Intermediation and Voluntary Exposure to Counterparty Risk *

Maryam Farboodi †

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Abstract

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a *core-periphery* network – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections. The predictions of the model are consistent with empirical evidence in the literature.

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†Email: farboodi@chicagobooth.edu

1 Introduction

The years following the financial crisis resulted in an intense scrutiny of the architecture of financial markets. Many prominent economists have argued that the existing financial structure was socially suboptimal due to high systemic risk that emerged from excessive interconnectedness between financial intermediaries.[1] A relatively new, but fast growing, body of work tries to understand the optimal regulatory response to such financial structure. This literature takes the financial structure as given, and assesses appropriate policy responses which minimize the systemic risk.[2] However, any policy which is implemented to mitigate the risk in the current financial architecture could feedback into bank decisions and influence the choice of inter-linkages. Alternative policy should account for endogenous changes to the financial architecture.[3] In this paper I develop a new model where the bilateral exposure of financial institutions emerges endogenously from their profit maximizing decisions. In doing so, I generate the underpinnings of interconnectedness in the financial structure which allows me to evaluate formally the efficiency of the current financial structure.

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk, which is measured as the distribution of total value lost due to bank failure. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By so doing, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a *core-periphery network* – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. In other words, my model predicts that there is a small number of very interconnected banks that trade with many other banks and a large number of banks that trade with a

[1]A high degree of interconnectedness among financial institutions has been frequently recognized by policy makers. Federal Reserve chairman Ben Bernanke and undersecretary of finance Robert Steel, in their senate testimony on April 3, 2008, alluded to potential risk of system wide failure due to mutual interconnections of financial institutions in defending Bear Stearns bailout.

[2]Notable examples of such policies are stress tests designed by the Fed. See Fed (2012), Fed (2013) for more detail.

[3]Policy makers have realized that effective policy design requires a much deeper understanding of how and why banks form their connections. As Donald Kohn, vice chairman of Federal Reserve Board, pointed out in his senate testimony on June 5, 2008: "Supervisors need to enhance their understanding of the direct and indirect relationships among markets and market participants, and the associated impact on the banking system. Supervisors must also be even more keenly aware of the manner in which those relationships within and among markets and market participants can change over time and how those relationships behave in times of stress–not just at banking institutions, but also at other financial firms that play prominent roles in financial markets and whose actions and condition can have an impact on financial stability." However, modeling interconnections has remained a challenge.

small number of counterparties.[4]

In the model, the financial network consists of banks and their lending decisions. Banks need to raise resources for investment either from households or from other banks through credit lines. My model endogenously generates indirect borrowing in the interbank market, which is a prominent feature of both the federal funds market and over-the-counter market for derivatives.[5] If the investment fails and the borrowing bank does not have sufficient funds to pay back her lender(s), it fails and potentially triggers a cascade of failures to the lenders, lenders of lenders and so on.

Banks are profit maximizers. There are two groups of banks in the model: those who have access to a risky investment opportunity, and those who do not. Each bank chooses its lending and borrowing relationships to get the highest expected possible rate on the funding it lends out and the investment it undertakes. When there are positive intermediation rents in the system, profit maximization creates private incentives to provide intermediation, which in turn leads to a particular structure for the equilibrium network. Since intermediation is profitable per-se, in equilibrium, competition implies that the banks who are able to offer the highest expected returns become intermediaries. These banks are exactly the ones who have access to the risky investment technology. On the other hand, a bank who is not an intermediator still wants to earn the highest possible returns, thus opting for the shortest connecting path to investing banks to avoid paying intermediation spread as often as possible. These two forces give rise to a core-periphery equilibrium network in which (a subset of) banks with risky investment opportunities constitute the core (theorem 1, section 5).

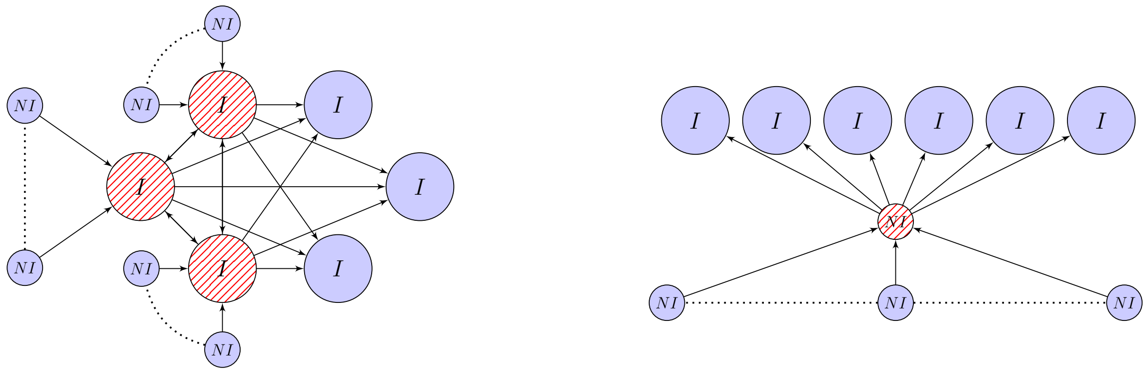
The network generated by the model is socially inefficient. Banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.[6] In other words, when default is costly, efficiency requires reaching the optimal scale of investment while minimizing the loss of failure, which leads to a different structure from the one which arises in equilibrium (theorem 2, section 5).

Finally, I explore diversification incentives of banks as another channel for inefficiency

[4]This structure is consistent with that in the calibrated model of Gofman (2012) as well as empirical evidence on intermediation in several markets, including the federal funds market (Bech and Atalay (2010), Allen and Saunders (1986),Afonso and Lagos (2012) and Afonso et al. (2011)), international interbank markets (Boss et al. (2004) for Austria; Chang et al. (2008) for Brazil; Craig and Von Peter (2010) for Germany and Van Lelyveld et al. (2012) for Netherlands), and the OTC derivatives market (Atkeson et al. (2013)).

[5]Bech and Atalay (2010), Gofman (2012) and Atkeson et al. (2013).

[6]The socially optimal structure is the one which maximizes the equally weighted sum of all bank expected profits.



(a) Equilibrium (b) Efficient

Figure 1: Equilibrium versus efficient structure of the interbank network[7]

due to provision of under-insurance in the network (a la Zawadowski (2013)). In contrast to Gale and Kariv (2007) and Blume et al. (2009) which suggest that the financial architecture does not matter for efficiency, my model predicts that the financial structure can lead to efficiency losses. The main driving force behind this difference is the presence of intermediation rents which prevent social and private incentives from aligning.

