the permanent component of these cohort effects. Covariances and variances are computed using the Newey-West approach with 10 lags. The data on return differentials are from the website of K. French (annual 1927-2007).

the analog of Figure 2 except that consumption is replaced with agents' disposable income net of dividends, rents, and interest. This picture confirms that the qualitative properties that hold for consumption cohort effects also hold for log (earned) income cohort effects, consistent with the model.

5.2 Calibration

The empirical results suggest that certain key predictions of the model are qualitatively consistent with the data. In this section we assess whether the model can account quantitatively for the joint properties of asset pricing moments, aggregate consumption, new company introduction, and the variation in cross sectional consumption cohort effects.

Our choice of parameters is in Table 4. The values of μ and σ are chosen so as to match aggregate consumption growth. In light of Equation (21), the parameter α controls the share of profits in aggregate income. We set $\alpha = 0.8$, which implies a profit share of 16%. In yearly NIPA data of the US since 1929 the average share of (after depreciation) profits and interest payments is about 15% of national income and it becomes 18% if one

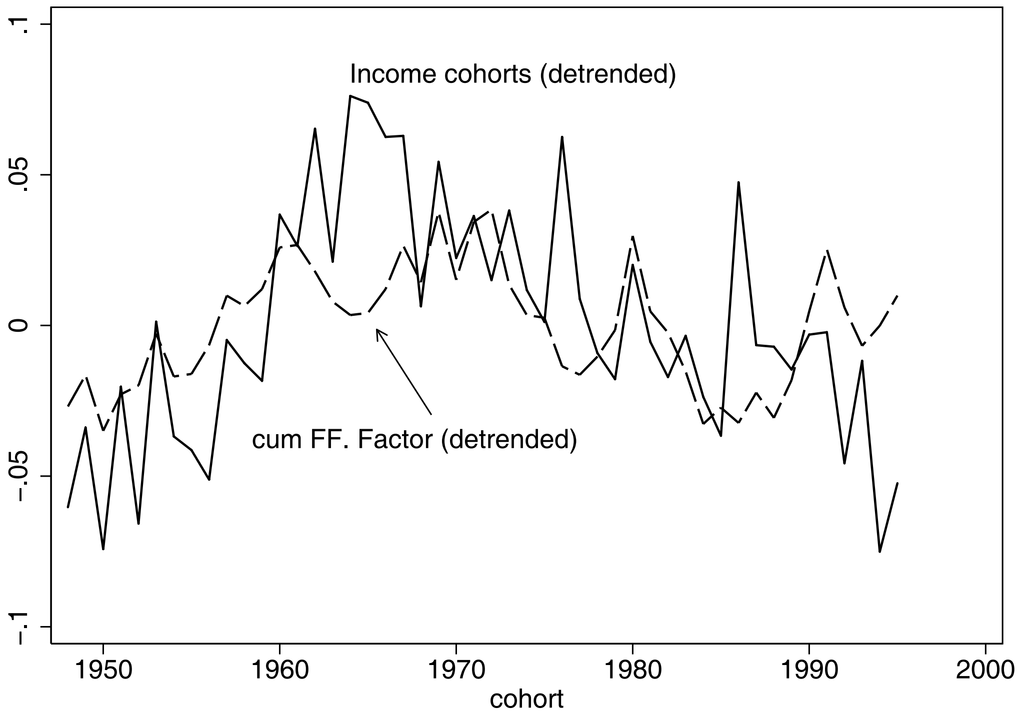

FIGURE 3: Earned (log) income cohort effects and cumulative returns on a growth-value portfolio (negative of the HML factor) after removing a constant time trend from both series and and multiplying the latter series by a scalar to fit in one scale.. A full set of age dummies was used in estimating the income cohort effects. The regression was estimated with post-war cohorts (1947 onward) to avoid discontinuities in earned income that arise at 65.

imputes that 1/3 of proprietor's income is due to profits.[18] The parameter λ is chosen so as to capture the arrival of new agents. In post war data the average birth rate is about 0.016. Immigration rates are estimated to be between $0.002 - 0.004$, which implies an overall arrival

[18]Since in our model there is no financial leverage, it seems appropriate to treat both dividend and interest payments as one entity. Moreover, it also seems appropriate to deduct depreciation from profits, because otherwise the relative wealths of agents e and w would be unduly affected by a quantity that shouldn't be counted as income of either. The real business cycle literature assumes a profit share of 1/3 but also models investment and depreciation explicitly and deducts investment from (gross) profits to obtain dividends. As a result the share of (net) output that accrues to capital holders is close to the number we assume here.

rate of new agents between 0.018 and 0.02. We take the discount factor to be close to 1, since in an OLG model the presence of death makes the "effective" discount factor of agents equal to $\beta(1-\lambda)$. Given a choice of $\lambda = 0.02$, the effective discount rate is 0.98, consistent with a standard choice in the literature.The constant ψ influences the growth of agents' marginal utilities, and hence is important for the determination of interest rates. We choose $\psi = 0.5$, in order to approximately match observed interest rates. On behavioral grounds, this assumption implies that an individual places equal weights on his own consumption and on his consumption relative to the aggregate.

