Dynamic Competition, Innovation and Strategic Financing

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ABSTRACT

This paper models the interactions among product market innovation, product market competition, and corporate financing decisions in the context of a dynamic duopoly. One competitor faces an opportunity to adopt a new technology. If adopted, the firm must also determine whether it will obtain public or private financing. Our results allow us to relate current firm and industry characteristics to these decision variables. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. The private versus public financing decision depends mainly on the magnitude of the technological improvement and length of the period during which private financing extends the innovator's product market advantage. Due to the model's formulation it is both tractable and amenable to empirical estimation. Tests bear out several of the model's predictions. An example of which is the relationship between the ease with which firms can entice away each others customers and the relative advantages to public or private financing.

How important are the interactions among innovation, product market competition and the going public decision?[1] This paper presents a tractable framework for examining financial decision making in dynamic competitive environments. In particular, it derives explicit, closed form solutions for a dynamic duopoly in which a firm decides whether to adopt an innovation. If adopted, the firm must determine whether it will obtain public or private financing. The model allows one to relate current firm and industry characteristics to these decision variables in ways that are empirically measurable.

Consider the package delivery market, Federal Express listed publicly on the NYSE in 1978, while UPS did not undertake its IPO until 1999 (in part, for currency to aid in its acquisition strategy). During the years preceding the IPO, there was significant industry investment in technological infrastructure (particularly logistics). We demonstrate how private status during this development stage can significantly impact value. Later, when the nature of the investment opportunities changes, public financing can become more attractive. The actual importance of competitive structure in financing decisions is an empirical question, and an important advantage of this paper is that it generates several testable hypotheses that can easily be matched with available data. Directly testable dynamic models are relatively rare in the capital structure literature although Leland (1994), Leland (1998), Goldstein, Ju and Leland (2001) are notable exceptions.[2]

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[1] In the results of their survey of CFOs, Brau and Fawcett (2006) report that "Disclosing information to competitors" and "SEC reporting requirements" ranked fourth and fifth, respectively, of 11 factors contributing to the firms' decisions to undertake an IPO. Not surprisingly, these factors were most important to firms actually undertaking the IPO as well as those that were large enough to go public but did not undertake an IPO. They were less important to firms with withdrawn IPOs.

[2] For example, Welch (2001, p. 11) writes: "Can we build a good model of, e.g., optimal corporate leverage decisions based on observable firm characteristics, and measure the effects of moving towards/away from this optimum?...I dream of models whose predictions are more quantitative than qualitative; models which are of direct use to empiricists."

Potential issuers face a tradeoff between releasing information and the promised return when deciding between a public or private financing. Public issuance in the United States involves the release of information that is potentially valuable to competitors and thus may hurt the future product market performance of the issuer.[3] On the other hand, private financing involves a limited number of investors who may require higher returns due to, for example, the relative illiquidity of their investment. Indeed, there is a well documented discount for private securities (see e.g., Hertzel and Smith (1993) for an analysis of private placements).

To address these issues, we solve a differential game based upon a variant of the Lanchester (1916) "battle" model. In our application two firms compete against each other for market share by spending funds to acquire each others customers. The adaptation we develop provides a simple, yet flexible structure for examining the dynamic interactions among product market competition, innovation and public versus private financing.

In the model, one firm has an opportunity to invest in a new value enhancing technology. Assuming the firm decides to innovate it then chooses between financing development costs with public or privately placed securities. The main benefit of private financing is that it allows the firm to extend the time during which it can hide its technological progress. This in turn extends the time it retains its competitive advantage over its rival which eventually catches up.[4]

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[3] The competitive effects of information-sharing are well-known in the industrial organization literature. Vives (1990) provides a survey. In general, sharing of cost information occurs in equilibrium under Cournot competition and does not occur under Bertrand (the results are the reverse for common value demand information). See also Li (1985) and Gal-Or (1985). There is a related accounting literature on disclosure in imperfectly competitive markets e.g., Darrough and Stoughton (1990); Wagenhofer (1990); Masako (1993)). With the exception of Bhattacharya and Ritter (1983) and the extension to debt markets in Bhattacharya and Chiesa (1995), in which a firm with private information takes into account the impact of disclosure on the ability to raise funds in financial markets and the probability of the success of a rival firm in an R&D game, these issues have received little attention in the finance literature. Note, also, that these papers do not model dynamic competitive interactions.

[4] Maksimovic and Pichler (2001) also model the advantage of privately placed securities as reducing the information available to competitors.

The model's duopoly setting allows it to produce predictions regarding the competitive environment's impact on innovation and financing decisions; insights that are obviously impossible to derive in a single firm model. It is also possible to examine how the type of innovation influences a firm's financial and marketing decisions. Profits in the model depend both upon the relative ability of each firm to lure away its rival's customers, and the profit per unit of market share the firm obtains from those customers. Consider, for example, an innovation of the former type that makes it easier for a firm to attract market share. In this case, the model finds that increasing a rival's ability to attract customers can encourage innovators to use public financing. Many other questions like these can be addressed as well, such as the impact of technological innovations and financing decisions by one firm on the value of another.

The paper's results indicate that the relative size of the firms within an industry plays an important role in the innovation decision. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. Intuitively, this is because small, less profitable firms are less able to withstand the aggressive competitive behavior by rivals that their own innovation triggers. The positive association between firm size and the number of innovations predicted by the model has been documented empirically (see e.g., Acs and Audretsch (1988), Henderson and Cockburn (1996), and Nahm (2001)).

Innovations that increase a firm's profits per unit of market share lead to dynamics that differ in fundamental ways from those that increase the attractiveness of its product. Once it becomes public knowledge that a firm has an innovation of the former type, then all firms in the industry increase their spending on market share acquisition. The innovator spends more because market share is now more valuable and the rival spends more to defend its position. Of course, both would prefer to spend less, but cannot credibly commit to doing so. A pooling equilibrium thus becomes both possible and attractive to *both* firms. In this case, the innovating firm secures private financing (which helps to hide its profitability and thus its new status) and then chooses the equilibrium spending of a less profitable firm (i.e., firm without the new technology). Here, private financing allows this technologically superior firm to remain within an equilibrium in which it spends less on acquiring market share. This in turn reduces the rival's market share spending leading to overall increased instantaneous earnings. Note it pays for the rival to adopt a "hear no evil, see no evil" strategy since finding out the truth would actually reduce its profits.

An important advantage of the model developed here is that it is well-suited for empirical analysis. The paper highlights this by using the model's structure to first estimate a measure of the ease with which firms can acquire market share (ϕ). Higher values of ϕ make it easier to acquire market share which the model indicates should generate stronger incentives to remain private. The empirical analysis seems to verify this. High ϕ industries attract more private financing rounds than low ϕ industries, which may help delay the date on which their firms need to go public. The results appear to be both statistically and economically significant. For example, SIC code 22 (tobacco) has a value of ϕ equal to about 0.21 while SIC code 70 (hotels and lodging) has an estimated ϕ of about 0.032. Based on our parameter estimates this then generates about a 1% greater chance per year that any one firm in SIC industry 22 will obtain an additional round of venture capital financing relative to a firm in SIC industry 70. Of course, this assumes that all the other empirical parameters are equal.

In addition to testing relationships between ϕ and financing patterns across industries, we also calibrate the model to examine differences in incentives to innovate and finance privately within industries. The calculations suggest that the value of private financing can be economically significant but that there is also substantial within-industry variation that depends on current market share, spending effectiveness and profitability.

Within the existing literature the paper that comes closest to this one is Maksimovic and Pichler (2001). They examine the public-private financing decision within an industry that produces a homogenous good over two periods and for which there is costly entry and exit. In contrast, this paper examines a duopoly in a heterogeneous goods industry over an infinite horizon. Other contrasts between the papers are discussed later on in the text.

The paper is organized as follows. Section I presents the basic model. Section II examines the solution in the infinite horizon case. Section III.A looks at the case where an innovation improves a firm's ability to attract customers away from its rival. Section III.B examines the case where the innovation increases a firm's profit per unit of market share. Section IV contains the empirical analysis. Section V examines the relationship between this paper and the prior literature. Section VI concludes. Finally, the Appendix contains details regarding the derivation of the model's equilibrium.

I. Model

A. Players, Timing, Dynamics and Strategies

The Lanchester (1916) battle model was originally designed to study military strategy. Since then variants have been widely used in the marketing literature to examine advertising strategies (see e.g., Erickson (1992); Erickson (1997); Fruchter and Kalish (1997); for a review, see Dockner, Jørgensen, Van Long, and Sorger (2000)).[5] Here it is adapted to produce a differential game within which to explore competition among duopolists over new innovations and financing choices.

Consider two risk neutral value maximizing firms battling for market share. Call the firm that faces the innovation and financing decision "Firm 1." The rival is "Firm 2." Let $u_i(t)$ be the dollars spent by firm $i \in \{1,2\}$ on gaining market share at instant t. Let s_i denote the effectiveness of spending. Note that spending to acquire a competitor's customers (u_i) can imply a wide range of activities including advertising, new product design, store openings and R&D. The s_i parameters can represent the relative attractiveness of each firm's product and/or the relative quality of their marketing campaigns.

The market share of Firm 1 at time t is denoted $m(t)$. Firm 2's market share is then $1 - m(t)$. Time is continuous and there is a finite starting point at $t = 0$. Given the initial condition $m(0)$, m evolves as follows:

$$dm = \frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{s_1u_1 + s_2u_2}dt \tag{1}$$

where ϕ represents the speed with which consumers react to each firm's entreaties. Intuitively, (1) says that the variation in Firm 1's market share is simply the difference between what it gains from Firm 2's market share and what it loses to Firm 2.[6] The market share of Firm 1 increases with its own spending and effectiveness (u_1 and s_1, respectively) and decreases with spending and effectiveness of the competitor's spending.

[5] Although, to our knowledge, not in the form presented here.
[6] The model can be modified to include a stochastic *dm*. The results are unchanged since the firms are risk neutral profit maximizers.

Note that high current $m(t)$ gives Firm 1 "more to lose" to Firm 2 and as a result makes it easier for Firm 2 to gain market share.[7] Since this paper seeks to examine economic outcomes within industries that are natural oligopolies an assumption about consumer behavior like this is needed. If the industry is characterized by positive network externalities then it is a natural monopoly. In this case once a firm's market share reaches a tipping point it eventually acquires all of the market. But, in a natural oligopoly that cannot be the case. Instead, it must be that some consumers naturally find a particular firm's product more attractive even if most use its rival's product. For example, McDonalds is the largest fast food restaurant chain in the U.S. Nevertheless, many people only eat at its rival Burger King. Thus equation (1) implies that this is a model where firms produce heterogeneous products and consumers have heterogeneous preferences.

Instantaneous profits are assumed to be proportional to market share. Let α_i denote the revenue generating ability of firm i per unit of market share. Profits π equal revenues minus both spending on market share competition and a fixed operating cost f_i:

$$\begin{aligned} \pi_1(t) &= e^{gt}\left(\alpha_1 m(t) - u_1(t) - f_1\right) \\ \pi_2(t) &= e^{gt}\left(\alpha_2(1-m(t)) - u_1(t) - f_2\right) \end{aligned} \tag{2}$$

The term g represents the industry's rate of growth. It is assumed that as the industry grows larger profits and costs grow proportionately. Firm 1 is currently financially constrained and has secured financing sufficient only to finance the current equilibrium path.[8]

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[7] In the marketing literature researchers tend to use as the law of motion either $dm/dt = u_1(1-m)-u_2m$ or $dm/dt = u_1\sqrt{1-m} - u_2\sqrt{m}$ (Dockner et al. (2000)). One advantage of using (1) instead is that it is unit free. This eliminates the problem that changing the unit of currency also changes the rate at which m changes over time.

[8] Note, that spending by each firm does not impact the industry growth rate. Thus, the model should be thought of as applying to an industry in which innovations tend to change customer loyalties rather than increase overall demand. For example, an easier to swallow aspirin will probably cause consumers to switch brands but seems

To help streamline the exposition details regarding the derivation the model's equilibrium conditions can be found in the Appendix. There a general version is solved. Each of the following sections then employs that general solution to discuss the interactions between firms and their financial structure in various special cases. Thus, in the main body of the paper equilibrium conditions are simply stated without proof except for occasional references back to the Appendix.