1.1 Model Implications

The model predicts that multiple banks with risky investment opportunities can be at the core of the financial system, with high gross and low net exposures among core banks. Consistent with this prediction, there is direct evidence from the financial crisis on substantial exposure among large financial institutions, which entailed runs and subsequent failure of one entity following its counterparty's failure.[8]

In equilibrium, the intermediaries are exposed to excessive risk since they do not contribute to the investment except through intermediation. The social planner prefers leaving such intermediaries out of the chain, replacing them with intermediaries who do not take any extra risk by intermediating. This minimizes the systemic risk without hurting the scale of investment. Thus social planner balances the net gain from investment with the

[7] The labels I and NI refer to banks with and without potential risky investment, the latter solely raising funds from households and intermediating them to Investing banks. See the model for the detail. The dots represent more NI banks.

[8] A prominent example, as reported in the FCIC report on the financial crisis, is the immediate run on holders of Lehman unsecured Commercial Paper (CP) and lenders to Lehman in tri-party repo, such as Wachovia's Evergreens Investment and Reserve Management Company's Reserve Primary Fund, after Lehman failed on September 15, 2008. The first wave of runs was followed by a second wave of withdrawal from Lehman OTC counterparties, most notably UBS and Deutche Bank. Fore more details please see FCIC (2011).

expected loss of default. In contrast, private incentives compare rents, partially in the form of intermediation spreads, with the cost of default. The cost of default is a real cost while intermediation spreads are a mere redistribution of surplus. Consequently, I illustrate that the social and private incentives diverge in several situations. The intuition can be obtained by focusing on Figure 1 that compares the equilibrium interbank network with the efficient one. Banks at the core are hatched in red in each structure.

One can also interpret the implications of the model in terms of *contagion*, the scenario in which shocks which initially affect only a few financial institutions spread to the rest of the financial sector. In the model, investment and funding opportunities arise at different banks, so funding should be channeled from banks with liquidity surplus to the ones with investment opportunities. This decentralized distribution of resources and investment opportunities gives rise to endogenous interbank intermediation. Moreover, the return to risky investment is not contractible, so all the bank liabilities are in the form of debt, which leads to failure if obligations are not met. As a result, lenders and intermediators are exposed to counterparty risk. Because investment is positive NPV, there is an *optimal level* of contagion in order to provide funding for the projects. In other words, even the financial structure chosen by a social planner involves a certain level of contagion when risky investment fails. The important prediction of the model is that the equilibrium interbank network involves *excessive* contagion, more than what is necessary to support the optimal level of investment.

The core-periphery structure implies that many banks are connected to each other only indirectly, a similar notion to weak ties as defined in Granovetter (1973). In the context of the model, the weak ties are intermediator's borrowing and lending relationships. As these relationships are associated with rents, every bank prefers to have many weak ties. In equilibrium, banks who are able to pledge the highest return to their investors have many weak ties and are in the core.

My model can be used to study several policies related to the architecture of the financial networks. In particular, the model provides a new rationale for the introduction of a Central Clearing Party (CCP), as depicted in Figure 1b. In addition, my model can be used to assess policy proposals to impose a cap on the number of counterparties and swaps. Section 6 describes policy implications of the model in detail.

1.2 Literature Review

This paper is related to multiple strands of the literature. First, as a model of interbank networks, this paper is closely related to application of networks in economics (two early seminal papers are Jackson and Wolinsky (1996) and Bala and Goyal (2000)). Jackson (2005), Jackson (2010) and Allen and Babus (2009) provide excellent reviews of the existing work. Most relevant to my paper are Hojman and Szeidl (2006), Hojman and Szeidl (2008) and Babus (2012), which predict star equilibrium equilibrium structures. These models are based on costly link formation, which calls for a minimally connected network. Hence, they cannot generate multi-core structures with high gross and low net exposures for the core, which is a key result of my model and relevant for the interbank model. As well, these models are all undirected networks, and as a result, the distinction between gross and net exposure is unclear.[9]

My work is also related to the literature on contagion and systemic risk in financial networks which started with the seminal work of Allen and Gale (2000) who consider the propagation of negative shocks in a simple financial network. Due to the complexity of modeling strategic interactions which lead to network formation, many papers have focused on properties of large networks or have taken the structure of the network as given.[10] Kiyotaki and Moore (1997) is one of the first papers that look at the formation of credit networks. Although the modeling assumptions of this paper are more closely related to supply chain networks, the implications for contagion and under-insurance can be interpreted in the context of financial networks.

Recent work in this area has also specifically focused on strategic network formation among financial institutions.[11] In particular, Acemoglu et al. (2013) predicts that the equilibrium interbank market can exhibit both under and over connection among banks. Furthermore, Zawadowski (2013) provides a rational for under-insurance (over exposure to risk) by banks due to the high market price of insurance. Neither model provides predictions on the overall structure of the financial network since in both models, banks are located on a ring. My paper best fits in this latter category, and contributes to the literature by allowing more general network structures to form endogenously.

Another line of literature which is closely related to my paper is the one on the role of banks as intermediaries, their optimal debt structure and issues related to insolvency.[12] I

[9]Babus (2012) can have an equilibrium which is an interlinked star network as well. Notably, one exception is Atkeson et al. (2013). While this paper does not explicitly model the banking network, their endogenous bilateral trade structure has similar implications, although through a very different mechanism.

[10]See Eisenberg and Noe (2001), Elliott et al. (2012), Gofman (2011), Gai and Kapadia (2010) and Caballero and Simsek (2012).

[11]See Acemoglu et al. (2013), Blume et al. (2011), Babus (2013), Allen et al. (2012), Moore (2011), Rotemberg (2008), Zawadowski (2011), Zawadowski (2013),Bluhm et al. (2013) and Cabrales et al. (2012).

[12]An incomplete list includes Diamond (1984), Rochet and Tirole (1996), Kiyotaki and Moore (1997), Moore (2011), Lagunoff and Schreft (2001), Leitner (2005), Cifuentes et al. (2005), Dang et al. (2010), Dasgupta (2004), Acharya et al. (2012), Acharya and Yorulmazer (2008) Bhattacharya and Gale (1987),

add to this literature by specifically modeling the role of banks as intermediaries among each other and the corresponding implications for the structure and efficiency of interbank market as well as systemic risk.

Finally, there is a large literature on bargaining and intermediation in general settings (Abreu and Manea (2012a), Abreu and Manea (2012b) and Elliott (2013)), as well as in (financial) networks (Gale and Kariv (2007), Manea (2013), Gofman (2011) and Babus (2012)). In all of these models except Babus (2012) intermediaries are determined exogenously. In my model, as well as Babus (2012), certain agents endogenously assume the role of intermediaries, which in my model, can lead to welfare losses in equilibrium.