In the model, the parameter δ controls age effects in income. In the real world, income is hump shaped as a function of age. However, for the purpose of calibrating the model, it is only the net present value of income at birth that affects the general-equilibrium outcomes. With this in mind, we use the estimated age-(log) earnings profiles of Hubbard et al. (1994) and determine δ so that

$$
\begin{aligned}
& E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1-\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho} \\
& \quad = \quad E_s \sum_{t=s}^{\infty} \Lambda_{t-s} G(t-s) \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) \left(\frac{A_t}{A_s}\right)^{-\rho},
\end{aligned}
$$

where Λ_{t-s} is an agents' survival probability at age $t-s$ obtained from the National Center for Health Statistics and $G(t-s)$ is the age-(log) earnings profile, as estimated by Hubbard et al. (1994).

To ensure positivity of u_t we choose a Gamma distribution with parameters k, ν. The parameters ρ and χ control the exposure of labor and dividend income to the shock u_t. We choose k, ν, ρ and χ so as to match a) the variation of permanent components of consumption cohort effects as reported in Table 2, b) the variation of the permanent components of income cohort effects, which we obtain by using earned log income (instead of consumption expenditure) on the left hand side of Equation (45), estimating the resulting cohort effects and isolating their permanent component, c) the volatility of aggregate dividends, and d) the correlation between aggregate dividends and aggregate consumption.

The parameter κ gives the proportion of growth options that are tradeable, while ϖ

β	0.999	k	0.25
ψ	0.5	ν	0.05
μ	0.015	ρ	0.9
σ	0.032	κ	0.9
α	0.8	χ	4
λ	0.018	ω	0.87
δ	-0.012	η	0.9

Table 4: Baseline parameters used in the calibration.

controls the timing of the exercising of these options (higher ϖ means that the options are exercised sooner). As a consequence, these two parameters jointly determine the aggregate price-to-earnings ratio, as well as the properties of growth firms. We therefore calibrate them to the aggregate price-to-earnings ratio and the covariance between HML returns and changes in the permanent component of consumption cohort effects.[19]

The parameter η does not affect aggregate valuation ratios, or the returns of a pure-growth firm, but it controls the proportions of value and growth firms, and thus the properties of the cross-section of valuation ratios. We calibrate it to the price-to-earnings ratio of the top X% of the distribution. [What should X be? Why 90 and not 50? etc]

Finally, we treat the parameter γ as a free parameter and examine the model's ability to match a wide variety of moments for values of γ that generate reasonable equity and value premia. Since we are attempting to match more moments than parameters, it is impossible to obtain an exact fit, but the deviations between the data and the simulated moments appear reasonably small. As can be seen in Table 5, with $\gamma = 10$ the model can match about 66% of the equity premium and about 80% of the value premium. As γ increases to 12 the model does well in almost all dimensions. In interpreting these results, it should be noted that the model has no role for financial or operating leverage, which, as Barro (2006)

[19]We chose this covariance over other moments because of its importance in the determination of the value premium, which is central to the paper.

	Data	$\gamma = 10$	$\gamma = 12$	$\gamma = 15$
Aggregate (log) Consumption Growth rate	0.017	0.017	0.017	0.017
Aggregate (log) Consumption Volatility	0.033	0.032	0.032	0.032
Riskless Rate	0.010	0.022	0.015	0.014
Equity premium	0.061	0.040	0.051	0.062
Aggregate Earnings / Price	0.075	0.103	0.108	0.119
Dividend Volatility	0.112	0.10	0.101	0.101
Correl. (divid. growth, cons.growth)	0.2	0.189	0.189	0.189
Std ($\Delta\alpha_s^{\text{perm}}$)	0.023	0.024	0.024	0.023
$\frac{cov\left(\Delta\alpha_s^{\text{perm}}, \log R^g - \log R^I\right)}{var\left(\Delta\alpha_s^{\text{perm}}\right)}$	3.92	4.226	4.378	4.598
Std (Δw_s^{perm})	0.022	0.023	0.023	0.023
Earnings / Price 90th Perc.	0.120	0.11	0.118	0.132
Earnings / Price 10th Perc.	0.04	0.041	0.039	0.041
Average Value premium	0.081	0.064	0.081	0.097
Std (Value Premium)	0.120	0.104	0.105	0.105
Expected Return Value - "Pure" Growth		0.102	0.121	0.141

TABLE 5: Data and model calibration for different values of risk aversion γ. Data on consumption, the riskless rate, the equity premium and dividends per share are from Campbell and Cochrane (1999). Data on the aggregate E/P ratio are from the long sample (1871-2005) on R. Shiller's Website. The E/P for value and growth firms are the respective E/P ratios of firms in the bottom 10% vs. top 90% of the cross sectional book to market distribution from Fama and French (1992). The value premium is computed as the difference in value weighted returns of stocks in the top 90%-bottom 10% of the cross sectional book to market distribution, as reported in the website of Kenneth French. The two numbers in square brackets for Std ($\Delta\alpha_s^{\text{perm}}$) refer to the standard deviation of the permanent components of consumption cohort effects as estimated in Table 2. Std (Δw_s^{perm}) refers to the respective numbers for the cohort effects of earned income.

argues, implies that the unlevered equity premium produced by the model compared to the levered equity premium produced by the data should be in a 2/3 ratio. Moreover, in the model there is no time variation in interest rates, volatility, and excess returns. Therefore, in light of the literature, it is not surprising that the model needs relatively high levels of risk aversion in order to explain the data. However, it should be noted that even in the absence of these effects, levels of risk aversion around 10 explain a non-trivial fraction of asset-pricing data. Therefore, the evidence in Table 5 suggests that the model's mechanisms are quantitatively powerful enough to explain a substantial fraction of the equity premium, the value premium, and the riskless rate.