B. The Equilibrium Value Functions

Let r denote the instantaneous discount rate. Assume $r>g$ and let $\delta=r-g$. Assume neither firm ever exits. Following standard practice in the literature on differential games the analysis seeks a Nash equilibrium in which the players use Markovian strategies (see Dockner, Jørgensen, Van Long, and Sorger (2000)). The Appendix shows that each firm's value function V_i at time t (i.e., the present discounted value of each firm's profit stream conditional on the equilibrium strategies) can be written as:

$$V_i\left(m,t\right)=a_i\left(t\right)+b_i\left(t\right)m \qquad (3)$$

within the scenarios considered in this paper. The terms a_i and b_i are functions of time and as shown in the Appendix equal:

$$a_1(t)=\delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2}-f_1\right]+C_1 e^{\delta t}, \qquad (4)$$

$$a_2(t)=\delta^{-1}\left[\frac{\phi\alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2}+\frac{\delta\alpha_2}{\phi+\delta}-f_2\right]+C_2 e^{\delta t}, \qquad (5)$$

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unlikely to lead to an overall increase in pill consumption. One can modify the model to allow g to depend on the u_i but at the cost of a closed form solution.

$$b_1(t) = k_1 e^{-(\phi+\delta)(T-t)} + \alpha_1(\phi+\delta)^{-1}, \tag{6}$$

and

$$b_2(t) = k_2 e^{-(\phi+\delta)(T-t)} + \alpha_1(\phi+\delta)^{-1} \tag{7}$$

where the constants C_i and k_i depend upon a particular problem's boundary value conditions (i.e., the value of the V_i terms at some terminal date T).[9]

II. Full Information, Infinite Horizon Equilibrium

The simplest version of the model involves two firms that do not innovate, do not need outside financing, and compete over an infinite horizon. Since the game lasts forever the solutions to the model must be time independent. Thus, in equations (4) through (7) the C_i and k_i must all equal zero.

Since $\partial V_i / \partial m$ equals b_i one can now plug equations (6) and (7) (with k_i equal to zero) into (34) and (35) to find each firm's equilibrium spending on customer acquisition of:

$$u_i^* = \frac{\phi \alpha_i^2 \alpha_j s_1 s_2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} \tag{8}$$

for i equal 1 or 2 and $j \neq i$ (see the Appendix for their derivation). The optimal controls (u_i^*) are dependent on the discount rate net of growth ($\delta = r - g$), each firm's revenue generating ability (α_i), their spending efficiency (s_i), and the speed with which consumers react to their attempts to gain market share (ϕ). Further, the marginal value of market share for a given firm (b_i) depends only on its own firm revenue generation ability α_i.

[9] There are, of course, boundary conditions under which the solutions given above will not hold. However, for the problems considered in this paper equations (4) through (7) fully characterize each firm's equilibrium value function.

To gain further insight into the impact of the model's parameters on equilibrium behavior consider what happens in the steady state: $dm = 0$. From (1) if dm equals zero, then the steady state market share m^* equals,

$$m^* = \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}. \qquad (9)$$

Clearly, both increased revenue generating ability and the efficiency of spending increases a firm's equilibrium market share. Overall industry concentration (sum of squared market shares) increases in $|\alpha_1 s_1 - \alpha_2 s_2|$. Thus one can think of the product of $\alpha_i s_i$ as a firm's competitive ability and the difference $\alpha_i s_i - \alpha_j s_j$ as i's competitive advantage relative to its rival. Note, that ϕ drops out of (9). In the long run it is irrelevant how long consumers take to react to each firm's attempts to acquire market share so long as they react at all.

Setting the C_i and k_i to zero in (4) through (7) and plugging the resulting values into (3) allows one to write out the explicit solutions for the value functions in the current case. An examination of the results yields the counter intuitive conclusion that if m is at its steady state value then decreasing consumer responsiveness (ϕ) increases the value of both firms. The reason for this can be found in the equilibrium values of u_i and the fact that the steady state value of m does not depend on ϕ. (For the latter see equation (9).) From (8) both firms will reduce their spending on market share competition if consumers become less reactive. Thus, both firms benefit from ϕ's reduction since they then earn the same steady state revenue stream while wasting fewer resources trying to lure away each other's customers. Formally then,

$\partial V_i / \partial \phi|_{m=m^*} < 0$ for both firms.[10] Effects of this type are easily seen in real industries. For example, if beer drinkers exhibited greater loyalty to particular brands brewers would undoubtedly advertise less, and collectively earn higher profits. From 1981 to 2004 per capita beer consumption in the U.S. fell from 24.6 to 21.6 gallons despite heavy product advertising (USDA, 2005). But, no one brewer can reduce its own spending without losing customers to competitors. Thus, in equilibrium, they end up advertising just to retain their current market share even amid stagnant sales. Compare this to the situation in, for example, natural gas distribution where consumers are locked into a single supplier and thus these firms do relatively little advertising.

III. Financing and Innovation

Within the model firms can improve their profitability by adopting innovations that increase their value of s and α. But changes in each offer potentially very different strategic options. The s_i represent the effectiveness of each firm's spending to garner market share. In real economic terms, an increase in s_i should thus correspond to an increase in the attractiveness of i's product. This can occur either through an innovation (such as a washing machine that cleans faster) or an improved advertising campaign (Nike's use of Michael Jordan as a spokesman). Whatever the case, it is clear that unless consumers are aware of the innovation or advertising campaign then s_i cannot change. But if changes to s_i can only occur if consumers know of it then it must also be the case that any rival firm must know as well. In line with this economic reality, within the model, the current value of each firm's s is always common knowledge. Now consider α_i which represents a firm's profits per unit of market share. Unlike s there is no reason α must be

[10]The basic model can be extended to examine competition among N firms. The resulting value functions are qualitatively similar to the two-firm case.

publicly known at all times, at least until some sort of financial report (like a 10-Q) is released. This means that changes in *s* and α can lead to very different equilibrium dynamics both because they influence profits in unique ways and because they have different implications regarding the equilibrium information sets. These issues are explored in the next two sections of the paper.

A. Innovations in s

The innovation game analyzed here assumes that at time 0 Firm 1 observes an opportunity to improve its spending effectiveness from s_1 to s_1^* at time T_1. To develop the technology Firm 1 must first incur a cost of Z. Firm 1 is currently private. To make the public/private financing question interesting, assume it only has enough capital to sustain equilibrium spending in the current competitive environment. If Firm 1 decides to innovate, it must secure financing of Z for the project.

At time zero Firm 1 must choose among the following strategies:[11] (i) innovate and finance publicly, (ii) innovate and finance privately, or (iii) do not innovate. If Firm 1 decides not to innovate spending effectiveness remains at S={s_1, s_2} forever. Firm values are then given by the solutions derived in Section II. Adopting the technology means that, after a period of development from T_0 to T_1, Firm 1 enjoys a first mover advantage in the product market. This competitive advantage does not last forever; Firm 2 eventually copies the technology and increases its spending effectiveness to $s_2^* > s_2$. The paper assumes that the relative spending effectiveness remains constant whenever both firms employ the same underlying technology. That is, $s_1 / s_2 = s_1^* / s_2^*$. Under this assumption, competition eventually drives each firm's

[11] We are assuming that the opportunity to innovate expires immediately if it is not taken. This might be expected in actual product markets. For example, when technologies can be patented, a firm's decision to abandon a potential development may open opportunities for other firms to profit from development. For a model that assumes that the option to go public at a later date is valuable, see e.g., Benninga, Helmantel and Sarig (2005).

profitability back to pre new technology levels. However, markets experience real efficiency improvements via the adoption of the innovation.[12]

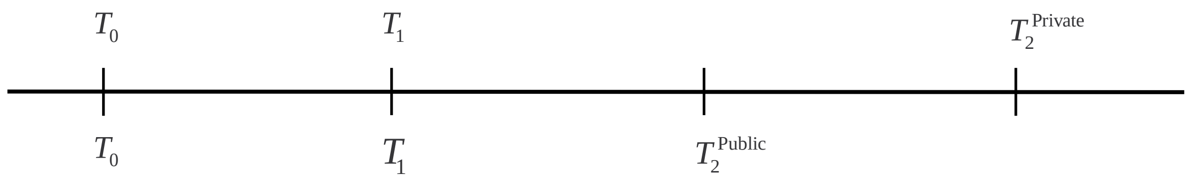
The public versus private financing decision has important value implications. Public financing is cheaper than private financing (i.e., there is a private market discount). This is due to the smaller pool of investors in private markets as well as the relative illiquidity of these markets. On the other hand, if Firm 1 decides to finance the project via a public offering, regulatory disclosure requirements force him to reveal investment behavior and financial data that increases the speed at which Firm 2 is able to successfully copy and adopt the technology.[13]

To capture the above conditions assume that if Firm 1 is privately financed Firm 2 will not be able to duplicate the technology until date T_2^{Private}. Conversely, if Firm 1 instead finances publicly then the required disclosures allow Firm 2 to begin the process of copying the technology somewhat earlier. In this case, Firm 2 successfully adopts the innovation by T_2^{Public} where the relative dates satisfy $T_2^{\text{Private}} > T_2^{\text{Pulic}} > T_1$. Upon Firm 2's successful adoption of the new technology s_2 increase to s_2^*.

[12] It is easy to relax this assumption. However, we believe that $s_1 / s_2 = s_1^* / s_2^*$ is more realistic since it captures the idea that, in competitive markets, innovations are eventually adopted by all firms in an industry.

[13] We assume that copying is costless. It might be more realistic to assume that the cost of copying is positive (though much less than *Z*). As long as the copying cost is sufficiently small that Firm 2 finds it worthwhile to copy, results and intuition regarding Firm 1's innovation and financing decision are identical to the costless copying case.

Figure 1: Innovation and Financing: Timeline



T_0: T=0 Firm 1 decides whether to adopt innovation s_1^*. If innovation occurs, he decides how to finance it (publicly or privately). Development stage begins and S={s_1, s_2}. If no innovation, firms play full information, infinite horizon differential game with S={s_1, s_2}.

T_1: Firm 1's innovation becomes effective. S={s_1^*, s_2}.

T_2: Firm 2 successfully copies and adopts the innovation. S={s_1^*, s_2^*}.

Consider Coley Pharmaceutical's recent IPO. (Lähteenmäk and Lawrence (2006) report that this was the sector's largest U.S. IPO in 2005). At time zero, Coley observes the opportunity for a breakthrough in cancer immune therapy; if financed, it takes until T_1 to bring the technology to market. If privately financed, it will retain a competitive advantage until date T_2^{Private}.[14] If publicly financed, rivals can observe Coley's investment patterns and perhaps glean other information from its mandated disclosures and "road show" documents to begin early development of the technology.[15] This then leads to the erosion of Coley's competitive advantage on date T_2^{Public}. In real life, Coley initially financed the development privately and waited until T_1 (market stage when there was no longer any information to hide) to undertake the

[14] Despite the fact that innovation abounds in the biotechnology industry this example fits well with the model's assumption that the innovation primarily draws customers from rivals rather than speeds up industry growth. While people with cancer may switch to Coley's treatment it seems highly improbable that consumers will now seek to increase their consumption of cancer related drugs.
[15] While patents help they do not block competitors from developing competing products. Once a drug shows promise rival firms see if they have drugs in their pharmacies that have similar effects and that might therefore generate similar treatment benefits. It is not a coincidence that there are now over a dozen cholesterol reducing statins on the market.

public offering.[16] Indeed, this is consistent with general empirical evidence suggesting the importance of private information in the entire biotech sector. Even for IPO firms, there is evidence that the amount of information that is disclosed at IPO is sensitive to proxies for the value of limiting disclosure. Guo, Lev and Zhou (2004) create an index of disclosure by biotech IPO firms in their prospectuses. They find a negative relationship between the amount of disclosed information and common proxies for information asymmetry.[17]

1. Value and Incentives to Innovate: Public Financing

To determine Firm 1's optimal decision rule one needs to compare its value under each of the three possible scenarios: (1) do not innovate; (2) innovate, finance publicly; and (3) innovate, finance privately. As usual, the solutions to each game are obtained by working backwards from when both firms successfully adopt the innovation. Since the intermediate steps provide limited economic insight they are relegated to the Appendix. After working through the algebraic details the value functions at t=0 under public financing are:

$$\begin{aligned} V_1^{Public}(T_0) &= \frac{\alpha_1}{\delta(\phi+\delta)} \frac{\phi\alpha_1^2 s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2} \\ &+ e^{-\delta T_1} \frac{\phi\alpha_1}{\delta(\phi+\delta)} \left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{\alpha_1^2 s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2} \right] \\ &+ e^{-\delta T_2^{Public}} \frac{\phi\alpha_1}{\delta(\phi+\delta)} \left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} \right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta} m(0) \end{aligned} \tag{10}$$

and

[16] Of the Top 10 IPOs of biotechnology firms of 2005, Lähteenmäk and Lawrence (2006) report that: 1 had a product at the market stage; 5 had products in Phase 3 development; and 4 firms had products in Phase 2. Importantly, none had products in Phase I development, pre-clinical testing or discovery. An important observation is that the industry has two large incumbents: Amgen and Genetech. Our model provides the testable implication that the relative dominance of these two firms plays an important role in the going public decisions made by others in the industry.

[17] The competitive benefits of private financing are not unique to the biotechnology sector. For example, Google's S-1 filing (for its 2004 IPO) contains Management's discussion of the advantages of initial growth as a private firm: "As a smaller private company, Google kept business information closely held, and we believe this helped us against competitors."

$$V_2^{Public}(T_0)=\frac{\alpha_2}{\delta(\phi+\delta)}\frac{\phi\alpha_2^2 s_2^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}$$
$$+e^{-\delta T_1}\frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}-\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right] \tag{11}$$
$$+e^{-\delta T_2^{Public}}\frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}\right]-\frac{f_2}{\delta}+\frac{\alpha_2}{\phi+\delta}\big(1-m(0)\big)$$

for Firms 1 and 2 respectively.