The rest of the paper is organized as follows. Section 2 lays out the basic environment. Section 3 provides a simplified version of the economy with four banks and solves for the equilibrium and social planner problem. Section 4 specifies the detail of the lending contracts and equilibrium concept. Section 5 provides general results for the economy with an unrestricted number of banks. Section 6 discusses policy implications of the model. Section 7 concludes.

2 Model

The model has three periods, $t = 0, 1, 2$, and one good, which I refer to as funding. There are two types of agents: banks and households. There are K banks in the economy: banks that randomly get risky investment opportunities (type I) and banks that do not (type NI). Let $\mathbb{I}$ and $\mathbb{NI}$ denote the set of I and NI banks, respectively, and let $\mathbb{N} = \mathbb{I} \cup \mathbb{NI}$. There are k_I banks of type I and k_{NI} banks of type NI.

The financial system consists of banks and their bilateral exposures. The bilateral exposures represent lending and borrowing relationships among banks. Note that bilateral exposures among banks are quite complex in reality. Banks can be exposed to each other through multiple channels: secured and unsecured lending, derivative contracts, and similar asset holding. For the purpose of this paper, I will restrict interbank relationships to exposures through debt contracts (lending). Bank i lending to bank j through a debt contract is exposed to bank j since if bank j fails, it will not be able to pay bank i back, which affects the balance sheet of bank i and might cause i to fail.

There is a second group of agents in the model, namely, households. There are K continuums of households in the economy. Continuum h of households has random size

Bolton and Scharfstein (1996), Diamond and Rajan (2005), Farhi and Tirole (2013) and Gorton and Metrick (2012).

$s_h \in [0, \bar{s}]$ distributed via CDF $H_h(.)$. Each household is endowed with a unit per-capita endowment. Throughout the paper, I will use the following convention: $\tilde{x}$ denotes a random variable, and x denotes the realization of that random variable.

The investment opportunity is a risky asset. Each bank I receives the opportunity to invest in the risky asset with probability q, which is iid across all I banks. Let $\tilde{\mathbb{I}}_R$ denote the random variable corresponding to the subset of $\mathbb{I}$ that receive the opportunity, and let $\mathbb{I}_R$ be the realization of such subset.

Let $\tilde{R}_i \in [0, \bar{R}]$ denote the (per-unit) random return of bank i's investment in the risky asset. The investment is linearly scalable. I assume the support of the asset return distribution has two mass points: the project succeeds with probability p and returns U, and fails with probability $1-p$ and returns 0:[13]

$$\tilde{R}_i = \begin{cases} R & \text{with probability } p \\ 0 & \text{otherwise.} \end{cases}$$

Besides the risky investment opportunity, each bank i (of type I or NI) has a value V_i, which is the value of the other businesses, assets, and services the bank provides. If the bank fails for any reason, this value is lost.[14][15] For simplicity, I will assume $V_i = V_I$ for every $i \in \mathbb{I}$ and $V_j = V_{NI}$ for every $j \in \mathbb{NI}$.

Bankers do not have any wealth. They can raise funding from two sources in the form of debt. First, at $t = 0$, a bank can raise resources from households if it gets a chance to do so. Both I and NI banks can potentially raise resources. Bank i can access a continuum i of households with probability p^f. Let $\mathbb{N}_F$ denote the subset of all banks that receive the opportunity to raise funding from households. Each bank who receives such an opportunity raises total s_i units from the continuum of households it meets. Because each set of households is a continuum, they are competitive and they lend their endowment to their corresponding bank if they break even.[16] Second, a bank can borrow from other banks at $t = 1$. To do so, at $t = 0$, it should enter into *potential borrowing contracts* with them. Potential borrowing agreements can be thought of as credit lines.

[13]The binomial nature of project return is purely for simplicity and is inconsequential for the results. Assuming $\tilde{R}_i$ is distributed according to a general cumulative distribution $F(.)$ does not change the results.

[14]This value accrues to the banker himself. This model is isomorphic to one with bankruptcy costs that are borne by the bankers in the event of failure.

[15]James (1991) finds that losses due to bank failure are substantial, losses on assets and direct expenses averaging 30% and 10% of the failed bank's assets, respectively.

[16]Households breaking even (i.e., zero rate of return) is a normalization. Any constant positive rate of return would work as well.

I model the financial system as a network. The financial network is a directed graph $G = (\mathbb{N}, E)$, where $\mathbb{N} = \{1, 2, \cdots, K\}$ is the set of nodes and $E = \{e_{ij}\}_{i,j \in V}$ is the set of edges. Each node is a bank, and edge $e_{ij} \in E$ is a credit line from bank i to bank j. Graph G represents the collection of banks and their credit lines.[17]

Each bank chooses its potential borrowing and lending relationships, that is, credit lines in which it is a borrower and a lender, to maximize its expected profit net of failure cost.

The timing of the model is as follows: At $t = 0$, the funding opportunities are realized and the potential lending and borrowing contracts are formed. A link e_{ij} means bank j *can* borrow from i in the period that follows. At $t = 1$, investment opportunities are realized and actual lending happens only along (some of) the links formed at $t = 0$.[18] At $t = 2$ random returns are realized, and banks that are not able to pay back their creditors fail. Hold precautionary liquidity is ruled out, so banks lend or invest as much resources as they are able to raise.

3 Economy with Four Banks

Starting with a simplified version of the model before formal description of the contracts is useful.[19] Here I will completely characterize all the equilibria in an economy with four banks, which illustrates the main forces of the model. Then I will provide a full description of the contracts under which the same intuition carries over to the general case.

Assume there are two I and two NI banks, $\mathbb{I} = \{I_1, I_2\}$ and $\mathbb{NI} = \{NI_1, NI_2\}$. Also, assume I banks raise no funding from households, whereas NIs raise one unit each.[20] As a result, each bank I needs to secure funding on the interbank market at $t = 0$ to be able to invest in its project later, at $t = 1$, if it gets an investment opportunity.

As explained in the previous section, to borrow on the interbank market at date $t = 1$, banks need to enter potential lending agreements (establish credit lines) at $t = 0$. Let $l \to b$ denote a credit line from bank l to bank b. Each credit line (opened at $t = 0$) is a promise by the lender to deliver at least one unit (at $t = 1$) if the borrower receives an investment opportunity, or if the borrower has a credit line to another bank that has

[17] $e_{ij} \in E$ only if at $t = 1$ funding is lent along this credit line with non zero probability. Otherwise e_{ij} is just removed from E.

[18] I should point out that the timing of the funding shock is inconsequential; that is, if one assumes funding shocks are realized after the network is formed, all the results remain intact. What is important is that investment opportunities are not known at the time lending decisions are made.

[19] Complicated network structures can arise and the contracts should determine the flow of funding in every possible network. As a result, full description of contracts is quite cumbersome.

[20] Zero is just a normalization. I will provide more general intuition later.