5.3 Inspecting the mechanism

As the calibration shows, the model is successful in matching unconditional return moments. A number of factors are responsible for this success, and here we take a closer look at the quantitative significance of each factor.

The model produces large equity and value premia for three reasons: First, current agents' consumption growth is more volatile than aggregate consumption growth, because of the displacement effect. Second, current firms' dividends are more sensitive to the displacement factor than current agents' consumption. And third, there is co-skewness between current firms' dividend growth and current agents' consumption growth.

A simple back of the envelope calculation helps illustrate the magnitude of each factor. Taking logarithms of the pricing kernel in equation (32), using (5), (27), and the definition of $\widetilde{v}(u_{t+1})$ in equation (33) leads to

$$\begin{aligned}
&\Delta \log \xi_{t+1} - \log \beta(1-\lambda)^{\gamma} - (-1+\psi\,(1-\gamma))\mu \\
&= \;(-1+\psi\,(1-\gamma))\varepsilon_{t+1} - \gamma\widetilde{v}(u_{t+1}) + (-1+\psi\,(1-\gamma))\,(1-\alpha)\,u_{t+1}. \qquad (51)
\end{aligned}$$

At the same time, the stochastic component of the aggregate dividend growth equals $\varepsilon_{t+1} + h(u_{t+1})$ for some function h.[20] Given our choice of parameters $\psi = 0.5$, $\gamma = 10$ and $\sigma = 0.032$,

[20] According to equation (28), $h(x) = -(1 + chi)x$ the existing value firms. However, since the dividends

the standard deviations of the first term in (51) is 0.18; the standard deviation of $\widetilde{v}(u_{t+1})$ is 0.023, so that of the second term is 0.23; and the last term has a standard deviation of 0.0275, which is an order of magnitude smaller and we therefore ignore for the purpose of our illustration. As for the dividends, since their standard deviation of growth is about 0.1 and that of ε is 0.032, $h(u_{t+1})$ has standard deviation of $\sqrt{0.1^2 - 0.032^2} = 0.095$. Given that the price-to-dividend ratio is constant, this is also the standard deviation of market (log) returns.

Based on these standard deviations, if ε_{t+1} and u_{t+1} (and therefore $\widetilde{v}(u_{t+1})$, approximately) were both normal, then the equity premium would equal $0.18 \times 0.032 + 0.23 \times 0.095 = 0.028$. The difference between this number and the 0.04 equity premium in our base-case calibration owes to the fact that u_{t+1} is a non-normal shock, making the consumption growth of existing agents and the returns in the stock market co-skewed.

This back-of-the-envelope calculation shows that the bulk of the equity premium in our model is *not* due to the fact that consumption of existing agents is excessively volatile. The total volatility of existing agents' consumption is approximately equal to 0.039, which is not far away from the aggregate volatility of 0.032.

We conclude by noting that the model not only produces a sizeable equity premium, but also an interest rate that is constant and low. The reasons are that a) current agents's consumption has a slightly lower mean growth rate (1.5%) than that of aggregate consumption (1.7%) and is more volatile (3.9%) than aggregate consumption (3.2%), and b) external habit formation (captured by ψ) implies that agents' marginal utility of consumption declines more slowly than suggested solely by risk aversion and consumption growth.

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of existing growth firms also include the dividends from the blueprints that are created within the period, h is generally different.

6 Extensions and Discussion

6.1 Frictions and blueprint endowments later in life

In order to be able to solve and analyze the model, we made several simplifying assumptions. Here we verify the robustness of our findings to some of these assumptions. Specifically, one of model's stylized assumptions is that innovating agents receive their blueprints once they get born. In reality, it may take some time to start a new firm, especially if frictions are present. Moreover, the shocks u_t may follow a moving average process, rather than being i.i.d. as we assumed.

We provide a simple example that illustrates that, while such frictions and modifications of the baseline model typically affect the short run dynamics of the equilibrium stochastic discount factor, the long-run properties of the model are likely to be unaffected.

To make this point, suppose that all agents are born as workers with an initial endowment of labor efficiency units equal to $\overline{h}\,(1-\phi)\,q_{s,s}$. Furthermore, suppose that a fraction ϕ of them receive blueprints in the second rather than the first period of their lives and become entrepreneurs. From the second period onwards, workers' endowment of efficiency units follows the same process as in the baseline model. Finally, assume that agents can only access financial markets in the second period of their lives, while in the first period they consume their wage income.

These simple assumptions capture the idea that an agent's "birth" cohort and the date at which that agent receives the property rights of blueprints may not coincide. Moreover, exclusion from markets captures in a stylized manner the idea that the agent cannot smooth consumption between the "birth" date and the innovation arrival date.

Repeating, essentially, the argument of Section 3.2, the equilibrium stochastic discount factor in this modified setup is obtained as

$$\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)} \hat{\upsilon}(u_{t+1}, u_t)^{-\gamma},$$

where

$$\hat{v}(u_{t+1}, u_t) = (1-\lambda)^{-1} \left(1 - \frac{\lambda y_t}{C_t}\right)^{-1} \left(1 - \lambda\,(1-\lambda) \sum_{i \in \{w,e\}} \phi^i \frac{c^i_{t+1,t}}{C_{t+1}} + \lambda \frac{y_{t+1}}{C_{t+1}}\right)$$

and y_t denotes an agent's initial (wage) income. Furthermore, the same reasoning as in the proof of Lemma 2 implies that the variance of the permanent component of (log) consumption cohort effects equals $Var\left(\hat{v}(u_{t+1}, u_t)\right)$.