While equations (10) and (11) look rather daunting they are in fact quite simple. The first term on the right represents the present value of each firm's profits until Firm 1's new technology comes on line. The second term equals the present value of each firm's profits during the period when Firm 1 has a technological advantage. The third term represents the final time period when Firm 2 catches up technologically. The final set of terms adjust the value function for the firm's fixed operating costs and current market share. Intuitively, one can think of the term $\alpha_i s_i/(\alpha_1 s_1+\alpha_2 s_2)$ as representing a firm's relative competitive strength ($\alpha_i s_i$) exclusive of the impact of its current market share. As time progresses, the s_i terms change in each fraction to track each firm's current technological level.

The observation that competitive environments greatly reduce the potential value of technological improvements routinely appears in the statements made by high-tech company executives. Their common complaint is that development of Microsoft compatible software is hindered by competition with Microsoft itself. The scenario typically outlined is that of a firm which produces an innovation and succeeds in acquiring customers. If this happens the claim is that Microsoft then copies the innovation (by building it into its existing products) and thereby steals away the innovator's market share. For example, a 2001 article on CNET News.com starts with,

> The battle over today's instant messenger market is vintage Microsoft, whose strategy enemies call "the three E's" in a parody of the company's marketing mantra: Embrace a rival's technology, extend it to work best with Windows, and extinguish the competition.
>
> Hu (2001)

This is the kind of industry dynamics captured in the value functions (10) and (11). Initially, a Microsoft rival creates an innovation at time T_0 that allows it to better capture market share. At time T_1, the innovation enters the firm's production function thus increasing s_1. Eventually, though, Microsoft discovers a way to incorporate the innovation into its own product line at date T_2 which then increases s_2 and eliminates the innovator's competitive advantage. Compared to a single firm setting, where T_2 is effectively set to infinity, the above scenario can greatly reduce, if not eliminate, an innovation's value to its discoverer.

2. Comparative Statics: Incentives to Innovate

Before examining the question of how an innovation should be financed the first question to be addressed is how competitive forces impact the decision to innovate at all. This section thus examines how the incentives to innovate vary with the size of the innovation and with the size/profitability of Firm 1 and Firm 2.

The main results presented so far demonstrate how competitive pressures in real markets can significantly reduce the value of innovations that might appear worthwhile if considered in isolation. The comparative statics analysis allows us to shed additional light on the issue of where (within industries) innovations can be expected to occur. To make it clear whether or not Firm 1 has a competitive advantage within each equation let $\psi_1 = s_1 / s_2 = s_1^* / s_2^*$ and $\psi_2 = s_1^* / s_2$. Then one can rewrite the value function of Firm 1 as:

$$V_1(T_0) = \frac{\phi\alpha_1^3\psi_1^2}{\delta(\phi+\delta)(\alpha_1\psi_1+\alpha_2)^2} +$$
$$e^{-\delta T_1}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{\alpha_1^2\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^2}\right] + \tag{12}$$
$$e^{-\delta T_2}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^2} - \frac{\alpha_1^2\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^2}\right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta}m(0).$$

The derivative of Firm 1's value function with respect to the magnitude of the competitive advantage during period T_1 to T_2 is thus:

$$\frac{\partial V_1(0)}{\partial \psi_2} = \left[\frac{\psi_2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{\alpha_1\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]\left[\frac{2\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})}{\delta(\phi+\delta)}\right]. \tag{13}$$

This is clearly positive (as would be expected). What is of greater interest is the question of how the incentive to innovate varies with Firm 1's characteristics, in particular, Firm 1's revenue generating ability (α_1):

$$\frac{\partial^2 V_1(0)}{\partial \psi_2 \partial \alpha_1} = \left[\frac{6\alpha_1^2\psi_2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{4\alpha_1^3\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3} - \frac{8\alpha_1^3\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3} + \frac{6\alpha_1^4\psi_2^3}{(\alpha_1\psi_2+\alpha_2)^4}\right] \times \left[(\delta(\phi+\delta))^{-1}\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})\right]. \tag{14}$$

This is always greater than zero and implies that larger, more profitable firms (recall that the equilibrium size of Firm 1 is $m^* = \alpha_1 s_1 / (\alpha_1 s_1 + \alpha_2 s_2)$), are more likely to adopt new technologies.[18] Basically, larger firms are already superior competitors: that is why they are large to begin with. The increase in *s* thus allows them to draw away a substantial number of

[18] The relationship between firm size and innovative activity has long been the subject of academic debate. See Kamien and Schwartz (1975) for a survey of early work and a discussion of the Schumpterian hypothesis that product market rivalry will impact innovation incentives. Acs and Audretsch (1987) find that large firms have the innovative advantage in industries that are capital-intensive with high concentration and advertising expenditure. They conclude that large firms have an advantage in imperfectly competitive industries, while small firms have a greater advantage in perfectly competitive industries. More recently, Hall (2005) traces some of the basic ideas investigated here to Schumpeter: that innovations can be copied by rival firms thereby decreasing incentives to invest. We not only explicitly model this possibility, but do so in a dynamic continuous time model.

additional customers. Then, even after they lose their competitive advantage at date T_2 it still takes the rival some time to reclaim its long run market share.

Now consider the incentive to innovate as a function of the rival firm's characteristics:

$$\frac{\partial^2 V_1(0)}{\partial \psi_2 \partial \alpha_2} = \left[\frac{-4\psi_2}{(\alpha_1\psi_2 + \alpha_2)^3} + \frac{6\alpha_1\psi_2^2}{(\alpha_1\psi_2 + \alpha_2)^4}\right]\left[\frac{\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})}{\delta(\phi+\delta)}\right]. \tag{15}$$

Equation (15) is greater than zero when $\alpha_1\psi_2 - 2\alpha_2 > 0 \quad \Rightarrow \quad \alpha_1 s_1^* - 2\alpha_2 s_2 > 0$. Therefore, for small firms, an increase in the competitors' profitability and size makes the innovation less valuable. This is because an increase in Firm 2's α or s increases its marginal value of market share and makes Firm 2 more aggressive. This can impose a cost that is greater than the potential gains from adopting the technology. A similar exercise shows that increasing consumer responsiveness increases the value of any new innovation; differentiating (13) with respect to ϕ yields $\partial^2 V_1(0)/\partial\psi\partial\phi > 0$. Also note that an increase in ϕ is more valuable to smaller rivals. Translating this to the available data, ϕ should play a stronger role for firms small enough to make good use of venture capital financing (which can be limited in size) than for firms big enough to consider going public. The empirical analysis presented later on appears to confirm this prediction. For succinctness, Table 1 summarizes the comparative static results given above and a few others as well.

Maksimovic and Pichler (2001) examine closely related issues. They consider financing choice in a growing industry and the potential for herding by potential entrants. The cost of disclosure is letting rivals know how lucrative the business is. Their main result is that private financing occurs when start-up costs are high and when there is a high probability of displacement by a superior rival; public financing occurs when the technology is not costly and when the probability of displacement is low. Our focus differs. We study financing choice in a model with dynamic competitive interactions and a finite interval over which a competitive advantage can be maintained. We also explicitly examine the potential value implications of the existence of a strong rival for a growing firm in an industry with heterogeneous goods.

An advantage of the model developed here is that it can easily be fit to data that are readily available. Besides offering a potentially rich set of cross sectional predictions for future testing it also allows for the investigation of competitive dynamics along the equilibrium path. This too is well suited for real data. Because competition evolves through time one can use the model to make better use of the available panel datasets chronicling stock returns and corporate accounting statements. One possible mapping between the model's parameter values and variables available on CRSP and COMPUSTAT can be found in Table 2.

3. Private versus Public Financing

We now turn to the question of how potential innovators will finance their investment. In this case the value functions at time zero are analogous to those under public financing and given by equations (10) and (11). The only difference is that the T_2^{Public} terms are replaced by T_2^{Private}. Label the value functions with this change V_i^{Private}. While the forms of the value functions are identical to the public financing case keep in mind that $T_2^{\text{Private}} > T_2^{\text{Public}}$ and thus Firm 1 enjoys first mover advantages for a longer time under private financing.

In concurrence with the empirical literature the model assumes that private financing is more costly than public financing (see e.g., Hertzel and Smith (1993) for evidence of this in the context of private placements). The simplest way to capture the increased cost of private financing is to build it directly into the technology's implementation cost. Thus, if the new technology costs Z to implement under public financing assume that its cost increases to $Z(1+D)$ under private financing. Here, D is the private market discount. Assuming the benefits from adopting the innovation exceed its costs then some algebra shows private financing is preferred when:

$$\left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right]\frac{\phi\alpha_1^3}{\delta(\phi+\delta)}\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2}\right] - ZD > 0. \tag{16}$$

4. Comparative Statics: Private versus Public Financing

This section examines how the incentives to finance publicly vary with firm characteristics. Recall from Section 2 above that $\psi_1 = s_1 / s_2 = s_1^* / s_2^*$ and $\psi_2 = s_1^* / s_2$. With this, rewrite the value of private financing as:

$$V_1^{\text{Private}} - V_1^{\text{Public}} = \left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right]\frac{\phi\alpha_1^3}{\delta(\phi+\delta)}\left[\frac{\psi_2^2}{(\alpha_1\psi_2 + \alpha_2)^2} - \frac{\psi_1^2}{(\alpha_1\psi_1 + \alpha_2)^2}\right] - ZD \tag{17}$$

We are interested in the cross-sectional relationships among the available financing choices and firm characteristics. First consider Firm 1's revenue generating ability, α_1. To simplify the notation, let $q = \left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right](\delta(\phi+\delta))^{-1}\phi$. Then:

$$\frac{\partial(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1} = q\alpha_1^2\left[\frac{\psi_2^2(\alpha_1\psi_2 + 3\alpha_2)}{(\alpha_1\psi_2 + \alpha_2)^3} - \frac{\psi_1^2(\alpha_1\psi_1 + 3\alpha_2)}{(\alpha_1\psi_1 + \alpha_2)^3}\right]. \tag{18}$$

As shown in the Appendix equation (18) is positive for all $\psi_2 > \psi_1$. Therefore, the incentive to secure private financing is increasing in Firm 1's revenue generating ability (α_1). Intuitively, this occurs because high α firms are in the best position to use any technological advance to aggressively pursue market share. Equation (18)'s prediction that the most lucrative projects should be financed privately is consistent with findings of operational underperformance of IPO firms in the years following issuance (e.g., Loughran and Ritter (1995)).[19] That is, initially firms finance their very high ψ_2 projects privately and then go to the public markets when only more modest ψ_2 innovations remain.

Now consider the private financing choice as a function of the rival's revenue generating ability (α_2):

$$\frac{\partial(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial \alpha_2} = 2q\alpha_1^3\left[\frac{\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^3} - \frac{\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]. \qquad (19)$$

The sign of equation (19) depends on the relative values of α_1 and α_2. Incentives to remain private are decreasing in α_2 when $\alpha_2 > \alpha_1\psi/2$, and increasing in α_2 when α_2 is small (i.e., $\alpha_2 < \alpha_1\psi/2$). This is because profitable rivals (high α_2) will spend more aggressively during the period of Firm 1's first mover advantage, making it difficult for Firm 1 to capture market share. On the other hand, when α_2 is small, the benefits from extending the period of first-mover advantage outweigh the costs of higher equilibrium spending.

There are several additional observations. First, Firm 1's incentives to finance privately increases in the technological advantage the innovation provides (i.e., in ψ_2). Second, as one might then expect, the opposite is true of ψ_1. An increase in Firm 1's initial (and final) spending effectiveness relative to Firm 2 decreases Firm 1's incentive to issue private securities. From equation (17), observe that the incentives to remain private decrease when real interest rates are high. This provides an explanation for expected IPO patterns based on economic fundamentals that is distinct from the market timing arguments in Baker and Wurgler (2000).

Third, the incentive to remain private increases as the time gained from delaying Firm 2's adoption increases (this can also be interpreted as an increased incentive to avoid disclosure

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[19] Chemmanur, He and Nandy (2007) report that total factor productivity of IPO firms increases prior to the IPO and then decreases in the years following the IPO. This peak in productivity at the time of the IPO is consistent with the prediction that the most lucrative projects are financed privately.

requirements).[20] That is: $\partial(V_1^{\text{Private}} - V_1^{\text{Public}}) / \partial T_2^{\text{Private}} > 0$ and $\partial(V_1^{\text{Private}} - V_1^{\text{Public}}) / \partial T_2^{\text{Public}} < 0$. This has implications for patent policy. Clearly, innovation can be good for consumers. Extending the incentive to innovate and finance publicly (via extending patent life and thus increasing T_2^{Public}) encourages innovation and eliminates the costs of private financing. Furthermore, because the model's parameters have real world analogs it can potentially help quantify to what degree an increase in patent protection will encourage the use of lower cost public financing.

Fourth, increasing consumer responsiveness (ϕ) increases the incentive to finance privately. The empirical section that follows offers some support to for this prediction. Fifth, the private market discount decreases Firm 1's incentives to remain private as it clearly makes such financing more costly. One can find a summary of the section's results as well as several additional comparative statics in Table 3.