(a) Infeasible set of credit lines

(b) Feasible set of credit lines

Figure 2: Two possible sets of credit lines between two NI banks and two I banks.

received an investment opportunity. Banks cannot default on their promises in the following sense: even if all the potential borrowers of a bank l draw on their credit lines, l must have sufficient funds to lend each of the one unit.[21] This restriction puts an endogenous limit on the number of credit lines a bank l can establish at $t = 0$. Specifically, the number of credit lines in which bank l serves as a lender is limited by the total amount of funding it has raised, either from households at $t = 0$ or through a credit line from other banks (in which l is a borrower). For a concrete example, consider Figure 2: in 2a, NI_1 has the unit it has raised from households, but no other source of funding. In particular, credit line $NI_2 \to NI_1$ does not exist. Moreover, NI_1 has a credit line to each I bank, so both $NI_1 \to I_1$ and $NI_1 \to I_2$ exist. In 2b, NI_1 has two units pledged to it, one from households and one through credit line $NI_2 \to NI_1$. Now consider the case in which both I banks receive investment opportunities. In both structures, NI_1 has promised one unit to each I bank. However, in 2a, it will not be able to keep its promise. As a result, 2a is not a feasible structure. The above restriction is formalized in the following assumption:

Assumption 1. *Each realized lending has a minimum size, normalized to one unit.*

This assumption implies a step cost function for number of lending contracts. If the maximum number of lending relationships that can simultaneously bind for bank i is less than the total funds pledged to it, zero cost is associated with each lending contract. The cost goes to infinity if the number grows beyond this threshold.

There is an exogenous division of expected net surplus that allocates a strictly positive share of net surplus, in expectation, to every bank involved in raising funding from households, intermediating funds on the interbank market, and investing in the risky project for any realization of investment opportunities. Here, I will use a specific rule that clarifies the intuition. The results go through for a much more general class of rules for surplus division,

[21] That is, I only allow for structures in which no lender would ever default on its promises.

and I will specify sufficient conditions when I provide the full description of contracts in section 4.1.

Specifically, I assume that if bank i raises funding from households and lends directly to bank j that makes the investment ($i \to j$), j and i receive in expectation a share $1-\alpha$ and α of expected net surplus of the project, respectively. Alternatively, if i raises the funding, lends to k which in turn lends to j which invests ($i \to k \to j$), then j, k, and i receive $1-\alpha$, $\alpha(1-\alpha)$, and α^2 shares, respectively.[22][23]

All the contracts are bilateral. The final return of the project at $t = 2$ is not contractible, so all the contracts are in the form of debt. However, the contract can be written contingent on all date $t = 0$ and $t = 1$ outcomes, specifically on the network structure as well as the realization of investment opportunities.[24] So bilateral contracts are *contingent debt* in which the face value of debt is set such that given the network and the realization of investment opportunities, each bank along the intermediation chain receives its appropriate share as described above.

A direct lending relationship between an I and NI is socially desirable if

$$pR - 1 > (1-p)(V_I + V_{NI}). \tag{1}$$

That is, the net expected return from the project should cover the expected cost, which is the expected loss in outside value of the two banks, because both the lender and borrower fail if the project fails (borrower cannot pay the lender and lender cannot pay the households from which he borrowed. Recall there is no liquidity choice). The participation constraints for the lender and borrower are the following, respectively:

$$(1-\alpha)(pR-1) > (1-p)V_I \tag{2}$$

$$\alpha(pR-1) > (1-p)V_{NI}. \tag{3}$$

Assume condition 1 as well as the participation constraint of both I and NI are satisfied, so that an NI bank always prefers direct lending to an I bank to staying in autarky.[25]

[22] There are multiple papers who provide evidence on existance of intermediation rents. For instance, Li and Schürhoff (2012) provides evidence that the dealer network for municipal bonds is a core-periphery network, that dealers charge spreads and spreads decrease as we move toward peripheral dealers.

[23] I will provide the precise generalization of this rule in section 4.3. The intuition is that starting from the bank that invests, the share of each bank along the chain falls at rate α and the bank furthest away from the investor receives the remaining share. The main property of this rule is that each borrower does not care (in terms of share he receives) where the funding come from, which helps me clearly convey the main mechanism of the paper very.

[24] That is which I banks get a chance to invest at date $t = 1$.

[25] Throughout, I would also assume $\alpha(pR-1) > (1-p)V_{NI}$, so that lending via one intermediator is

Note that with only two banks, whenever direct lending happens, it is efficient. As a result, with only one lender and one borrower, equilibrium can only exhibit under-investment, in the form of under-lending,[26] compared to the socially optimal outcome. Surprisingly, I will show that with more banks and the possibility of multiple investment opportunities, the equilibrium involves over-lending among a certain group of banks.

I first analyze the model under the following assumption. Later I relax this assumption and incorporate the additional force it introduces.

Assumption 2. *If a bank i has credit lines to multiple I banks with realized investment opportunities, all of its funding is allocated randomly with equal probability to exactly one of them. An I bank that receives an investment opportunity invests all of its funds in its own project.*

I start with the above assumption for two main reasons: first, it simplifies the exposition. More importantly, it allows me to analyze pure intermediation and diversification separately. Pure intermediation, which survives under the above assumption, is to channel funds to different points of the financial system where investment opportunities arise, and avoid under-investment. However, this assumption disables diversification. Intuitively, when a bank j randomly channels all of its available funds to a single one of its borrowers, if that borrower fails, j fails as well. By induction on the length of path, when an I bank fails, any bank that has lent to it (directly or indirectly) fails. Because diversification is fairly well-studied in a number of different contexts, I choose to abstract away from it in order to focus on the novel point of the model. I will re-introduce diversification in section 5.1 and show how it interacts with the mechanism introduced in this paper.[27]

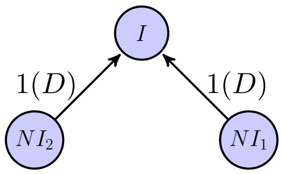
3.1 Equilibrium

I borrow the equilibrium concept from the matching literature as defined in Roth and Sotomayor (1990). It can be thought of as an extension of pairwise stability defined in Jackson and Wolinsky (1996), generalized to allow for any number of banks to participate

always viable as well.

[26] Or equivalently, under-exposure to banks that have investment opportunities. This phenomena is the classic trade break down due to high outside options, as pointed out in Gofman (2011).

[27] Random allocation of all funds is not sufficient to kill diversification at the level of an I bank because I banks can have cross-lendings. I need to make an assumption that implies each I bank is involved in at most one project, which is achieved through the second clause of the above assumption.