This simple example illustrates that the frictions likely to be relevant early in an agent's life are likely to affect at most the transitory dynamics of cohort effects, returns, and the stochastic discount factor. Therefore, they do not affect our main conclusion that the permanent component of cohort effects provide an accurate measurement of the permanent component of the displacement factor.

Even though the simplifying assumptions of the model imply that all shocks are permanent, in the calibration exercises we first isolated the permanent component of cohort effects and then used those as an input to the model. In light of the results of recent literature, our choice to match permanent components implies that the introduction of frictions with transitory effects may alter the properties of short run returns, but is unlikely to affect the model's quantitative conclusions about long-run returns. Additionally, by focusing on the permanent component of cohort effects, the model is robust to how "finely" one chooses to define cohorts.[21]

[21] A finer definition of cohorts will result in more cohorts, but less variation in the first differences of cohorts. Therefore the overall long-run variance of cumulative cohorts will be unaffected by the exact cohort definition.

Appendix

A Auxiliary Results and Proofs

Proposition 1 *Let* ζ *be defined as*

$$\zeta \equiv \beta\left(1-\lambda\right)^{\gamma} e^{\mu\psi(1-\gamma)+\frac{\sigma^2}{2}\psi^2(1-\gamma)^2}$$

and consider the solution to the following system of three equations in three unknowns θ^e, θ^w, *and* θ^b

$$\theta^e = \frac{1-\zeta E_t\left[e^{\psi(1-\alpha)(1-\gamma)u_{t+1}}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)^{1-\gamma}\right]}{1-\zeta E_t\left[e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{t+1}}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\right]} \tag{52}$$

$$\theta^w = \frac{1-\zeta E_t\left[e^{\psi(1-\alpha)(1-\gamma)u_{t+1}}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)^{1-\gamma}\right]}{1-\zeta\left(1-\lambda\right)\left(1-\delta\right)E_t\left[e^{((1-\alpha)\psi(1-\gamma)-\rho)u_{t+1}}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\right]} \tag{53}$$

$$\theta^b = \frac{\varpi\zeta E_t\left[e^{(1-\alpha)\psi(1-\gamma)u_{t+1}}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\left(1-e^{-(1+\chi)u_{t+1}}\right)(1-\kappa)\right]}{1-\varpi\zeta E_s\left[e^{(1-\alpha)\psi(1-\gamma)u_{t+1}}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\right]}. \tag{54}$$

Here,

$$\widetilde{\upsilon}(x;\theta^e,\theta^w,\theta^b) \equiv 1-\theta^e\alpha\left(1-\alpha\right)\left(\kappa\left(1-e^{-(1+\chi)x}\right)+\left(\frac{1-\varpi}{\varpi}\right)\theta^b\right) -\theta^w\left(\alpha^2+1-\alpha\right)\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho x}\right). \tag{55}$$

Assuming positivity of the numerators and denominators in (52) and (53) and positivity of the denominator in (54), there exists an equilibrium with stochastic discount factor

$$\frac{\xi_{t+1}}{\xi_t} = \beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)}\left[\frac{1}{1-\lambda}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)\right]^{-\gamma}. \tag{56}$$

Proof of Proposition 1. To prove Proposition 1 we conjecture that the expression $\frac{c^i_{t+1,t+1}}{Y_{t+1}}$ is exclusively a function of u_{t+1}, and then confirm our conjecture based on the resulting expression for $\frac{\xi_{t+1}}{\xi_t}$. To start, we note that if $\frac{c^i_{t+1,t+1}}{Y_{t+1}}$ is exclusively a function of u_{t+1}, then an appropriate function $f(u_{t+1})$ exists such that the stochastic discount factor is given by $\frac{\xi_{t+1}}{\xi_t} = \beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)} \times f(u_{t+1})$.

To determine $c^i_{t+1,t+1}$ for a worker ($i = w$) under this conjecture for $\frac{\xi_{t+1}}{\xi_t}$ we use (29), (11), and the fact that $h_{t,s} = \overline{h}\,(1-\delta)^{t-s}$ inside the intertemporal budget constraint (13) to obtain

$$c^w_{s,s} = h_{s,s} q_{s,s} w_s \left[\frac{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1-\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho}}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}} \right]. \tag{57}$$

Under our conjecture the expression ξ_{t+1}/ξ_t is a deterministic function of ε_{t+1} and u_{t+1}, and it follows that the distribution of $\frac{\xi_t}{\xi_s}$ for $t \geq s$ depends *only* on $t-s$. The same is true for A_t/A_s and for w_t/w_s (by equation [24]). Therefore, the expectations in both the numerator and the denominator inside the square brackets of equation (57) are time-invariant constants. Hence, using (11) we can express (57) as

$$c^w_{s,s} = \overline{h}\left(1-(1-\lambda)(1-\delta)\,e^{-\rho u_s}\right) w_s \theta^w, \tag{58}$$

where θ^w is defined as

$$\theta^w \equiv \frac{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1-\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho}}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{(Y_t/N_t)^{(1-\psi)(1-\gamma)}}{(Y_s/N_s)^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}}. \tag{59}$$

The initial consumption of new business owners, who are born at time s, can be computed in a similar fashion. To start, we observe that

$$\Pi_s = \pi^I_s \left[E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{\pi^I_t}{\pi^I_s}\right) \right], \tag{60}$$