B. Innovations in α: Hear no Evil, See no Evil.

The prior sections have dealt with an innovation that provides Firm 1 with an opportunity to profit by increasing s_1. While changes to s_1 must be immediately observable if consumers are going to react, the same does not hold for changes in α. This brings up new strategic possibilities for firms that may wish to hide their new and improved status.

Assume that Firm 1 has an opportunity to immediately raise α_1 to α_1^* at some cost Z which is assumed to be low enough that it pays to innovate. As before, Firm 1 can finance the innovation publicly or privately. For simplicity assume that if it is publicly financed, all

[20] An obvious implication is that, if the private market discount is sufficiently high (making private financing prohibitive), disclosure requirements can inhibit innovation.

information regarding fundamentals (α_1) becomes common knowledge immediately. If the technology is privately financed, Firm 1 can try to "hide" and operate as though it were a low type firm (i.e., it pretends no innovation has occurred) until some finite time T.[21] At time T the true α_1 becomes common knowledge (e.g., through taxes or some other public signal revealed to the market). From there on in both firms play the infinite horizon game knowing the true alpha values.[22]

In the pooling equilibrium focused on here Firm 2 behaves as if Firm 1 is a low type for sure, and Firm 1 behaves as if it is the low type whether or not it truly is. (See the Appendix for a proof that no firm will defect from the proposed equilibrium strategies.) There are two reasons to concentrate on this particular pooling equilibrium: First, under the sufficient condition for its existence given below it is the Pareto dominant equilibrium. Second, if one thinks of the innovation as a true surprise then it is the only Markov equilibrium. Since many innovations are indeed surprises (very low probability events) then this may be the economically most important case to consider when looking at data.

The potential incentive for the high type to hide comes from the fact that optimal spending (u_1^*) is increasing in α_1. The high type firm would like to credibly commit to spend less on gaining market share (as long as Firm 2 also does so), but cannot do so when there is full information. However, if the conditions for a pooling equilibrium exist then the lack of complete information can make lower spending individually rational. Firm 1 can then continue to hide its

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[21] With some additional algebra (but little additional insight) one can assume that private financing delays the date on which α_1^* becomes common knowledge until date $T^{Private}$ while with public financing the information is revealed at date T^{Public} with $T^{Public}<T^{Private}$.

[22] Unlike the analysis of innovations in s found in Section III.A, this section assumes that firm 2 cannot copy the innovation. It is, however, easy to extend the analysis to case where it can by simply adding a date T_2 at which, for example, α_1^* drops back to α_1. In this case copying does not make the industry more profitable. Rather it just eliminates any competitive advantage one firm has over the other. Other scenarios are also possible.

new status by spending less on obtaining market share. Pooling will be profitable if the gains from keeping u_1 low outweigh the opportunity cost of a more aggressive campaign. Naturally, the cost is that Firm 1's market share increases at a slower rate. As the Appendix shows a sufficient condition for the pooling equilibrium to exist is that

$$\frac{(\alpha_1^* - \alpha_1)\alpha_1}{(\alpha_1 s_1 + \alpha_2 s_2)} - s_1\left[\frac{\alpha_1^{*3}}{(\alpha_1^* s_1 + \alpha_2 s_2)^2} - \frac{\alpha_1^3}{(\alpha_1 s_1 + \alpha_2 s_2)^2}\right] > 0 \tag{20}$$

holds. When equation (20) holds a high type Firm 1 is better off under pooling than under separation. Note that it is more likely to hold when α_1^* is small and for very large α_2. Thus, relatively small competitors will choose to hide by adopting the pooling equilibrium strategy. Interestingly the consumer responsiveness parameter (ϕ) does not enter into the conditions under which Firm 1 prefers pooling. This is because the innovation impacts profitability, not s_1, Firm 1's ability to attract customers. Finally, observe that the conditions for the pooling equilibrium hold for any time t. As long as the parameters in the model are such that pooling is preferred, Firm 1 will choose to secretly profit.

Another empirically useful result that comes directly out of equation (20) is that for large enough innovations ($\alpha_1^* \to \infty$) the inequality never holds. Thus, if a firm finds that it can greatly increase its profitability it acts more aggressively right away. Oddly, it is the smaller innovations that firms seek to hide.

An interesting aspect of the pooling equilibrium is that it is *always* preferred by the rival Firm 2. The proof is straightforward and the intuition is as follows: A high type Firm 1 pools by keeping u_1 low. Firm 2 is therefore able to maintain a larger market share while spending less money to compete with Firm 1. In addition, Firm 2's market share during the period of asymmetric information is larger than its equilibrium market share when α_1^* is common knowledge. These two facts mean Firm 2's profits are strictly higher in the pooling than the separating equilibrium.

Given U.S. regulations regarding disclosure for public firms, it can be very difficult for them to withhold information from rivals while complying with the requirement that they keep investors informed. The results in this section show that private financing may provide a mechanism through which firms can commit to less aggressive spending. The intuition developed here can be compared to the capacity pre-commitment in Gelman and Salop (1983) in that by pre-committing (via private financing) to withhold information, an equilibrium outcome is one in which both firms spend less on market share gains. It is also analogous to the "Fat Cat Effect" in Fudenberg and Tirole (1984) in that spending on market share is a strategic complement and private financing provides a commitment device for less aggressive behavior.[23]

IV. Empirical Evidence

One important advantage of the main model is that it is well-suited for empirical analysis. Equation (1) provides a mechanism through which the consumer responsiveness parameter ϕ can be estimated. Because accounting data arrive in discrete time intervals the equation must be adapted. Also, real industries generally have more than two firms so the equation needs to be extended to accommodate this as well. For simplicity, and to avoid too high a ratio of parameters to data points, the estimates assume that $\mathrm{s_i} = 1$ for all firms. In this case, the law of motion for market share for firm *i* in an industry with *J* competitors becomes:

$$dm_i = \frac{\phi u_i}{\sum_{j=1}^{J} u_j} - \phi m_i dt. \tag{21}$$

[23] Spulber (1995) also shows how, in Bertrand competition, not knowing rivals' costs implies equilibrium prices that are above marginal costs (i.e., information asymmetry softens product market competition).

Since firm spending on u cannot be observed between filings the empirical work assumes it is constant during each filing interval. If so, then observations at time 1 and 0 for firm i should be related by

$$m_i(0) - \frac{u_i}{\sum_{j=1}^{J} u_j} = e^{\phi}\left(m_i(1) - \frac{u_i}{\sum_{j=1}^{J} u_j} \right). \tag{22}$$

With some simple algebra (22) can be rearranged into

$$\left[m_i(0) - m_i(1)e^{\phi} \right]\left[\sum_{j=1}^{J} u_j \right] = u_i\left(1 - e^{\phi}\right). \tag{23}$$

Unfortunately, the accounting statements do not reveal the true value of u_i but rather a noisy version μ_i equal to u_i-ε_i where ε_i is a white noise error term. Replacing the u_i in (23) with the μ_i+ ε_i yields:

$$\left[m_i(0) - m_i(1)e^{\phi} \right]\left[\sum_{j=1}^{J} \mu_j + \varepsilon_j \right] = \left(\mu_i + \varepsilon_i\right)\left(1 - e^{\phi}\right). \tag{24}$$

Equation (24) is homogenous of degree 0 in the μ_i+ε_i since the market shares have to add to one meaning there are only J-1 independent equations. Thus, define μ_J+ε_J as 1. Next, note that (24) is *linear* in the ε_i terms and one can write the above as:

$$\begin{aligned} &\left[m_i(0) - m_i(1)e^{\phi} \right]\left[\sum_{\substack{j=1 \\ j\neq i}}^{J-1} \varepsilon_j \right] + \left[m_i(0) - m_i(1)e^{\phi} + e^{\phi} - 1 \right]\varepsilon_i = \\ &\mu_i\left(1 - e^{\phi}\right) - \left[m_i(0) - m_i(1)e^{\phi} \right] - \left[m_i(0) - m_i(1)e^{\phi} \right]\left[\sum_{j=1}^{J-1} \mu_j \right], \end{aligned} \tag{25}$$

where the second to last term follows from $\mu_J+\varepsilon_J\equiv1$ Given the system of J-1 independent equations this means it is possible to solve for the J-1 error terms. To do so write (24) in matrix form as

$$\begin{bmatrix} m_1(0)-m_1(1)e^{\phi}+e^{\phi}-1 & m_1(0)-m_1(1)e^{\phi} & \cdots & m_1(0)-m_1(1)e^{\phi} \\ \vdots & \ddots & & \vdots \\ m_{J-1}(0)-m_{J-1}(1)e^{\phi} & \cdots & & m_{J-1}(0)-m_{J-1}(1)e^{\phi}+e^{\phi}-1 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{J-1} \end{bmatrix} = \begin{bmatrix} \mu_1\left(1-e^{\phi}\right)-\left(m_1(0)-m_1(1)e^{\phi}\right)-\left(m_1(0)-m_1(1)e^{\phi}\right)\sum_{j=1}^{J-1}\mu_j \\ \vdots \\ \mu_J\left(1-e^{\phi}\right)-\left(m_{J-1}(0)-m_{J-1}(1)e^{\phi}\right)-\left(m_{J-1}(0)-m_{J-1}(1)e^{\phi}\right)\sum_{j=1}^{J-1}\mu_j \end{bmatrix}. \qquad (26)$$

One can more compactly write (26) as $A\varepsilon=b$, where ε is the J-1×1 vector of the individual ε_i's. Assuming the elements of ε are independent with means of zero one can then estimate the value of ϕ via either maximum likelihood or non-linear least squares. Further assuming that ϕ remains constant over the sample period one can then improve the estimate by stacking the ε vectors and minimizing the total sum of squares.

A. Data and Hypotheses

1. Data and Sample Selection

To estimate ϕ, it is necessary to obtain data on both market shares and spending by all firms in the industry. Market share, $m_{i,t}$, is defined as share of sales of all *CRSP/COMPUSTAT* firms in the *COMPUSTAT* 4-digit SIC code. While broader industry categories are often used in the finance literature (e.g., 2-digit and 3-digit SIC codes), we choose to use the finer 4-digit codes in an effort to capture industry dynamics in the best possible way.[24] Spending, u_i, is defined as the sum of: capital expenditures, research and development and advertising. The analysis covers a broad cross-section of industries and the spending proxy is chosen to be sufficiently general to capture market share competition in a wide range of competitive structures.[25]

The initial sample consists of all *COMPUSTAT* 4-digit SIC codes for which there is non-missing information on sales and spending in *COMPUSTAT* and for which we obtain estimates for the consumer responsiveness parameter ϕ.[26] This results in a final sample of 299 industries and 7,761 industry-year observations for the period 1972 through 2005.

The estimates of ϕ are used in empirical tests of currently private firms' decisions to obtain private versus public equity financing. IPO financing data are from *SDC-Platinum* and venture capital financing information comes from *Venture Economics*. New venture capital financing rounds and dollars is the sum of all VC financing rounds and dollar amounts, respectively, in industry i as reported in *Venture Economics*. This eliminates LBOs and other acquisition activity. One issue is that the Venture Economics data do not provide SIC code information for portfolio companies; however, they do provide three levels of industry description. We use the fact that

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[24] *COMPUSTAT* codes are used due to findings in the literature that linkages among firms (e.g., return correlations) are higher than with CRSP SIC codes. For example, see Guenther and Rosman (1994).

[25] Advertising has been used to estimate variants of Lanchester models the marketing literature. See e.g., the empirical specification in Chintagunta and Vilcassim (1992).

[26] We exclude financial services sectors (SIC codes 6000-6999) and conglomerates (code 9997). We also exclude all industries with less than two publicly traded firms during the entire sample period (1972-2005). The main model assumes no exit; however, due to changing product mix and SIC code re-classification for some firms, we adjust for "entry" and "exit" in the data by assuming that each industry firm loses and gains market from the entrant and exiting firm, respectively, in proportion to their current market share. That is, let m_C_i equal each firm's market share calculated from the firms in the industry listed on COMPUSTAT at date *t*. Using this notation, the estimates assume that: $m_i(t) = m\_C_i(t)\left[\frac{Exiting\ Firm\ Share(t)}{1 - Exiting\ Firm\ Share(t)} - New\ Firm\ Share(t+1)\right]$ for each industry..

many private firms eventually undertake IPOs to map the descriptions in Venture Economics for the IPO firms to the industry classifications in *COMPUSTAT*.