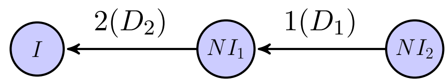


Figure 3: Different lending structures among two NI and one I bank. Each NI bank has one unit of funding and I has zero. 3a: Direct lending from two NI banks to an I bank. 3b: Intermediation chain among two NI banks.

in the deviation.[28][29]

Definition 1. *A network structure G is blocked by a coalition B of banks if there exists another (feasible) network structure G' and a coalition B such that*

(a) G' can be reached from G by a set of bilateral deviations by $b, b' \in B$ and unilateral deviations by $b \in B$.

(b) Every bank $b \in B$ is strictly better off in G' than in G.

Definition 2. *A* **group stable** *network is one that is not blocked by any coalition of banks.*

Before moving to equilibrium characterization, consider the two lending structures depicted in Figure 3. The two lending arrangements differ in that in 3a there is no intermediation, while in 3b there is. It is straightforward to verify the face values of debt in each

[28]In other words, the notion of equilibrium is equivalent to Strong strict Nash equilibrium of Aumann (1959) adopted for a network.

[29]Two points are worth mentioning here: first, a subtlety exists in adopting a concept from matching to a network framework. In most matching models, the utility of each agent is only own-match dependent and does not depend on the rest of the matching. So the utility of the blocking coalition only depends on the matches among them, as if they go to autarky together, and whether or not a coalition blocks a matching does not depend on what the rest of the agents outside the coalition do. However, in a network model, utility of the blocking coalition can still depend on the rest of the network, so what banks outside the coalition do matters. I am assuming they don't change their actions. This equilibrium concept is different from the one in which the blocking coalition goes to autarky, which is referred to as the β-core in the network literature.

Second, why would I use this more complex equilibrium concept? The main idea is that this paper is about *interbank intermediation*, and such intermediation requires at least one intermediary, a group of banks to be intermediated, and a group of banks to be intermediated to. As a result, pairwise stability is not the appropriate notion for addressing the relevant deviations.

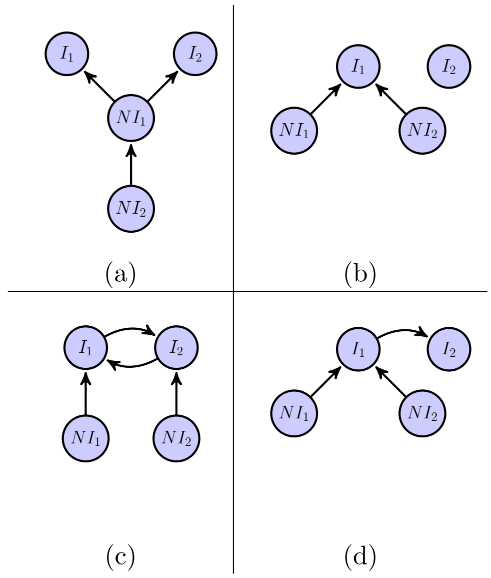


Figure 4: Possible equilibria for an economy with two I and two NI banks

structure:

$$\text{3a: } D = \frac{\alpha(pR-1)+1}{p}$$

$$\text{3b: } D_2 = D$$

$$D_1 = \frac{\alpha^2(pR-1)+1}{p} \leq D$$

$D_2 - D_1$ represents the *intermediation spread*. Simply put, the face value of debt is set to ensure that in expectation, each party (including the intermediator) receives its share of expected net surplus. However, given that inability to repay debt obligation entails costly default, rents come at a cost.

Now consider the network structures in Figure 4. These structures are the only possible equilibria of the economy with four banks. Every other feasible structure is not even pairwise stable; that is, they are blocked by a coalition of size two.[30]

To study the individual incentives in equilibrium, let us focus on two specific equilibria in Figure 4, namely, 4a and 4c. Intuitively, the main difference among the two structures 4a and 4c is the following: in 4c, regardless of which bank(s) have the investment opportunity, all the banks are involved as either investor, intermediator, or final lender in every investment, whereas in 4a, if only one I invests, the other I bank is not exposed to the risk of investment failure. Given that the two banks' project returns are iid, the expected

[30]The proof of this claim as well as all other proofs are provided in the appendix.

return in a financial structure only depends on the scale of investment and not on how it is distributed among investors. Because in both 4a and 4c, both units are always invested, the net expected return of the risky investment is the same and the two structures only differ in cost of default, which I argued is lower in 4a.

Now assume we are in 4a and consider the joint deviation by $\{I_1, I_2, NI_2\}$ leading to 4c, as depicted in 5. Observe that when only I_2 receives the investment opportunity, I_1 serves as the intermediator for NI_1 and captures the intermediation rents. However, it fails if I_2 fails. So the incremental cost of default in 4c (compared to 4a) is born by I banks, that is, precisely the banks that can choose to be out of the chain of intermediation (as in 4a). However, if the intermediation spread $(D_2 - D_1)$ is high enough, I banks would want to deviate from 4a to 4c in order to earn the spread. In other words, I banks intentionally choose to expose themselves to this incremental cost, which is counterintuitive. The insight is that the financial system contains rents that can be captured only through voluntary exposure to counterpart risk, and if these rents are high enough, banks would choose to incur the additional risk in order to capture them.

Finally, one can ask why an NI bank agrees to be part of the deviation. The answer is that each bank chooses to lend to counterparties that offer the highest rate of return. Given that intermediation spreads exists, being close to the bank that invests translates into higher returns, and in 4a, NI_2 is always far from the bank that invests. As a result, it also has an incentive to join a deviation which leads to a structure in which it is as high as possible in the intermediation chain.

In sum, if the ratio of intermediation rent associated with one unit relative to expected cost of default is higher than a certain threshold, 4a ceases to be an equilibrium and 4c becomes an equilibrium instead.

Alternatively, assume we are in 4d. Now whenever I_1 does not get an investment opportunity but I_2 does, I_1 receives an intermediation spread for two units, whereas his cost of default stays the same as in 4c. So the per-unit intermediation rent he would require to maintain the link $e_{I_1 I_2}$ is lower compared to 4c. Nevertheless, if the intermediation rents are too low, I_1 would unilaterally deviate and stop lending to I_2, and 4d would not be an equilibrium anymore.

Formally, let $X = pR - 1$ be the net expected return of the project. Also, let $\kappa = \frac{\alpha(1-\alpha)X}{(1-p)V_I}$, which is the ratio of the intermediation spread per unit intermediated over the expected cost of default due to intermediation for an I bank. Note that the participation constraint of the I bank (2) implies $\kappa > \alpha$. Finally, let $\bar{\kappa} = 1 + \frac{q}{2(1-q)} > 1$ and $\underline{\kappa} = \frac{(1-q)(1-\alpha)}{\alpha(\alpha(2-q)-1)} \frac{V_{NI}}{V_I}$. The next proposition characterizes the range of parameters for which each financial structure is

(a) Stable (b) Unstable

Figure 5: Joint deviation by $\{I_1, I_2, NI_2\}$ leading from structure 4a to 4a.

an equilibrium.