Our conjecture on $\frac{\xi_{t+1}}{\xi_t}$, together with (28), implies that the expression inside square brackets in (60) is a constant. Observing that $A_s - A_{s-1} = A_s\left(1-e^{-u_s}\right)$ (from [9]) and that $\int_{A_{s-1}}^{A_s} \pi^I_{j,s} dj = \left(1-e^{-u_s}\right) \alpha\,(1-\alpha)\, Y_s$ (from [28]), and using (60) inside (16) gives

$$\begin{aligned} V_{s,s} &= \alpha(1-\alpha) Y_s \times \left\{ E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{\pi^I_t}{\pi^I_s}\right) \right\} \\ &\quad \times \left\{ \kappa\left(1-e^{-(1+\chi)u_s}\right) + \left(\frac{1-\varpi}{\varpi}\right) E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t}{Y_s}\right) (1-\kappa)\left(1-e^{-(1+\chi)u_t}\right) \varpi^{t-s} \right\}. \end{aligned}$$

It will be useful to define

$$\theta^e \equiv \frac{E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{\pi_t^I}{\pi_s^I}\right)}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s}\left(\frac{\xi_t}{\xi_s}\right)\left(\frac{(Y_t/N_t)^{(1-\psi)(1-\gamma)}}{(Y_s/N_s)^{(1-\psi)(1-\gamma)}}\beta^{-(t-s)}\frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}} \tag{61}$$

and

$$\theta^b \equiv E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{Y_t}{Y_s}\right)(1-\kappa)\left(1-e^{-(1+\chi)u_t}\right)\varpi^{t-s}. \tag{62}$$

The maintained conjecture that ξ_{t+1}/ξ_t is a deterministic function of ε_{t+1} and u_{t+1} and equation (28) imply that θ^e and θ^b are both constants. Using (29) inside (14),

$$c_{s,s}^e = \frac{\theta^e}{\phi}\frac{\alpha(1-\alpha)Y_s}{\lambda}\left\{\kappa\left(1-e^{-(1+\chi)u_s}\right)+\left(\frac{1-\varpi}{\varpi}\right)\theta^b\right\}. \tag{63}$$

Combining (58) and (63) and noting that s in equations (63) and (58) is arbitrary, we obtain

$$\begin{aligned} \sum_{i\in\{w,e\}} \phi^i \frac{c_{t+1,t+1}^i}{Y_{t+1}} &= \theta^e \frac{1}{\lambda}\left\{\kappa\left(1-e^{-(1+\chi)u_{t+1}}\right)+\left(\frac{1-\varpi}{\varpi}\right)\theta^b\right\}\alpha(1-\alpha) \\ &\quad +\overline{h}\theta^w\left(1-(1-\lambda)(1-\delta)e^{-\rho u_{t+1}}\right)\left(\alpha^2+1-\alpha\right), \end{aligned} \tag{64}$$

which is a deterministic function of u_{t+1}. Using (64) and $\overline{h}=1/\lambda$ inside (32) verifies the conjecture that there exists an equilibrium with $\frac{\xi_{t+1}}{\xi_t}=\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)}\times f(u_{t+1})$ where $f(u_{t+1})$ is given by $f(u_{t+1})=\left[\frac{1}{1-\lambda}\widetilde{\upsilon}(u_{t+1};\theta^e,\theta^w,\theta^b)\right]^{-\gamma}$. This proves equation (56).

To obtain equations (52), (53) and (54) we start by using (56) to compute the term inside square brackets in equation (60). Since $\left(\frac{\xi_{i+1}}{\xi_i}\right)\left(\frac{\pi_{i+1}^I}{\pi_i^I}\right)$ is an i.i.d random variable for any i, it follows that

$$\begin{aligned} E_s\sum_{t=s}^{\infty}\left(\frac{\xi_t}{\xi_s}\right)\left(\frac{\pi_t^I}{\pi_s^I}\right) &= \sum_{t=s}^{\infty}E_s\left(\frac{\xi_t}{\xi_s}\right)\left(\frac{\pi_t^I}{\pi_s^I}\right)=\sum_{t=s}^{\infty}E_s\prod_{i=s}^{t-1}\left(\frac{\xi_{i+1}}{\xi_i}\right)\left(\frac{\pi_{i+1}^I}{\pi_i^I}\right) \\ &= \sum_{t=s}^{\infty}\prod_{i=s}^{t-1}E_s\left(\frac{\xi_{i+1}}{\xi_i}\right)\left(\frac{\pi_{i+1}^I}{\pi_i^I}\right)=\sum_{t=s}^{\infty}\left[E_s\left(\frac{\xi_{s+1}}{\xi_s}\right)\left(\frac{\pi_{s+1}^I}{\pi_s^I}\right)\right]^{t-s} \\ &= \frac{1}{1-E_s\left(\frac{\xi_{s+1}}{\xi_s}\right)\left(\frac{\pi_{s+1}^I}{\pi_s^I}\right)} \\ &= \frac{1}{1-\zeta E_s\left[e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{s+1}}\widetilde{\upsilon}(u_{s+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\right]}, \end{aligned} \tag{65}$$

where the last equality follows from (56):

$$\begin{aligned} \mathrm{E}_s\left[\frac{\xi_{s+1}}{\xi_{s+1}}\frac{\pi_s}{\pi_s}\right] &= \beta \mathrm{E}_s\left[\left(Z_{s+1}e^{(1-\alpha)u_{s+1}}\right)^{\psi(1-\gamma)-1}\left(Z_{s+1}e^{-(\alpha+\chi)u_{s+1}}\right)\right] \\ &= \beta \mathrm{E}_s\left[Z_{s+1}^{\psi(1-\gamma)}e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{s+1}}\right]. \end{aligned}$$