2. Hypotheses

The analysis focuses on both venture capital financing and public equity offerings to identify potential differences between the factors contributing to observed financing choices. These differences can help distinguish the innovation incentives due to idiosyncratic industry factors from those influenced by private and public financing (the latter of which is this paper's primary topic). The following model of equity financing by currently private firms is estimated via OLS:

$$\frac{\text{Venture Firms}_{it}}{\text{Public Firms}_{i,t-1}}, \text{or} \frac{\text{IPO Firms}_{it}}{\text{Public Firms}_{i,t-1}}$$
$$= \gamma + \beta_1 R_{it} + \beta_2 RD_{it} + \beta_3 \phi_i + \beta_4 \text{HighHHI}_i + \beta_5 \phi_i * RD_{it} + \beta_6 \phi_i * \text{HighHHI}_i \qquad (27)$$
$$+ \beta_7 \text{Volatility}_{it} + \beta_8 \text{MTB}_{it} + \beta_9 \text{Size}_{it} + \varepsilon_{it}$$

Where: R_{it} is the real industry discount rate, RD_{it} is total industry R&D expenditures divided by book value of assets in industry *I*, ϕ_i is estimated based on the law of motion for market share in equation (21) , *HighHHI* is an indicator variable equal to 1 if the median sum of squared market shares (HHI) of industry i over the sample period is in the top tercile of all industries in the sample*; Volatility* is the average standard deviation of equity returns in industry i, calculated over the 60 months ending in year t-1, MTB_{it} is the average industry equity market-to-book ratio, and *Size* is the (natural log) market value of industry equity plus debt.

The coefficient on the estimated ϕ is of particular interest. Given the predictions presented in Table 2, all else equal, a larger consumer responsiveness parameter implies greater benefits associated with innovation and remaining private. Therefore, one expects β_3, the coefficient on ϕ, to be positive (i.e., more private financing in industries with high ϕ). For firms choosing private equity financing, the sensitivity to ϕ should be particularly high (e.g., $\beta_3>0$ in the venture financing equation). The industry discount rate is included as a proxy for δ. The model implies less innovation and more public financing when δ is high ($\beta_1<0$). R&D, Volatility and MTB are control variables for innovation opportunities, industry risk and potential attempts by firms to attempt to time the market, respectively. We also examine whether the marginal effect of ϕ varies with innovation opportunities (R&D). The intuition is that if innovation opportunities are high the impact of ϕ on the decision to innovate or obtain private financing may be particularly strong. If this is the case, then the coefficient on the interaction term (β_5) will be positive. HighHHI and the HighHHI*ϕ are analyzed since, given that the main model describes oligopolistic competition, one might expect that the sensitivity of financing by currently private firms to the ability to steal rivals' customers (ϕ) would be most evident in highly concentrated industries. If so, then the coefficient on β_6, the HighHHI*ϕ interaction term, will be positive.

B. Results

1. ϕ and Industry Half Life Estimates

Table 4 lists the median estimated industry ϕ's based on all four digit industries within a particular two digit industry. While equation (1) makes it possible to estimate the consumer responsiveness parameter ϕ, to understand the economic implications it helps to transform it into a market share "half life." Here half life refers to the estimated time it would take a firm, in years, to lose half of its market share if it ceased spending money to attract and keep customers. Setting u_i equal to zero in the multiple firm version of (1) generates a half life of $\ln(2)/\phi$.

The values in Table 4 range from 1.6 years in the "Coal Mining" to a (likely overestimated) value of 35.1 years in the "Hotels and Other lodging" industry. The median four digit industry has a half life of 11.8 years and the interquartile range is 5.1 to 19.4 years. Cell by cell *t*-statistics are not reported since it is unclear what the null hypothesis should be. Furthermore, the primary goal is simply to provide some economic intuition as to what parameter estimates imply, since they are not used in any of the formal estimates, rather than pin down any particular number.

Economically, the question is whether the half lives in Table 4 are "reasonable." Recall, that setting u_i to zero does not imply that the firm ceases operations, maintenance activities, or eliminates all customer service. Rather, it means that it does not actively compete for customers through things like advertising, R&D, and the construction of new outlets. In this light the estimates seem plausible. For example, the estimated half life for firms in the publishing industry is 8.1 years. Here, long term subscriptions and consumer loyalties would seem to make this a reasonable estimate. Indeed publications like the *New York Times* often come close to giving their product away to college age consumers in an attempt to develop what the firm hopes will be a life time loyalty. Thus, while there are clearly some industries with estimated half lives that appear to be either too high or low most seem within the range one expects.

2. New Financing

Given ϕ, Equation (27) can be estimated. Table 5 contains summary statistics of the industries and variables used. Results of estimating Equation (27) are presented in Table 6. *Model 1* presents estimates without the $\phi \times \text{RD}$ interaction and industry concentration (HighHHI) analysis. Model 2 includes the High HHI indicator variable and *Model 3* adds to this both the $\phi \times \text{RD}$ and $\phi \times \text{HighHHI}$ interactions.

The results from *Model 1* suggest that venture and IPO financing are positively and significantly related to both consumer responsiveness, ϕ, and innovation opportunities (R&D). Importantly, the estimated coefficients are significantly greater for venture financing than for IPO financing (at the 5% and 1% levels, for ϕ R&D, respectively). This suggest that the benefits of private financing are particularly great when consumer responsiveness and opportunities to innovate are high, consistent with some of the primary empirical implications from main model.

As reported in Table 6, the sign on the real discount rate is negative for both private and public equity financing (as predicted) and significantly more negative for private equity financing. Within the model this suggests that an increase in the discount rate encourages firms to shift from private to public financing. The coefficient estimates are compatible with this conjecture if one also allows for the fact that the first order effect may be to reduce overall project financing. Under this scenario, both sets of coefficients would be negative but the shift from private to public financing would then increase the parameter estimate somewhat for the IPO sample.[27]

[27] In addition to the main results in Table 6, the regressions were also estimated year-by-year. Inference based on the thirty-four annual coefficient estimates yields similar results. Also, given that the dependent variable is left-

Model 2 is presented to describe the independent impact of concentration on financing by private firms. However, the interaction between concentration and ϕ in (*Model 3)* is of greater interest. The *Model 3* results provide additional insights into the mechanisms through which higher ϕ positively impacts private financing (as suggested by the *Model 1* results). There is a positive and significant coefficient on the $\phi \times \mathrm{RD}$ interaction term (β_5) in the venture capital financing equation, and a negative impact of this interaction in the IPO equation. The venture capital findings are consistent with the conjecture that the marginal effect of a higher ϕ on the decision to obtain private financing might be high when R&D is higher. In other words, public financing in high R&D industries may cause firms to lose due to aggressive spending on new developments by rivals. In addition, consistent with the observation that the main model is one of oligopoly, marginal effect of a higher ϕ on the decision to obtain private financing is higher when industries are highly concentrated (i.e., $\beta_6 > 0$). Both these channels through which the ability to steal customers can impact financing appear to be important, as omission of these terms (*Models 1 and 2)*, causes substantial reduction in R-squared.

To summarize, the empirical tests use the basic structure provided by the main model. The results provide evidence consistent with the idea that current competitive structure plays an important role in firms' decisions to remain private. Private equity (venture capital) financing is more common in industries with higher consumer responsiveness, innovation opportunities and industry risk.

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censored at zero, the model was also re-estimated using a Tobit specification. All signs and significance levels are consistent with the OLS results presented in Table 6.

C. Calibrations

In this final stage of the analysis, we calibrate the model. The main focus is on the economic value of remaining private. To analyze this value both between and across industries, we relax the assumption in the prior sections that $s_i = 1$. To calculate individual s_i for each firm in the sample, we first estimate parameters α_i and fixed cost f_i based on Equation (2): $\pi_i(t) = \alpha_i m_i(t) - u_i(t) - f_i$. Where $\pi_i(t)$, $m_i(t)$ and $u_i(t)$ are earnings, market share and spending of firm i during year t, respectively. To account for the possibility that profitability might be time-varying, we estimate rolling α_i based on profitability, spending and market share data for the years t-5 through t-1. Given α_i, we use Equation (9) solve for s_i (setting s_2=1 and α_2 = the mean of all competing firms in the industry for which we have valid α estimates). Given these parameters and Equation (17), we calculate $\Delta V = V_{private} - V_{public}$ under the assumptions that: the firm invests in an innovation that increases its *s* by 20%; industry growth is 2%; T_1=2 and T_2^{Public} =5. Real discount rates are calculated using equity betas from a market model estimated over the 60 months preceding year t (unlevered betas are calculated using book values of debt and assuming β_D=0).

We obtain estimates for all industries in the sample; however we present two sample industries in Table 7 for illustrative purposes. We focus on Carpets and Rugs (SIC 2273) and Pumps and Pumping Equipment (SIC 3561). These industries reflect significant between-industry variation in consumer responsiveness (φ of .06 and .49, respectively), as well as within-industry variation in profitability. It is important to note that we assume ZD=0; however, from Table 7, it is easy to apply a private market discount. Given the evidence in Hertzel and Smith (1993), 20% would be an appropriate approximation. This implies, that for a required investment of $30 million, firms in pump industry would finance privately, while carpet and rug industry firms would typically not find it profitable do so if it only extends competitive advantage by two years (ZD = $6Million>4.76 Million). Interestingly, the mean relative profitability (α) of firms that go public in industry 2273 is 30% greater than the profitability of currently public firms, whereas IPO firms are less than 2% more profitable than existing public firms in SIC code 3561. This is consistent with the idea that profitable firms in high φ industries remain private.

Figure 2 shows individual firm values for four sample firms within these industries. From the figure, it is easy to see that current spending effectiveness, market shares and profitability and all have important impacts on firms' incentives to remain private. Assuming a 20% private market discount, the calibrations suggest that a firm like Mohawk would be willing to privately finance an investment in s that costs less than $17M, even if it extended the first-mover advantage period by only one year. A private firm similar to its competitor, Dixie, would only be willing to finance the investment privately if it cost less $1 million.

V. Relationship to the Prior Literature

IPO activity has been extensively studied in both the theoretical and empirical literature. In their survey paper, Ritter and Welch (2002) provide several reasons for IPOs, including a role for product market competition (such as gaining a first-mover advantage via being the first firm in an industry to have an IPO). Our paper adds to this literature by highlighting that dynamic interactions between rival firms can influence when and if a firm goes public. Ritter and Welch (2002) also note the significant time series variation in IPO activity (e.g., low issuance in the 1980s and high issuance in the 1990s). Shocks to common variables (at the industry or macro-economic level) can be added to our model and would generate patterns in issuance that are consistent with these IPO waves, both in the aggregate economy (through changes in the discount rate) as well as within industries (through changes in industry-level variables). In recent work, Pastor and Veronesi (2005) also link IPO waves to economic fundamentals. They show how decreases in expected returns or increases in future aggregate profitability can cause increases in issuance; however, their focus is on the timing of aggregate IPO activity, not issuance within industries. Further, while a special case of their model might include industry level variables, there is no explicit role for industry competitive dynamics.

This is, of course, not the first paper to posit a relationship between the product and financial markets. However, the previous literature has tended to focus on the strategic use of debt in a firms' capital structure to obtain a competitive advantage. For example, the "limited liability" effect of debt, in which debt commits firms to more aggressive product market behavior is described in Brander and Lewis (1986) and Maksimovic (1988). Bolton and Scharfstein (1990) show how debt can soften competition. Further examples and discussion can be found in the survey by Maksimovic (1990). On the empirical side of the literature, Chevalier (1995) and Leach, Moyen, and Yang (2006) provide evidence on the interaction between leverage and corporate behavior. Chevalier looks at a sample of supermarkets following LBOs and finds that debt "softens" product market competition. In contrast, Leach, Moyen, and Yang look at telecommunications firms and reach the opposite conclusion. Thus, it may be that as yet unmodeled industry characteristics influence the degree to which the predictions in Brander and Lewis (1986) and Maksimovic (1988) are borne out.

MacKay and Phillips (2005) document significant variation in financial structure within industries and that the role of financial structure varies with industry concentration. In particular, in competitive industries, firms with capital-to-labor rations that are close (far away from) to the industry median use less (more) financial leverage. Leverage is higher and more uniform in less competitive industries. In contrast to this literature our paper's focus has been on the public-private decision rather than the leverage decision.

With the exception of Maksimovic and Pichler (2001) in the theoretical literature and recent empirical work Chemmanur, He and Nandy (2007), little has been done to improve our understanding of the potential strategic role played by the private versus public financing decision. Given the size of the private equity (and debt) markets it is important to identify the factors that encourage a firm to use this source of financing rather than the public markets. Indeed, over the past decade or so the private equity market has grown significantly. It is reported to have peaked at $160 billion in 2000, up from $10 billion in 1991 was $40 billion in 2003. (See e.g., “The New Kings of Capitalism,” The Economist, 11/27/2004, 373 (8403), 3-5.)

VI. Conclusions

The paper’s main goal has been to answer the following questions: First, what are the characteristics of firms that benefit most from innovation and from private financing? Second, how important are industry structure, rival characteristics, and the nature of the innovation? In the context of a dynamic duopoly, we provide closed form solutions for the values of two competing firms, in a setting in which one firm faces an opportunity to innovate. If the technology is adopted, the firm must also determine whether it will obtain public or private financing. Our results relate current firm and industry characteristics to these decision variables. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. The private versus public financing decision depends mainly on the magnitude of the technological improvement and length of the period during which private financing extends the innovator’s product market advantage. The results from our model suggest that future empirical work examining financing patterns should also explicitly consider competitive dynamics.