Proposition 1. *For every set of parameter values $(q, p, R, \alpha, V_I, V_{NI})$, the following conditions characterize all the equilibria:*

(a) 4a is an equilibrium when $\kappa \leq \bar{\kappa}$,

(b) 4b is an equilibrium when $\kappa \leq \frac{1}{2}$,

(c) 4c is an equilibrium when $\kappa \geq 1$,

(d) 4d is an equilibrium when $\kappa \geq \frac{1}{2}$.

Figure 6 depicts the range of equilibria as a function of κ. Note that except for 4b, all the other equilibria are core-periphery structures.[31]

The interesting observation is the range in which 4c is an equilibrium. One would intuitively think that 4c will not be an equilibrium below $\bar{\kappa}$. However this is not the case. The problem is that although each I prefers to be in 4a but in order to deviate to that structure they need both NI banks to agree, in particular they need one NI to be the periphery. No NI agrees to be part of such deviation. In other words there is no way to make both NIs better off than what they get in 4c and 4d[32], so there is no way to get the NI banks to deviate. The only remaining deviation is unilateral (or bilateral) by the two I banks, which requires the intermediation spread not to cover the cost of default. This implies a lower threshold of 1 for κ.

The following numerical example is useful to clarify the above result.

[31] 4b corresponds to trade breakdown due to high outside options, as pointed out by Gofman (2011), which is a well-known result in the bargaining literature.

[32] They each get the same under both structures.

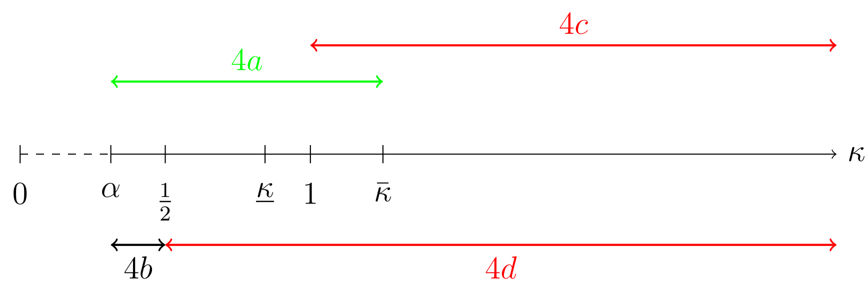


Figure 6: Equilibria of the economy with four banks as a function of κ, the ratio of per-unit intermediation spread over the expected cost of default for I bank due to intermediation. The green equilibrium is efficient. The two red ones are inefficient due to overconnection, and the black one is inefficient due to underconnection.

Let $q = 0.5$, $\alpha = 0.4$, $V_I = 10$, and $V_{NI} = 5$. We have $\bar{\kappa} = 1.5$ and $\alpha < \frac{1}{2-q}$. Let $p = 0.8$ and $R = 20$. Then $\kappa = 1.8 > 1.5$, so both 4c and 4d are equilibria. Note that in 4c, either I bank would like to deviate jointly with the NI periphery of the other I bank and deviate to 4d, but because NI banks are equally well-off in the two structures, this deviation is not valid, so the two equilibria coexist.

Now consider $R = 15$, where $1 < \kappa = 1.32 < 1.5$. In this range intermediation rents are sufficiently low that I banks prefer 4a to 4c, so 4a is an equilibrium. However, they are not able to jointly deviate back to 4a if they start from 4c.[33] As a result 4c is an equilibrium as well. Finally, intermediation rents are sufficiently high that an I bank is willing to be exposed to counterparty risk if he intermediates two units, so 4d is also an equilibrium.

Take an even lower return, $R = 10$, for which $0.5 < \kappa = .84 < 1$. In this range, intermediation rent associated with one unit is not sufficient to cover the incremental cost of default due to exposure to counterparty risk for an I bank, so 4c ceases to exist. Finally, note that in order to have participation constraint of I bank satisfied and $\kappa < \frac{1}{2}$ it is necessary that $\alpha < 0.5$. Now consider an even lower $R = 6$,[34] $\kappa = .456 < 0.5\bar{\kappa}$ and 4d is not an equilibrium either.

One would wonder which structure is efficient, that is, maximizes the total surplus.[35] Under (1), maximizing scale of investment is efficient because the return on the asset exhibits constant return to scale. So the social planner's problem reduces to minimizing expected loss of default due to failure of project(s). Note that when a project fails, not only

[33] In fact there is no deviation which leads to 4a for any parameter values in this model.

[34] Note that the both participation constraints are still satisfied.

[35] To be precise, I am interested in the constraint efficient solution, that is, the one that maximizes the total surplus without violating the participation or minimum size constraint.

does the I bank that has undertaken the project fail,[36] but *contagion* also occurs: I will not be able to pay its lenders back, and based on the lenders' portfolios, they sometimes fail as well. Consequently, the solution to the social planner's problem is to have one NI bank be the intermediary, borrow from the other NI, and lend to both Is (4a).

This result is quite intuitive: the social planner's objective is to maximize total net return from the projects minus the expected loss, and he does not care about the division of surplus. Assume NI_1 is the NI bank chosen as the "intermediator." Given that maximizing the scale of invest requires that NI_1 lends to I_1 and I_2 when either of them has an investment opportunity, it can as well intermediate the funding raised by NI_2, and this intermediation does *not* expose NI_1 to any extra risk.[37] As such, the scale of investment is maximized and the cost in the event of failure is minimized. A similar intuition goes through in the general case.

Knowing the efficient network provides more insight on the structure of the equilibria depicted in 4. First note that in the above numerical example, there are always multiple equilibria. If $\kappa < \bar{\kappa}$ efficient and inefficient equilibria coexist. Otherwise, both equilibria are inefficient.

Region $1 < \kappa < \bar{\kappa}$ is interesting: in this region, there is an inefficient equilibrium in which the banks with risky investment opportunities are in fact willing to decrease their (inefficient) exposure to counterparty risk and deviate to a more efficient equilibrium. However, they are not able to convince their lender banks to agree to a lower rate and keep funding them, so they are *stuck* in the bad, high-risk equilibrium.

Now consider the financial network in 4b. Note that this structure differs from the other three in that it exhibits under-investment due to under-lending: bank I_2 never gets to invest even if it receives an investment opportunity, because it does not have access to any source of funding. If intermediation spreads are high enough to cover the expected cost of default[38] borne by the *well-funded* I bank (here, bank I_2), I_2 and I_1 will deviate by adding $e_{I_2 I_1}$, and the economy switches from one inefficient equilibrium to the other. This scenario happens if $\kappa \geq \frac{1}{2}$. One would think that the two NI banks can also jointly deviate with I_1 to 4a, if being intermediated does not dramatically decrease the rate the lender receives (weighted by how often it gets it).[39]. Such deviation would be possible if $\alpha > \frac{1}{2-q}$ and $\kappa > \underline{\kappa}$. I show that this deviation is never feasible.