Following a similar reasoning,

$$\begin{aligned} & E_s \sum_{t=s}^{\infty}(1-\lambda)^{t-s}\left(\frac{\xi_t}{\xi_s}\right)\left(\frac{w_t}{w_s}\right)(1-\delta)^{t-s}\left(\frac{A_t}{A_s}\right)^{-\rho} \\ = & \frac{1}{1-(1-\lambda)(1-\delta)\zeta E_s\left[e^{((1-\alpha)\psi(1-\gamma)-\rho)u_{s+1}}\widetilde{\upsilon}(u_{s+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\right]} \end{aligned} \tag{66}$$

and

$$\begin{aligned} & E_s \sum_{t=s}^{\infty}(1-\lambda)^{t-s}\left(\frac{\xi_t}{\xi_s}\right)\left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}}\beta^{-(t-s)}\frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}} \\ = & \frac{1}{1-\zeta E_s\left[e^{\psi(1-\alpha)(1-\gamma)u_{s+1}}\widetilde{\upsilon}(u_{s+1};\theta^e,\theta^w,\theta^b)^{1-\gamma}\right]}. \end{aligned} \tag{67}$$

Finally,

$$\begin{aligned} & E_s \sum_{t=s+1}^{\infty}\left(\frac{\xi_t}{\xi_s}\right)\left(\frac{Y_t}{Y_s}\right)\left(1-e^{-(1+\chi)u_t}\right)(1-\kappa)\varpi^{t-s} \\ = & E_s \sum_{t=s+1}^{\infty}\left[\prod_{i=s+1}^{t}\left(\frac{\xi_i}{\xi_{i-1}}\right)\left(\frac{Y_i}{Y_{i-1}}\right)\varpi\right]\left(1-e^{-(1+\chi)u_t}\right)(1-\kappa) \\ = & \sum_{t=s+1}^{\infty}E_s\left\{\left[\prod_{i=s+1}^{t-1}\left(\frac{\xi_i}{\xi_{i-1}}\right)\left(\frac{Y_i}{Y_{i-1}}\right)\varpi\right]\left(\frac{\xi_t}{\xi_{t-1}}\right)\left(\frac{Y_t}{Y_{t-1}}\right)\varpi\left(1-e^{-(1+\chi)u_t}\right)(1-\kappa)\right\} \qquad (68) \\ = & \left\{\sum_{t=s+1}^{\infty}\left\{\prod_{i=s+1}^{t-1}E_s\left(\frac{\xi_i}{\xi_{i-1}}\right)\left(\frac{Y_i}{Y_{i-1}}\right)\varpi\right\}\right\}\times E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\varpi\left(1-e^{-(1+\chi)u_{t+1}}\right)(1-\kappa)\right] \\ = & \frac{\varpi\zeta E_s\left[e^{(1-\alpha)\psi(1-\gamma)u_{s+1}}\widetilde{\upsilon}(u_{s+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\left(1-e^{-(1+\chi)u_{s+1}}\right)(1-\kappa)\right]}{1-\varpi\zeta E_s\left\{e^{(1-\alpha)\psi(1-\gamma)u_{s+1}}\widetilde{\upsilon}(u_{s+1};\theta^e,\theta^w,\theta^b)^{-\gamma}\right\}} \end{aligned}$$

Combining (68) with (62) leads to (54). Similarly, combining (61) with (67) and (65) leads to (52), while combining (59), (66), and (67) implies (53). ■

Proof of Lemma 1. To establish that the equity premium is non-zero in the limit, it suffices to show that

$$\lim_{\alpha\to 1}\operatorname{cov}\left\{R_t,\left(\xi_{t+1}/\xi_t\right)\right\}\neq 0. \tag{69}$$

Since $\kappa = 1$, all stocks have rate of return

$$R_t = \frac{\left(\pi^I_{t+1}/\pi^I_t\right) + \left(\Pi_{t+1}/\pi_{t+1}\right)}{\Pi_t/\pi_t}.$$

Equation (60) implies that (Π_t/π_t) is a constant. Therefore, in order to establish (69) it suffices to show that $\lim_{\alpha \to 1} \operatorname{cov}\left\{\left(\pi^I_{t+1}/\pi^I_t\right), \left(\xi_{t+1}/\xi_t\right)\right\} \neq 0$. To see that this is the case note that $\lim_{\alpha\to 1}\left(\pi^I_{t+1}/\pi^I_t\right) = e^{\mu-(1+\chi)u_{t+1}}$. Hence, in order to establish (69), we need to show that ξ_{t+1}/ξ_t is a non-degenerate function of u_{t+1} as $\alpha \to 1$. Given that

$$\lim_{\alpha\to 1}\left(\frac{\xi_{t+1}}{\xi_t}\right) = \beta\left(1-\lambda\right)^{\gamma} e^{\mu(-1+\psi(1-\gamma))}\left[1-\theta^w\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho u_{t+1}}\right)\right]^{-\gamma}, \quad (70)$$

the lemma holds as long as a solution $\theta^w > 0$ exists to Equation (53), an equation that simplifies to

$$\begin{aligned}&\theta^w\left(1-\zeta\left(1-\lambda\right)\left(1-\delta\right)E\left[e^{-\rho u}\left(1-\theta^w\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho u_t}\right)\right)^{-\gamma}\right]\right)\\ &= 1-\zeta E\left[\left(1-\theta^w\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho u_t}\right)\right)^{1-\gamma}\right]. \quad (71)\end{aligned}$$