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Appendix

A. The General Model and its Solution

This appendix contains the solution to the most general version of the model discussed in this paper. For the problem described in Section I the value functions for the firms at time 0 are:

$$V_1(m,t) = \int_0^T (\alpha_1 m(t) - u_1 - f_1)e^{-\delta t}dt + B_1 \quad (28)$$

and

$$V_2(m,t) = \int_0^T (\alpha_2(1-m(t)) - u_2 - f_2)e^{-\delta t}dt + B_2 \quad (29)$$

respectively. Here *T* is a terminal date on which the game ends, and B_i the present value of each firm's value at the terminal date. Note, because the game ends at date *T* the value functions (V_i) depend both on *m* and the time remaining until date *T*.

The analysis seeks a Nash equilibrium in which the players use Markovian strategies. For each firm the instantaneous value functions given by (2) imply that in a Markovian Nash equilibrium the following Hamilton-Jacobi-Bellman (HJB) equations must hold:

$$0 = \max_{u_1} \quad \alpha_1 m - u_1 - f_1 + \frac{\partial V_1}{\partial m}\left\{\frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{u_1s_1 + u_2s_2}\right\} + \frac{\partial V_1}{\partial t} - \delta V_1 \quad (30)$$

and

$$0 = \max_{u_2} \quad \alpha_2(1-m) - u_2 - f_2 + \frac{\partial V_2}{\partial m}\left\{\frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{u_1s_1 + u_2s_2}\right\} + \frac{\partial V_2}{\partial t} - \delta V_2 \quad (31)$$

subject to the terminal condition that $V_i(T)$ equals $B_i(T)$.

The first order condition for Firm 1 is:

$$\frac{\partial V_1}{\partial m}\phi u_2 s_2 s_1 = (u_1s_1 + u_2s_2)^2 \quad (32)$$

and the correspondingly for Firm 2:

$$-\frac{\partial V_2}{\partial m}\phi u_1 s_1 s_2 = (u_1 s_1 + u_2 s_2)^2. \tag{33}$$

Equations (32) and (33) yield equilibrium spending of u_1^* and u_2^*:

$$u_1^* = -\frac{\left(\frac{\partial V_1}{\partial m}\right)^2 \frac{\partial V_2}{\partial m}\phi s_1 s_2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2} \tag{34}$$

and

$$u_2^* = \frac{\frac{\partial V_1}{\partial m}\left(\frac{\partial V_2}{\partial m}\right)^2 \phi s_1 s_2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2}. \tag{35}$$

Plugging the solutions for u_1^* and u_2^* into the HJB equations (30) and (31) yield the two differential equations (after some extensive algebra):

$$0 = \alpha_1 m - f_1 + \frac{\left(\frac{\partial V_1}{\partial m}\right)^3 \phi s_1^2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2} - \frac{\partial V_1}{\partial m}\phi m - \frac{\partial V_1}{\partial t} - \delta V_1 \tag{36}$$

and

$$0 = \alpha_2(1-m) - f_2 - \frac{\left(\frac{\partial V_2}{\partial m}\right)^3 \phi s_2^2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2} + \frac{\partial V_2}{\partial m}\phi(1-m) - \frac{\partial V_2}{\partial t} - \delta V_2 \tag{37}$$

that need to be solved.

The solutions for the value functions in (36) and (37) are determined by guessing and verifying that they take on the time dependent forms given by (3) at any date *t*. Those functional forms then imply that the derivatives with respect to *m* and *t* of the value functions equal:

$$\frac{\partial V_1}{\partial m} = -b_1, \qquad \frac{\partial V_1}{\partial t} = -\frac{\partial a_1}{\partial t} - \frac{\partial b_1}{\partial t} m \tag{38}$$

for firm 1 and

$$\frac{\partial V_2}{\partial m} = -b_2, \qquad \frac{\partial V_2}{\partial t} = -\frac{\partial a_2}{\partial t} - \frac{\partial b_2}{\partial t} m \tag{39}$$

for firm 2.

Plugging equations (38) and (39) into (36) and (37) yields a set of differential equations that need to be solved for the a_i and b_i terms subject to the boundary conditions $B_i(T)$. However, since the equalities have to remain true for all m there are in fact four equations that must hold, one for each a_i term and one for each b_i term. After some algebra this implies that solutions need to be found for the following four ordinary differential equations (ODE). For a_1 the ODE is

$$0 = -f_1 + \frac{\phi \alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} + \frac{\partial a_1}{\partial t} - \delta a_1 \tag{40}$$

while for a_2 it is

$$0 = \alpha_2 - f_2 + \frac{\phi \alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - \frac{\alpha_2}{\phi+\delta} + \frac{\partial a_2}{\partial t} + \delta a_2. \tag{41}$$

Solving for the a_i in the above two equations yields (4) and (5). Next, collect the terms multiplying m to yield for b_1 the ODE:

$$\alpha_1 - \phi b_1 - \frac{\partial b_1}{\partial t} - \delta b_1 = 0 \tag{42}$$

and for b_2:

$$-\alpha_2 - \phi b_2 + \frac{\partial b_2}{\partial t} - \delta b_2 = 0. \tag{43}$$

Solving these last two equations for b_i produces equations (6) and (7).

Given equations (4) through (7) a particular problem's boundary conditions then determine the C_i and k_i terms and thus provide a full characterization of the economy's equilibrium behavior. The main body of the text presents various scenarios, their boundary value conditions, and the solutions they impose on the C_i and k_i terms. There, one can also find the paper's analysis of the economy's overall behavior.

B. Solving for $a_i(t)$ and $b_i(t)$

1. Time Independent Case

In the time independent case the values a_i and b_i do not depend on t. Simple inspection of (4) through (7) then implies that the C_i and k_i terms must equal zero.

2. Time Dependent Case: Value Functions for $t \in [T_1, T_2)$

At time T_2 the value functions given by the time independent case must hold with the s_i set to s_i^* since no further changes in the s_i take place within the model. Based on this one now needs to find solutions for the a_i and b_i as functions of t for the period $t \in [T_1, T_2)$. To do so, the value

functions for the period $t \in [T_2, \infty)$ evaluated at T_2 serve as the boundary conditions:

$$\begin{aligned} a_1(T_2) &= \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2}\right\} - f_1\right\}; \quad b_1(T_2) = \alpha_1(\phi+\delta)^{-1} \\ a_2(T_2) &= \delta^{-1}\left\{\alpha_2 + (\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2}{\phi+\delta} - f_2\right\}; \quad b_2(T_2) = -\alpha_2(\phi+\delta)^{-1}. \end{aligned} \tag{44}$$

Given the above boundary conditions, one can solve for the constants C_i and k_i in (4) through (7) to yield the equilibrium value functions:

$$\begin{aligned} V_1(T_1) &= \frac{\alpha_1}{\delta(\phi+\delta)}\frac{\phi\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + \\ & e^{-\delta(T_2-T_1)}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_1^2 s_1^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta}m(T_1) \end{aligned} \tag{45}$$

and

$$V_2(T_1) = \frac{\alpha_2}{\delta(\phi+\delta)} \frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + \tag{46}$$
$$e^{-\delta(T_2-T_1)} \frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] - \frac{f_2}{\delta} + \frac{\alpha_2}{\phi+\delta}\big(1 - m(T_1)\big).$$

Note that, except for the second terms in each equation, the value functions are of the same form as in the infinite horizon base case. These additional terms can be interpreted as the "first mover advantage" and "second mover disadvantage." Importantly, the marginal value of market share (and the optimal control) is time independent.

3. Time Dependent Case: Value Functions for t∈[0, T_1)

From the solutions (45) and (46) the boundary value conditions for the b_i are $b_1(T_1) = \alpha_1(\phi+\delta)^{-1}$ and $b_2(T_1) = -\alpha_2(\phi+\delta)^{-1}$ respectively. Plugging these into (42) and (43) yields the result that the k_i terms must equal zero. Thus, $b_1(t) = \alpha_1(\phi+\delta)^{-1}$ and $b_2(t) = -\alpha_2(\phi+\delta)^{-1}$ for all t. Similarly, for the a_i the equilibrium the $C_{i,T1}$ must satisfy:

$$\delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^{*2}}{(\phi+\delta)(\alpha_1 s_1^* + \alpha_2 s_2)^2} - f_1\right] +$$
$$e^{-\delta(T_2-T_1)}(\delta(\phi+\delta))^{-1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] \tag{47}$$
$$= \delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - f_1\right] + C_{1,T1}e^{\delta T_1}$$

and

$$\delta^{-1}\left[(\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}+\frac{\delta\alpha_2}{\phi+\delta}-f_2\right]+$$

$$e^{-\delta(T_2-T_1)}(\delta(\phi+\delta))^{-1}\phi\alpha_2^3\left[\frac{s_2^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}\right] \tag{48}$$

$$=\delta^{-1}\left[\frac{\phi\alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2}-f_2\right]+C_{2,T1}e^{\delta T_1}.$$

Solving for $C_{1,\mathrm{T}1}$ and $C_{2,T1}$ produces:

$$\begin{aligned}C_{1,T_1}&=(\delta(\phi+\delta))^{-1}e^{-\delta T_1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}-\frac{s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right]\\&+(\delta(\phi+\delta))^{-1}e^{-\delta T_2}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}\right]\end{aligned} \tag{49}$$

and

$$\begin{aligned}C_{2,T_1}&=(\delta(\phi+\delta))^{-1}e^{-\delta T_1}\phi\alpha_2^3\left[\frac{s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}-\frac{s_2^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right]\\&+e^{-\delta T_2}(\delta(\phi+\delta))^{-1}\phi\alpha_2^3\left[\frac{s_2^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}\right].\end{aligned} \tag{50}$$

C. Comparative Statics: Private versus Public Financing

1. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\alpha_1$

Equation (18) characterizes Firm 1's incentives to finance publicly as we vary its revenue-generating ability: $\dfrac{\partial(V_1^{\text{Private}}-V_1^{\text{Public}})}{\partial\alpha_1}=q\alpha_1^2\left[\dfrac{\psi_2^2(\alpha_1\psi_2+3\alpha_2)}{(\alpha_1\psi_2+\alpha_2)^3}-\dfrac{\psi_1^2(\alpha_1\psi_1+3\alpha_2)}{(\alpha_1\psi_1+\alpha_2)^3}\right].$

To sign this, note that at $\psi_1=\psi_2$, (18) equals 0. Since $\dfrac{\partial\left(\dfrac{\alpha_1\psi^3+3\alpha_2\psi^2}{(\alpha_1\psi+\alpha_2)^3}\right)}{\partial\psi}=(\alpha_1\psi_2+\alpha_2)^{-4}6\alpha_2^2\psi$ is clearly positive, then for all $\psi_2>\psi_1$, $\dfrac{\psi_2^2(\alpha_1\psi_2+3\alpha_2)}{(\alpha_1\psi_2+\alpha_2)^3}>\dfrac{\psi_1^2(\alpha_1\psi_1+3\alpha_2)}{(\alpha_1\psi_1+\alpha_2)^3}$. Thus, (18) is positive and the incentive to secure private financing for an innovation is increasing in Firm 1's revenue generating ability (α_1).

2. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\alpha_2$

Equation (19) shows how Firm 1's incentives to finance publicly vary with the profitability of Firm 2: $\dfrac{\partial(V_1^{\text{Private}}-V_1^{\text{Public}})}{\partial\alpha_2}=2q\alpha_1^3\left[\dfrac{\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^3}-\dfrac{\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]$. Observe that at $\alpha_2=0$, Equation (19) is greater than zero since $\psi_2>\psi_1\Rightarrow\dfrac{1}{\psi_1}-\dfrac{1}{\psi_2}>0$. Also note that

$\dfrac{\partial\left(\dfrac{\psi^2}{(\alpha_1\psi+\alpha_2)^3}\right)}{\partial\psi}=(\alpha_1\psi_2+\alpha_2)^{-4}\left(2\alpha_2\psi-\alpha_1\psi^2\right)$. This means that when $\alpha_2>(\alpha_1\psi/2)$, Equation (19) is negative. Firm 1's incentives to obtain private financing are decreasing in α_2 when α_2 is large relative to α_1. The opposite is true for small α_2.

3. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\psi$

The impact of the size of Firm 1's relative spending advantage (ψ parameter) on incentives to finance publicly is also of interest. Since $\dfrac{\partial\left(\dfrac{\psi}{(\alpha_1\psi+\alpha_2)}\right)}{\partial\psi}=(\alpha_1\psi+\alpha_2)^{-2}\alpha_2>0$, Equation (17) implies that increasing Firm 1's spending advantage due to the technology (high

ψ_2) will increase incentives to finance publicly. Conversely, increasing Firm 1's current spending advantage (ψ_1) decreases its incentive to finance innovations with privately issued securities.

4. $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right)/\partial\phi$

Since $\partial q/\partial\phi>0$ and since ϕ does not enter (17) outside of q one has $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right)/\partial\phi>0$.

5. $\partial^2\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right)/\partial\alpha_1\partial\alpha_2$

We now consider the impact of an increase in the profitability of a rival on the value of an increase in an innovator's profitability (and incentive to remain private):

$$\frac{\partial^2(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1\partial\alpha_2} = 6q\alpha_1^2\alpha_2\left[\frac{\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^4} - \frac{\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^4}\right].$$ Since

$\dfrac{\partial\left(\dfrac{\psi^2}{(\alpha_1\psi+\alpha_2)^4}\right)}{\partial\psi} = (\alpha_1\psi\ +\alpha_2)^{-5}2\psi(\alpha_2-\psi\alpha_1)$ is greater than zero when α_1 is small (i.e., $\alpha_{2/}\ \psi >\alpha_1$), this implies that increasing the profitability of a rival increases the value of a profitability increase for less profitable innovators.