[36] I will consider a variant of the model in which banks diversify, so failure of one's own project does not always lead to the failure of the bank. A similar argument is applicable there.

[37] And it also enables him to form lending contracts with both I banks.

[38] due to exposure to counterpart risk

[39] That is, if one NI bank agrees to be a leaf lender, receives a lower rate but instead more frequently

In other words the relative position of $\underline{\kappa}$ and $\frac{1}{2}$ depends on the particular choice of parameters. So the question arises whether there can be a non empty range $(\underline{\kappa}, \frac{1}{2})$ such that $\frac{1}{2-q} < \alpha < \underline{\kappa}$, where 4a, which is the sole efficient equilibrium, is the unique equilibrium. Interestingly the answer is no since $\frac{1}{2-q} > \frac{1}{2}$. As a result there are always multiple equilibira: for low intermediation rents there are both efficient and inefficient equilibria, and when the intermediation rents become sufficiently large, $\kappa > \bar{\kappa}$, all the equilibria become inefficient.

Finally, note the role of the particular division of surplus used here: given that a borrower (final and intermediate) does not care about the source of the funding, for him, the only gain to shortening (or any change) in the intermediation chain is if he becomes an intermediary and is able to absorb intermediation rent, without a change in his share in other states where they don't intermediate. This particular choice allows me to characterize all the equilibria as a function of a single variable κ. For more general rules (explained in the next section), a similar argument applies but the regions are more complex.

3.2 Discussion

Here I discuss the modeling assumptions and how they induce the results, and then provide some explanation for efficiency versus stability in the context of the model.

The markets are incomplete: first, the return on the investment is not contractible, so the only instrument for borrowing funds is debt. Second, the decision to open a credit line is made before the realization of investment opportunity, and once investment opportunities are realized, lending can only happen through established credit lines. In other words, banks cannot change their lending decisions based on the realization of the investment opportunities.[40] This assumption is to capture the long-term relationship between lenders and borrowers in the inter-bank market, and is consistent with the findings of Afonso et al. (2011), which shows that in the federal funds market, approximately 60% of the funds an individual bank borrows in one month persistently come from the same lender.

Third, there is a bargaining friction of the following form: for every unit of funds originated[41] at bank l and intermediated to bank b through a sequence of intermediaries, every intermediary along the chain, as well as the initial lender and final borrower, get strictly positive shares of the surplus, and every bank's share is decreasing in the number of intermediaries. Positive intermediation spreads are reported in both federal funds market (Gofman (2011) and Gofman (2012)) and OTC market (Atkeson et al. (2013)).

[40] To be precise, they can choose among their already established credit lines but cannot add a new line.
[41] That is, raised from households.

Finally, I assume that each lending relationship has a minimum size constraint, so ex-ante each bank effectively faces a constraint on the number of credit lines it can pledge to other banks based on the funds it has available (either borrowed from households or from other banks).[42] This constraint implies that not every bank with excess funding can lend to every bank with an investment opportunity.

The minimum size constraint combined with the ex-ante formation of credit lines implies intermediation is necessary to increase the scale of investment. Strictly positive intermediation rents imply banks compete to be the intermediator to capture the rents, and if they cannot, they choose to be as close as possible to investor banks to avoid paying intermediation rents. As a result, in equilibrium, banks that are able to offer the highest rents, which are exactly the banks with potential risky investment opportunities, become the intermediators and they get many direct lenders. This implies a core-periphery structure.

Furthermore, given that contracts are debt and default is costly, efficiency requires maximum scale of investment while minimizing the cost of failure, which would imply that for any investment, every bank in the intermediation chain should either provide sufficient funds from outsiders or make the investment. This turns out not to be the case when intermediation spreads are high, which leads to excessive exposure to counterparty risk. The main source of inefficiency is that the gains from intermediation are purely redistributional, whereas the loss is incremental.

In this section, I abstracted away from diversification. Shutting down diversification allowed me to clearly demonstrate an important force of the model that has been addressed before, and has important implications for the equilibrium structure of the interbank network. Each bank's motive to absorb intermediation spreads, along with differential abilities of different banks in offering rates of return on their borrowing, leads to a specific equilibrium structure. Moreover, the same rent-seeking behavior mis-aligns private and social incentives to form lending and borrowing relationships, which exposes the banking system to an excessive loss of default. What makes this channel particularly interesting is that it differs in spirit from the classic channel for *contagion*, which has been studied in recent years (Kiyotaki and Moore (1997), Allen and Gale (2000) and Acemoglu et al. (2013), among many others). Here, banks choose to expose *themselves* to an excessive probability of failure in order to be able to absorb intermediation rent. So the additional loss borne by the system is failure of this particular bank, above and beyond any contagion from this bank to other banks. In other words, the above assumption allows me to fix all other banks

[42]Bech and Atalay (2010) reports that only loans above \$1M are reported in their sample of federal funds transactions.

that fail due to failure of an investment, and only focus on the possible incremental cost of the particular intermediator bank.

Note that three roles exist in the financial system: lender (i.e. raise money from households), borrower (i.e., invest in the risky project), and borrower-lender (i.e., intermediator). The problem is set such that lender and borrower are *inherent* properties of a bank. Then the question of efficiency boils down to which bank is more suitable to take on the third role. In other words, which ability, being a lender-by-design (NI) or borrower-by-design (I), legitimizes (from a social perspective) being an intermediator. The answer is that although an intermediator both borrows and lends, a suitable one is a lender-by-design, and the intuition is interesting: the social incentive for having a bank involved in an investment depends on the total contribution of the bank minus its expected loss due to failure. If either party is not making its fundamental contribution but is taking on a secondary role, the social planner might be able to enhance the surplus by removing him completely and allocating the secondary role to a different bank.

For a borrower, the fundamental contribution is the access to investment opportunity, whereas for the lender, it is the ability to provide funds otherwise not available for investment. If a borrower I_1 is not investing, his presence in the intermediation chain offers no social gain if all the funds can still be channeled to the borrower that is taking on the investment (through a different lending structure). Moreover, exposing I_1 to risk of default by I_2 leads to a social loss. In contrast to borrowers, lending is optimal for each lender regardless of who is making the investment, so if one of them acts as intermediator, the exposure to risk does not increase.