By expanding the right-hand side of (71), the equation further simplifies to

$$1 = \zeta E\left[\left(1-\theta^w\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho u_t}\right)\right)^{-\gamma}\right]. \quad (72)$$

As the right-hand side increases in θ^w, and the probability of the event $\{u_t \in (0,\epsilon)\}$ is strictly positive for all $\epsilon > 0$, conditions (34) and (35) are necessary and sufficient for the existence of a solution $\theta^w > 0$. (Note that $\theta^w \leq 1$.)

At $\theta^w = 0$, the left hand side of (71) is zero while the right hand side is positive, since $\zeta < 1$. Now let $\theta^* > 0$ be the smallest θ such that $1-\zeta\int_0^\infty\left[1-\theta^*\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho x}\right)\right]^{1-\gamma}g(x)dx = 0$, so that the right hand side of (71) is positive for all $\theta \in (0,\theta^*)$ and becomes zero when $\theta^w = \theta^*$. Note that $\theta^* < 1$, since at $\theta^w = 1$ the right hand side of (71) is negative by (35).

In order to establish the existence of a root to θ^w in the interval $(0,\theta^*)$ it suffices to show that the left hand side of (71) is positive for all $\theta \in (0,\theta^*]$. Since the term inside square brackets of (71) is decreasing in θ^w, it suffices to show that

$$1-\zeta\left(1-\lambda\right)\left(1-\delta\right)\int_0^\infty e^{-\rho x}(1-\theta^*\left(1-\left(1-\lambda\right)\left(1-\delta\right)e^{-\rho x}\right))^{-\gamma}g(x)dx > 0. \quad (73)$$

To that end, note that the integral in equation (73) can be expressed as

$$\int_0^\infty \frac{e^{-\rho x}}{1-\theta^*\left(1-(1-\lambda)(1-\delta)e^{-\rho x}\right)}\left(1-\theta^*\left(1-(1-\lambda)(1-\delta)e^{-\rho x}\right)\right)^{1-\gamma}g(x) \tag{74}$$

$$< \int_0^\infty \frac{e^{-\rho x}}{1-\left(1-(1-\lambda)(1-\delta)e^{-\rho x}\right)}\left(1-\theta^*\left(1-(1-\lambda)(1-\delta)e^{-\rho x}\right)\right)^{1-\gamma}g(x)$$

$$= \int_0^\infty \frac{1}{(1-\lambda)(1-\delta)}\left(1-\theta^*\left(1-(1-\lambda)(1-\delta)e^{-\rho x}\right)\right)^{1-\gamma}g(x) = \frac{1}{\zeta(1-\lambda)(1-\delta)}, \tag{75}$$

where the first inequality follows because $\theta^* < 1$ and the last equality follows by construction of θ^*. Inequality (75) establishes (73), which in turn implies that $\lim_{\alpha\to 1}\theta^w$ exists and is positive. ■

Proof of Lemma 2. The value of a growth firm is given by the value of all its assets in place and all its growth options.

$$\begin{aligned} P_{t,s} &= (1-\eta)\kappa\left(\int_{A_{s-1}}^{A_s}\Pi^I_{j,t}dj\right) + \sum_{n=s+1}^{t}(1-\varpi)(1-\kappa)\varpi^{n-(s+1)}\left(\int_{A_{n-1}}^{A_n}\Pi^I_{j,t}dj\right) \\ &\quad + (1-\varpi)\left[E_t\sum_{n=t+1}^{\infty}\left(\frac{\xi_n}{\xi_t}\right)\varpi^{n-(s+1)}\left(\int_{A_{n-1}}^{A_n}\Pi^I_{j,n}dj\right)(1-\kappa)\right] \end{aligned}$$

Using the definition of q, and noting that $\int_{A_{s-1}}^{A_s}\pi^I_{j,t}dj = \left(\frac{A_s}{A_t}\right)^{1+\chi}\left(1-e^{-(1+\chi)u_s}\right)$ along with the definition of θ^b in equation (62) leads to (39). ■

Proof of Lemma 3. Equation (46) implies that

$$a^i_{s+1}-a^i_s = \log c^i_{s+1,s+1}-\log c^i_{s,s}+\frac{1}{\gamma}\log\left(C_{s+1}^{(1-\psi)(1-\gamma)}\beta^{-(s+1)}\xi_{s+1}\right)-\frac{1}{\gamma}\log\left(C_s^{(1-\psi)(1-\gamma)}\beta^{-s}\xi_s\right). \tag{76}$$

Using (56) inside (76) along with $C_s = Y_s$ and simplifying give

$$a^i_{s+1}-a^i_s = \log c^i_{s+1,s+1}-\log c^i_{s,s}-\log\left(\frac{C_{s+1}}{C_s}\right)-\log\left[\frac{1}{1-\lambda}\widetilde{v}(u_{s+1};\theta^e,\theta^w,\theta^b)\right].$$

Using the definitions of a_s and z_s along with (58), (63), noting that $C_s = Y_s$, and simplifying gives

$$a_{s+1}-a_s = -\log\left[\frac{1}{1-\lambda}\widetilde{v}(u_{s+1};\theta^e,\theta^w,\theta^b)\right]+z_{s+1}-z_s. \tag{77}$$

Iterating (77) forward leads to (49). ■

B Labor input as a composite service and human capital depreciation

This section provides a more elaborate model of the labor market, that reproduces the path of labor income over an agent's life, as postulated in equations (11) and (12). The main difference between the baseline model, and the model of this section, is that the labor income process results from general equilibrium wage effects, rather than assumptions on agents' endowments of labor efficiency units.