6. $\partial^2\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right)/\partial\alpha_1\partial\delta$

We now consider the impact of an increase discount rate on the value of an increase in an innovator's profitability (and its impact on incentive to remain private):

$$\frac{\partial^2(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1\partial\delta} = -(\phi+2\delta)(\delta(\phi+\delta))^{-2}\left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right]\phi\alpha_1^2\left[\frac{\psi_2^2(\alpha_1\psi_2+3\alpha_2)}{(\alpha_1\psi_2+\alpha_2)^3} - \frac{\psi_1^2(\alpha_1\psi_1+3\alpha_2)}{(\alpha_1\psi_1+\alpha_2)^3}\right]$$

As we showed in Case 1 above, the last term is positive, so this expression is negative. An increase in the discount rate decreases the impact of an increase in the innovator's profitability on its incentive to remain private.

7. $\partial^2\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right)/\partial\alpha_2\partial\delta$

Similarly, consider the impact of an increase discount rate on the value of an increase in a rivals's profitability (and its impact on the innovator's incentive to remain private):

$$\frac{\partial^2(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_2\partial\delta}$$

$$= -2(\phi+2\delta)(\delta(\phi+\delta))^{-2}\left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right]\phi\alpha_1^3\left[\frac{\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^3} - \frac{\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]$$

This sign depends on the relative values of α_1 and α_2 as in Case 2 above.

D. Changes in α: Pooling versus Separating

Proposition: A Pareto optimal pooling equilibrium exists whenever equation (20) holds. In this pooling equilibrium the following conditions hold: (1) The low type Firm 1 optimizes as if it is in the full information case. (2) The high type Firm 1 mimics the low type's actions along all game paths. (3) Firm 2 optimizes as if Firm 1 is a low type for sure. (4) If Firm 1 deviates from the equilibrium path then Firm 2 plays as if Firm 1 is a high type for sure.

Proof: As will be shown the equilibrium holds up under the standard refinements, properly interpreted for the model's continuous time setting. It is easiest to begin with Firm 2. Consider some non-degenerate prior over Firm 1's type. In the pooling equilibrium Firm 2 believes its value function (3) at time T will equal $V_2\left(m,T\middle|\alpha_1\right)$ or $V_2\left(m,T\middle|\alpha_1^*\right)$ for the infinite horizon case with probabilities based on its priors. Note, that in either case the solution to b_2 for $t \geq T$ is identical since b_2 does not depend on α_1. Thus, the terminal condition for the differential equation yielding b_2 for $t<T$ is independent of Firm 1's realized type. Given that Firm 2 believes Firm 1 will play as a low type firm regardless of Firm 1's true type the HJB equation (31) must continue to hold for $t<T$. From here it is simple to show that the solution to b_2 for $t<T$ is unchanged from the full information case. Thus, for Firm 2, all that changes in equilibrium is the constant a_2 which does not influence its behavior. Since Firm 2 was optimizing in the full information case it must remain optimal to follow the same strategy in the pooling equilibrium. The only remaining issue for Firm 2 is whether it proposed out of equilibrium beliefs are "credible." To answer this question turn now to the incentives for each Firm 1 type.

Proving the low type Firm 1 does not wish to defect from the equilibrium is trivial. Under the pooling equilibrium the low type receives the same profits as it would under full revelation. Thus, the only question is whether a defection that leads Firm 2 to believe Firm 1 is a high type can make the low type better off. In the candidate equilibrium a defection causes Firm 2 to act as though Firm 1 is a high type for sure. Firm 2's equilibrium spending levels on market share will therefore be based on the solution to (31) with $\alpha_1 = \alpha_1^*$. Some algebra shows that this just leads to higher spending on market share acquisition by Firm 2 which can only make Firm 1 worse off.

The final player is the high type Firm 1. The Nash equilibrium of the full information game at the time of the improvement in α (time 0) is

$$V_1^{FullInfo}(0) = \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^{*3}s_1^2}{(\alpha_1^* s_1+\alpha_2 s_2)^2}\right\} - f_1\right\} + (\phi+\delta)^{-1}\alpha_1^* m(0). \tag{51}$$

Consider now the value function when Firm 1 decides to privately finance in order to hide its true revenue generating capacity until T. In this case, Firm 1 obtains the profits of the low-type firm, plus $\left(\alpha_1^* - \alpha_1\right)m(t)$ at each instant between 0 and *T*. If it pools, the value function for Firm 1 is:

$$\begin{aligned} V_1^{Pool}(0) &= \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right\}-f_1\right\}+(\phi+\delta)^{-1}\alpha_1 m(0) \\ &-e^{-\delta T}\left[\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right\}-f_1\right\}+(\phi+\delta)^{-1}\alpha_1 m(T)\right] \\ &+e^{-\delta T}\left[\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^{*3} s_1^2}{(\alpha_1^* s_1+\alpha_2 s_2)^2}\right\}-f_1\right\}+(\phi+\delta)^{-1}\alpha_1^* m(T)\right] \\ &+\int_0^T e^{-\delta t}(\alpha_1^*-\alpha_1)m(t)dt \end{aligned} \tag{52}$$

The first two terms of (52) represent the full-information profits of a low-type firm from 0 to T, the third term is the full-information profit of a high-type firm from T to ∞ and the last term represents the discounted profits associated with a high-type firm hiding and pretending to be a low-type firm.

To find the conditions under which $V_1^{Pool} - V_1^{FullInfo} > 0$ the first step is to find a solution for the last term in (52). Note that, while hiding, equilibrium spending is given by (8) with the values of alpha given by α_1 and α_2. Therefore, *dm* must equal:

$$dm = \frac{\phi s_1 \alpha_1}{s_1\alpha_1 + s_2\alpha_2} - \phi m. \tag{53}$$

Solving this ordinary differential equation for m(t) yields $m(t) = Ce^{-\phi t} + \frac{s_1\alpha_1}{s_1\alpha_1 + s_2\alpha_2}$ where *C* is a constant whose solution is given by the boundary value. At t=0, $m(0) = C + \frac{s_1\alpha_1}{s_1\alpha_1 + s_2\alpha_2} \Rightarrow C = m(0) - \frac{s_1\alpha_1}{s_1\alpha_1 + s_2\alpha_2}$. The general solution is:

$$m(t) = \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2} + e^{-\delta t}\left[m(0) - \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}\right]. \tag{54}$$

Substitution into $\int_0^T e^{-\delta t}(\alpha_1^* - \alpha_1)m(t)dt$ gives:

$$\begin{aligned}&\int_0^T e^{-\delta t}(\alpha_1^* - \alpha_1)\left[\frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2} + e^{-\phi t}\left[m(0) - \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}\right]\right] dt = \\ &(\alpha_1^* - \alpha_1)\left[(1 - e^{-\delta T})\delta^{-1}\frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2} + (1 - e^{-(\delta+\phi)T})(\delta + \phi)^{-1}\left(m(0) - \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}\right)\right].\end{aligned} \tag{55}$$

Plugging this into the pooling value function (52) and subtracting $V_1^{FullInfo}$ implies that pooling is preferred to separation when (20) holds. Thus, when (20) holds any defection that leads Firm 2 to believe Firm 1 is a high type firm must make Firm 1 worse off. Given the conjectured equilibrium strategies and beliefs it follows that a high type Firm 1 will not defect from the proposed equilibrium.

The last issue is whether the proposed out of equilibrium beliefs for Firm 2 are credible under the usual refinements. If Firm 2 sees a defection the set of best responses lie within those that correspond to the actions it would take under some equilibrium with the belief that Firm 1 is a high type with a nonzero probability. It is easy to show that any such best response involves higher spending on u_2 which makes both the high and low Firm 1 types worse off. Thus when (20) holds, there does not exist a best response by Firm 2 that can make either Firm 1 type better off. Under the standard refinements Firm 2 can then act as though it believes Firm 1 is a high type for sure thereafter. QED

Table 1: Change in the Value to the Innovator from an Innovation

Derivative	Economic Interpretation	Sign	Condition
$\partial^2 V_1(0)/\partial\psi_2 d\alpha_1$	The impact of an increase in the innovator's profitability on the value of any innovation.	+	All firms.
$\partial^2 V_1(0)/\partial\psi_2 d\alpha_2$	The impact of an increase in the rival's profitability on the value of any innovation.	-	Small firms.
$\partial^2 V_1(0)/\partial\psi_2 \partial\alpha_2$	The impact of an increase in the rival's profitability on the value of any innovation.	+	Large firms.
$\partial^2 V_1(0)/\partial\psi_2 \partial\delta$	An increase in the discount rate reduces the value of any innovation.	-	All firms.
$\partial^2 V_1(0)/\partial\psi\partial\phi$	An increase in consumer responsiveness increases the value of any innovation.	+	All firms.

Table 2: Possible Empirical Proxies for the Model's Parameters		
Parameter	Description	Possible Empirical Proxies
m	Market share	Share of total industry: ▪ Sales ▪ Assets ▪ Market Value Equity + Book Vale of Total Debt
u	Spending to gain market share	▪ Advertising ▪ R&D ▪ Capital Expenditures
ϕ, s	Effectiveness of spending, and consumer responsiveness.	▪ Estimation based on the discrete time version of equation (1) : $m_{t+1} - m_t = \frac{\phi u_1 s_1}{u_1 s_1 + u_2 s_2} - \phi m_t$
f	Costs of Operations	▪ Operating Expenses (net of proxy for market share spending)
α	Revenue-generating ability	▪ Sales ▪ Operating Profit
δ	r-g: discount rate minus industry growth rate	▪ Interest Rates ▪ Industry Growth Rate
T_1, T_2		▪ Patent protection periods ▪ R&D expenditures
Other Variables of Interest		
	Opportunities to Innovate	▪ Number of Patents ▪ R&D Expenditures
$\alpha_1 - \alpha_2$	Relative competitive advantage	▪ Difference in market shares ▪ Industry HHI

Table 3: Change in the Value of Remaining Private ($V_1^{\text{Private}} - V_1^{\text{Public}}$) when Financing an Innovation

Derivative w.r.t.	Economic Interpretation	Sign	Condition
α_1	The impact of an increase in the innovator's profitability.	+	All firms.
α_2	The impact of an increase in the rival's profitability.	-	Small innovator.
α_2	The impact of an increase in the rival's profitability.	+	Large innovator.
ψ_1	The relative ability of the innovator to take market share relative to its rival.	-	All firms.
ψ_2	The innovation's improvement in the innovator's ability to increase its market share during the period it has a technological advantage, T_1 to T_2.	+	All firms.
ϕ	Consumer responsiveness to corporate spending seeking to increase market share.	+	All firms.
δ	An increase in the real interest rate.	-	All firms.
T_2^{Public}	Increase in the time during which the innovating firm can maintain its advantage relative to the rival in a full-disclosure setting.	-	All firms.
T_2^{Private}	Increase in the time during which the innovating firm can maintain its advantage relative to the rival when private financing is chosen.	+	All firms.

Table 4: Estimated Market Share Half Lives By Two Digit SIC Industry

The ϕ parameter is estimated for each individual 4 digit SIC industry (as classified by COMPUSTAT) using equation (26) – a discrete time version of the law of motion for market share. The estimated value of ϕ is the one that minimizes the sum of squared errors, ε_i. Firm i's market share, $m_{i,t}$ is defined as the share of sales of all *CRSP/COMPUSTAT* firms in the industry. The values of u_i are taken to be either capital expenditures or the sum of capital expenditures, research and development and advertising (based on the lowest sum of squared errors). The half life listed below uses medians of all estimated ϕ's and half lives within that 2 digit industry. Based on equation (1) the half life equals $\ln(2)/\phi$. This half life represents, in years, the time it would take a firm that spends nothing on customer recruiting to lose half its current market share.