To draw this distinction, we continue to assume that workers' endowments of hours evolve deterministically over their life according to $h_{t,s} = \overline{h}\,(1-\delta)^{t-s}$. However, the innovation shocks u_t no longer have any effects on agents' endowment of labor efficiency units.

Assume moreover, that labor is not a homogenous service. Instead, the units of labor that enter the production function of final goods and intermediate goods are measured in terms of a composite service, which is a constant elasticity of substitution (CES) aggregator of the labor units provided by workers belonging to different cohorts. Specifically, one unit of (composite) labor is given by

$$L_t = \left(\sum_{s=-\infty}^{t} v_{t,s}^{\frac{1}{b}} \left(l_{t,s}\right)^{\frac{b-1}{b}} \right)^{\frac{b}{b-1}}, \tag{78}$$

where $l_{t,s}$ denotes the labor input of cohort s at time t, $v_{t,s} > 0$ controls the relative importance of this input and $b > 0$ is the elasticity of substitution. The production function of final goods continues to be given by (3) and it still takes one unit of the composite labor service to produce one unit of the intermediate good. Equation (78) captures the idea that different cohorts have different skills and hence they are imperfect substitutes in the production process. Next, we let

$$v_{t,s}^{\frac{1}{b}} \equiv (1-\phi)^{\left(\frac{b-1}{b}\right)} q_{t,s} h_{t,s}^{\frac{1}{b}}. \tag{79}$$

Before proceeding, we note that using (79) inside (78), recognizing that in equilibrium $l_{t,s} = h_{t,s}$, and noting that $\sum_{s=-\infty}^{t} q_{t,s} h_{t,s} = 1$ implies that the aggregate supply of (composite) labor units is constant and equal to $(1-\phi)$.

Since labor inputs by agents belonging to different cohorts are imperfect substitutes, we need to solve for the equilibrium wage $w_{t,s}$ of each separate cohort. This process is greatly facilitated by first constructing a "wage index", i.e. taking a set of cohort-specific wages as given, and then minimizing (over cohort labor inputs) the cost of obtaining a single unit of the composite labor input. As is well established in the literature, this wage index for CES production functions is given by

$$\overline{w}_t = \left(\sum_{s=-\infty}^{t} v_{t,s} \left(w_{t,s} \right)^{1-b} \right)^{\frac{1}{1-b}} .$$

With this wage index at hand, the cohort specific input demands for a firm demanding a total of L_t units of the composite good are given by

$$w_{t,s} = \overline{w}_t v_{t,s}^{\frac{1}{b}} \left(\frac{l_{t,s}}{L_t} \right)^{-\frac{1}{b}} . \tag{80}$$

It is now straightforward to verify that an equilibrium in such an extended model can be determined by setting $\overline{w}_t = w_t$ (where w_t is given by [24]) and then obtaining the cohort-specific wages by setting $l_{t,s} = h_{t,s}$, and $L_t = (1 - \phi)$ in equation (80) and solving for $w_{t,s}$. To see this, note that by making these substitutions and using (79) inside (80) leads to the per-worker income process

$$\frac{w_{t,s} h_{t,s}}{(1 - \phi)} = \overline{w}_t q_{t,s} \ , \tag{81}$$

which coincides with the labor income process in the baseline model. Furthermore by setting $w_t = \overline{w}_t$, the market for total (composite) labor units clears by construction, whereas the cohort specific wages implied by (81) clear all cohort specific labor markets, since they satisfy equation (80) for all markets.

C Data Description

The CEX data are from the NBER website as compiled by Ed Harris and John Sabelhaus. See http://www.nber.org/ces_cbo/Cexfam.pdf for a detailed description of the data. In short, the data set compiles the results from the four consecutive quarterly interviews in the CEX into one observation for each household. We follow a large literature (see e.g. Vissing-Jogensen (2002)) and drop from the sample households with incomplete income responses, households who haven't completed one of the quarterly interviews, and households that reside in student housing. To ensure that data selection does not unduly affect the results, we also ran all the regressions on the raw data including dummies for reporting status and the number of completed interview quarters. The results were not affected in any essential way.

A more delicate issue concerns the definition of consumption. As Fernandez-Villaverde and Krueger (2007), we used a comprehensive measure of consumption that potentially includes durables. Specifically, we used exactly the same definition as Harris and Sabelhaus. Our choice is motivated by our model; according to the model, cohort effects are determined by the intertemporal budget constraint of agents born at different times. Accordingly, as Fernandez-Villaverde and Krueger (2007), we use total consumption expenditure in the estimation of cohort effects. To test if this choice materially affects our results, we also ran the results using non-durable consumption (total consumption expenditure excluding medical and educational expenses, housing, furniture and automotive related expenses). Using non-durable consumption the volatility of cohort effects was larger; however, there was not a big difference between the variance of the permanent components of the cohort effects, no matter which concept of consumption (total or only non-durable) we used. This is to be expected, as one would expect the two notions of consumption to share the same permanent component.

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