SIC	Half Life	Industry	SIC	Half Life	Industry
1	10.7	Agriculture Production-Crops	40	11.1	Railroads, Line-Haul Operating
2	21.0	Agric Prod-Lvstk,Animal Spec	41	12.4	Transit & Passenger Trans
7	26.5	Agricultural Services	42	8.6	Trucking
10	6.0	Metal Mining	44	11.7	Water Transportation
12	1.6	Coal Mining	45	18.6	Air Transport
13	6.0	Oil And Gas	47	18.9	Transportation Services
15	3.5	Building Operations And Contracting	48	7.4	Television, Radio And Broadcast Services
16	10.0	Heavy Constr-Not Bldg Constr	49	7.2	Gas, Electrical, Refuse Services
17	2.2	Construction-Special Trade	50	5.5	Durable Goods-Wholesale
20	15.6	Food And Kindred Products	51	6.9	Nondurable Product Wholesalers
21	3.0	Tobacco Products	52	10.8	Bldg Matl,Hardwr,Garden-Retl
22	11.9	Textile Mill Products	53	9.8	General Merchandise Stores
23	15.0	Apparel & Other Finished Pds	55	8.1	Auto Dealers, Gas Stations
24	4.5	Lumber And Wood Pds, Ex Furn	56	26.0	Apparel And Accessory Stores
25	21.2	Office And Home Furniture	57	15.2	Home Furniture & Equip Store
26	14.2	Paper And Allied Products	58	18.8	Eating Places
27	8.1	Publishers	59	10.4	Miscellaneous Retail
28	11.6	Chemicals & Allied Prods	70	35.1	Hotels, Other Lodging Places
29	5.5	Petroleum And Coal Products	72	8.8	Personal Services
30	10.7	Rubber, Foam, And Plastic Products	73	8.7	Advertising, Business Matching Services
31	33.8	Leather And Leather Products	75	24.7	Auto Repair,Services,Parking
32	12.6	Glass Clay And Concrete Products	76	11.4	Misc Repair Services
33	8.0	Iron And Steel Products	78	11.2	Motion Pictures
34	10.5	Other Metal Products	79	4.8	Amusement & Recreation Svcs
35	17.6	Machinery, Bearings, Comp.Equip.	80	3.8	Health Services
36	12.5	Electr, Oth Elec Eq, Ex Cmp	82	12.4	Educational Services
37	17.1	Truck, Aircraft, Train Equip & Parts	83	6.4	Social Services
38	15.5	Technical And Lab Instruments	87	23.9	Engr,Acc,Resh,Mgmt,Rel Svcs
39	16.7	Jewelry, Toys, Sporting Goods	40	11.1	Railroads, Line-Haul Operating

Table 5: Summary Statistics – Industries

This table presents summary statistics for the industries (4-Digit *COMPUSTAT* SIC codes) in the sample. The sample period is 1972 through 2005. *Industry sales* are the sum of sales of all CRSP/COMPUSTAT firms in industry i during year t. φ is the consumer responsiveness parameter, estimated from the discrete time version of the law of motion of market share (Equation 1 in the text). *MTB* is equity market capitalization, divided by book value of equity of *CRSP/COMPUSTAT* firms. *R&D/Assets* is equal to the sum of R&D expenditures, divided by the book value of total industry assets. *HHI* is the sum of squared market shares of CRSP/COMPUSTAT firms. The *discount rate* is the estimated real industry discount rate. This is calculated using equity betas from a market model estimated over the 60 months preceding year t (unlevered betas are calculated using book values of debt and assuming β_D=0).
Standard Deviation Returns is the standard deviation of all industry equity returns, calculated over the 60 months ending in year t-1. *Value* is defined as market value of equity plus debt, scaled by the total book value of assets. New venture capital financing rounds and dollars is the sum of all VC financing rounds and dollar amounts, respectively, in industry i as reported in *Venture Economics*. *Number of IPOs* and *IPO* are the number and dollar values of IPOS in industry i, as reported in *SDC*, respectively. Venture Capital rounds are from *Venture Economics*; IPO data are from *SDC* . Venture Economics industry descriptions are matched to *COMPUSTAT* SIC codes by matching the *Venture Economics* firms that eventually went public to *CRSP/COMPUSTAT*. There are 7761 industry-year observations and 299 industries.

	Mean	**Median**	**Min**	**Max**	**Standard Dev.**
Number of Firms	11.67	8.00	2.00	382.00	16.49
Industry Sales (2005$, Thousands)	14,557.58	5,262.37	4.09	444,026.60	27,694.23
φ	0.15	0.05	0.00	2.35	0.28
R&D/Assets	0.02	0.01	0.00	0.33	0.03
Real Industry Discount Rate	6.06	5.24	(3.53)	35.61	3.58
Industry HHI	3,938.58	3,379.83	504.89	9,995.58	2,180.54
Standard Deviation Returns	0.12	0.11	0.04	0.45	0.04
MTB	1.22	1.02	0.03	22.95	0.77
VC Rounds/# Public Firms	0.17	0.00	0.00	18.00	0.58
Later Stage VC Rounds/# Public Firms	0.07	0.00	0.00	8.00	0.27
IPO Firms/# Public Firms	0.04	0.00	0.00	2.50	0.11
VC Rounds ($M 2005)	37.53	0.00	0.00	31,602.65	487.47
Later Stage VC Rounds ($M 2005)	20.11	0.00	0.00	15,527.57	253.32
IPOs ($M 2005)	31.00	0.00	0.00	11,125.97	196.44
Number of VC Rounds	4.86	0.00	0.00	2,082.00	43.59
Number of Later Stage VC Rounds	2.29	0.00	0.00	784.00	19.75
Number of IPOs	0.56	0.00	0.00	80.00	2.29
Industry Size (Ln $M)	8.34	8.38	1.60	14.18	1.76

Table 6: Dependent Variable = Firms Obtaining New Financing$_{it}$/Total Public Firms$_{it-1}$

This table presents results of estimation of an OLS model in which the dependent variable is the number of currently private firms obtaining new finance, scaled by the number of public firms in industry i (4-Digit *COMPUSTAT* SIC code) during year t-1. Venture Capital rounds are from *Venture Economics* ; IPO data are from *SDC* . *Venture Economics* industry descriptions are matched to COMPUSTAT SIC codes by matching the *Venture Economics* firms that eventually went public to *CRSP/COMPUSTAT*. The sample period is 1972 through 2005. New venture capital financing is the sum of all VC financing rounds in industry *i* as reported in Venture Economics. IPO is number of IPOS in industry i, as reported in *SDC*.

The dependent variables are as follows: The *discount rate* is the estimated industry real discount rate. This is calculated using equity betas from a market model estimated over the 60 months preceding year t (unlevered betas are calculated using book values of debt and assuming β_D=0). *R&D/Assets* is equal to the sum of R&D expenditure divided by the book value of total industry assets). φ is the consumer responsiveness parameter, estimated from the discrete time version of the law of motion of market share (Equation 1 in the text). Control variables are: *Standard Deviation* of industry equity returns; *High HHI* is an indicator variable equal to 1 if the median HHI of industry i over the sample period is in the top tercile of all industries in the sample; *MTB* is equity market capitalization, divided by book value of equity of currently public firms; *Size* is defined as market value of equity plus debt.

	Dependent Variable=# VC Rounds/Total Public Firms						*Dependent Variable=# IPO Firms/Total Public Firms*					
	Model 1		*Model 2*		*Model 3*		*Model 1*		*Model 2*		*Model 3*	
	Coefficient	*t-stat*	*Coefficient*	*t-stat*	*Coefficient*	*t-stat*	*Coefficient*	*t-stat*	*Coefficient*	*t-stat*	*Coefficient*	*t-stat*
Intercept	-0.683	-15.66	-0.668	-14.63	-0.674	-14.8	0.051	5.67	0.060	6.37	0.064	6.69
Real Discount Rate	-0.012	-5.80	-0.012	-5.82	-0.012	-5.9	-0.002	-3.95	-0.002	-3.99	-0.002	-4.05
R&D/Assets	4.624	19.57	4.621	19.56	3.266	12.55	0.124	2.54	0.123	2.51	0.194	3.57
φ	0.050	2.32	0.051	2.35	-0.062	-2.21	0.009	2.01	0.009	2.1	0.005	0.79
High HHI			-0.016	-1.12	-0.039	-2.5			-0.010	-3.23	-0.013	-3.84
φ*R&D/Assets					15.366	11.58					-0.906	-3.27
φ*High HHI					0.092	2.12					0.022	2.45
Std. Equity Returns	2.880	16.38	2.864	16.23	2.834	16.2	0.072	1.96	0.062	1.69	0.060	1.64
MTB	0.160	18.13	0.160	18.16	0.153	17.44	0.026	14.4	0.027	14.53	0.027	14.7
Industry Size (ln)	0.035	8.45	0.034	7.95	0.038	9.1	-0.005	-5.96	-0.006	-6.55	-0.006	-6.9
N	7,761		7,761		-0.674	-14.8	7,761		7,761		7,761	
R-Square	0.1649		0.1651		0.1806		0.0335		0.0348		0.0366	
Adj. R-Square	0.1643		0.1643		0.1797		0.0327		0.0339		0.0355	

Table 7

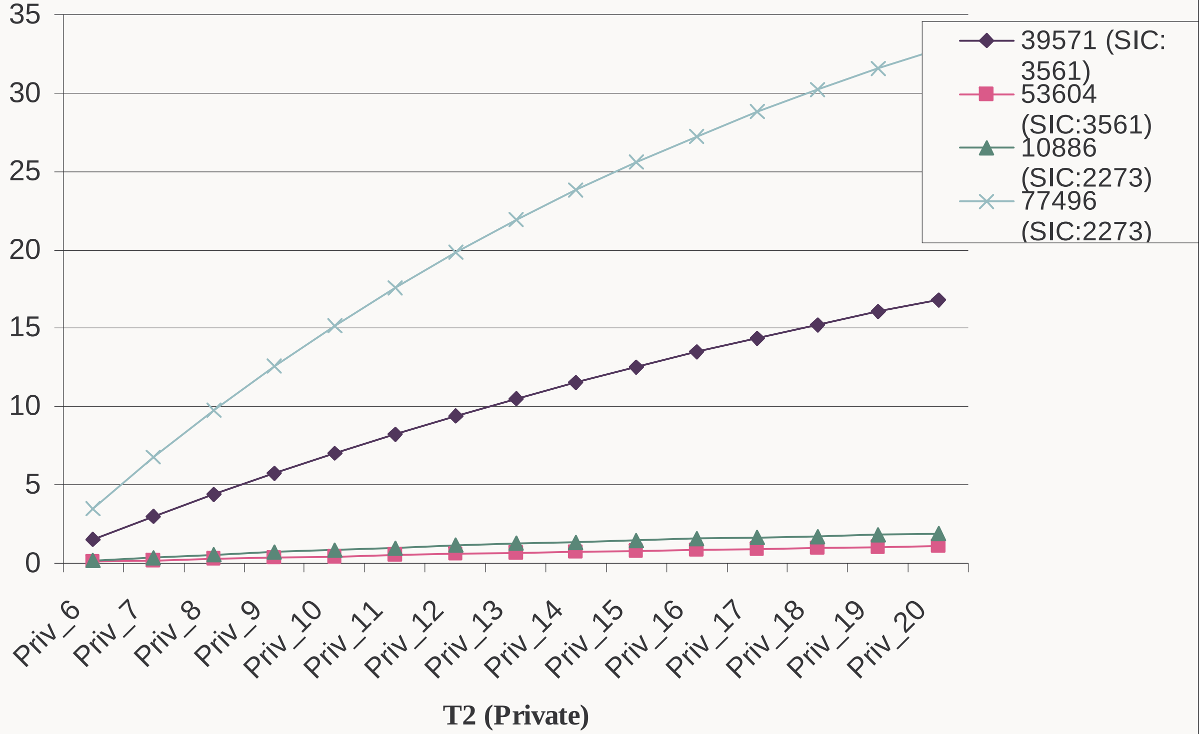
Calibrations: Estimated Value of Obtaining Private Financing in Two Industries (opportunity to increase *s* by 20%)

This table presents the mean difference in the values of firms financed publicly versus privately for two industries, given an opportunity to increase *s* by 20%. This is $\Delta V = V_{private} - V_{public}$, as specified in (17). ZD is set to zero, for straightforward comparisons. (An alternative would be to apply a 20% private market discount to the cost of the innovation. See Hertzel and Smith (1993)). Within each industry, firm-specific α's and s's and were estimated over rolling 5-year periods based on equations (2) and (9), respectively. In estimating s for firm i, s_2 is set to 1 and α is the mean α of all competing firms in the industry. All firms for which we were able to obtain estimates of α and s are included.

Industry Name	Carpets and Rugs	Pumps and Pumping Equipment
SIC Code	2273	3561
φ	0.06	0.49
Mean Value $Mil from Private Financing:		
2 additional years of competitive advantage	4.67	91.65
5 additional years of competitive advantage	11.03	223.10
10 additional years of competitive advantage	20.22	427.74
15 additional years of competitive advantage	28.04	616.57
Mean Alpha of IPO firms Relative to existing firms	0.30	0.02

Figure 2

Calibration: Value of Remaining Private ($Mil), given opportunity to increase s by 20%



This figure presents results from a calibration that estimates each firm's value change given an opportunity to innovate that results in a 20% increase in spending effectiveness (s) for four firms. The Y-axis represents the number of years of first-mover advantage under private financing. Public financing allows for a competitive advantage for five years. The estimated parameters for the firms and industries are as follows:

PERMNO	Company	Data Yr.	s	α	SIC	Ind. Name	φ	Market Share
39571	Graco, Inc	2003	0.027	1,743.34	3561	Carpets and Rugs	0.490	0.056
53604	Gorman Rupp	2003	0.029	802.01	3561	Carpets and Rugs	0.490	0.022
10886	Dixie Group	1999	0.066	411.73	2273	Pumps and Pumping Equipment	0.059	0.062
77496	Mohawk Industries	1999	0.547	392.25	2273	Pumps and Pumping Equipment	0.059	0.332