Furthermore, I bank raising no funds from households is not necessary for the inefficiency result. What is important is that I bank's contribution to scale of investment should not be sufficient to justify its risk-taking behavior. To be more precise, assume I_1 has raised $\epsilon < 1$ funds from households. Recall that without intermediation, the participation constraint of a direct lender (I_1 in this case) to I_2 requires that $\epsilon\alpha(pR-1) \geq (1-p)V_I$. Let $\hat{\epsilon}$ be the amount of funds for which the above inequality holds with equality. Then for any $\epsilon < \hat{\epsilon}$, it is more efficient that an NI bank with one unit raised from households do the intermediation as opposed to I_1.

Finally, note that three of the four equilibria, 4a, 4c, and 4d cannot be Pareto ranked. The main intuition is that the rent-seeking behavior of banks implies that they attempt to change the distribution of surplus in their own favor and create inefficiency while doing so; as a result, not everyone's profits are hurt. So neither network structure Pareto dominates the other.

4 General Specification

Because few constraints are imposed on the structure of the interbank network, complex structures can form. In particular, multiple intermediation chains might exist between two banks. As a result, I need a rich set of contracts to specify how the funds flow in the network given a network structure and a realization of investment opportunities.

4.1 Lending Contracts

Lending contracts are formed before banks receive their investment opportunities. e_{ij} represents bank i's conditional commitment to lend to bank j, subject to the terms of the lending contract, to be explained shortly. In this sense, lending contracts are *conditional credit lines*. In short, bank i has to lend at least one unit to bank j if j receives an investment opportunity, or if j is able to intermediate the fund to an investment opportunity that i cannot access through fewer intermediaries. In other words, set $\{j|e_{ij} \text{ exists}\}$ is the set of all banks that i chooses to be able to lend to, or bank i *relationship borrowers*, and set $\{j|e_{ji} \text{ exists}\}$ is the set of bank i *relationship lenders*.

Each lending contract has a minimum size constraint, normalized to 1, as explained in section 3 and formalized in assumption 1.[43]

There is perfect information: every bank knows the set $\mathbb{I}$ and $\mathbb{NI}$, the structure of the formed lending contracts, the realization of the investment opportunities and the realization of final returns. However, as already mentioned, markets are incomplete: first, the realization of returns are not contractible, so all the contracts are of the form of debt. Second, the potential lending contracts are formed before investment opportunities are realized.

Given that the only restriction on lending relationships is 1, financial network G can be quite complex. The following definitions are useful to explain the remainder of the model.

Definition 3. *Given financial network G, a "path" from bank i to bank j is a sequence of banks $\{i_1, \cdots, i_m\}$ such that $e_{i_d i_{d+1}} \in E$ for $\forall d = 1, \cdots, m-1$.*

[43] To be precise, i needs to have sufficient units pledged to him such that for every realization of investment opportunities and for any set of binding contracts, he is able to lend at least one unit to each borrower whose contract is binding. The idea is that each unit of funding comes into the financial system at some bank j, meaning bank j raises that unit from households. In equilibrium, for every realization of investment opportunities, each unit of funding can be channeled to the set of banks with investment opportunities through multiple routes. I assume each unit is intermediated along the shortest path, namely, the path that has the minimum number of intermediaries. As a result, the existence of link e_{ij} does not necessarily mean at least one unit of funds is pledged to j *for every realization of investment opportunities*, because the shortest path of i to some of I banks might not go through j. This assumption reflects the fact that given the set of intermediaries to which bank i can lend, he would like to choose the one that provides him the highest rate of return.

A "cycle" is a closed path; that is, $i_m = i_1$.

A "leaf" bank is a bank that only lends to other banks and does not borrow.

Bank i is "connected" to bank j if a path exists from bank i to bank j.

For every unit of money raised from households at bank i, invested by bank j, and intermediated through a number of intermediaries $\{i_1, \cdots, i_m\}$, the sequence of banks involved $\{i, i_1, \cdots, i_m, j\}$ (or any subsequence of it) is called an "intermediation chain" (or simply a "chain"). Banks $\{i_1, \cdots, i_m\}$ are "intermediators" along the chain.

The "shortest path" from bank i to j, $SP(i, j)$, is the path that involves the minimum number of intermediaries. With some abuse of notation, I use $SP(i, \mathbb{J})$ to denote the collection of shortest paths of i to every bank $j \in \mathbb{J}$, $SP(i, \mathbb{J}) = \{SP(i, j)\}_{j \in \mathbb{J}}$.

The "distance" from bank i to j is the number of edges along the shortest path between i and j, denoted by $dist(i, j)$.

Banks are not competitive. For each set of funding and investment opportunities, $\mathbb{N}_F$ and $\mathbb{I}_R$, and set of lending contracts, E, a subset of lendings will be realized. There is a fixed distribution of expected total surplus over all the banks involved in raising, intermediating, and investing the funds, denoted by $\mathcal{L}(G, \mathbb{N}_F, \mathbb{I}_R)$, which is a primitive of the model. With a slight abuse of notation, let $\mathcal{L}(i; G, \mathbb{N}_F, \mathbb{I}_R)$ denote the share of bank i.

$\mathcal{L}(.)$ satisfies the following properties:[44] First, the rule is anonymous, and the net expected surplus from each unit of investment[45] is divided only among the banks in the corresponding intermediation chain, as a function of length of the chain and bank position. Second, for every unit of funds, every member of the corresponding *intermediation chain* receives strictly positive shares of net surplus generated by that unit.[46] Third, eliminating an intermediator from an intermediation chain weakly increase the share of every other bank along the chain, and strictly increases the share of the initial lender. Moreover, renegotiation and side payments are ruled out.[47]

[44] These conditions are sufficient but not necessary.

[45] $\mathbb{E}[R] - 1$.

[46] In other words, *intermediation rents* exists. Theoretically, one way to motivate the constant bargaining shares is as a commitment problem. Whereas an unconnected agent would like to commit to offer generous terms to the other agents in the deviating coalition, he can't commit not to hold out for a large share of the surplus once the potential connections are in place. From an empirical prospective, intermediation spreads are prominent in both federal funds market and OTC trades.

[47] One can think that once the actual investment and funding opportunities arrive, banks would want to change their lending decisions. As I will explain in the text shortly, the current model is a reduced form model for a dynamic game. When funding and investment opportunities arrive at different times, and the cost of finding, verifying, and matching with borrowers is sufficiently high, a lender prefers to be intermediated through its current connections to a bank that has an investment opportunity, as opposed to searching and switching every period. In this sense, renegotiation is not a big issue. In addition, because investment happens at $t = 1$ and non-contractible return is realized at $t = 2$, the borrower cannot commit

Every lending contract is subject to the following simple rule, which specifies the conditions under which the lending is realized, as well as the face value of debt that should be paid back to the lender. Let $B(i;G)$ and $C(i;G)$ denote the set of borrowers and creditors (lenders) of bank i in interbank network G, respectively. Note that for every realization of $\mathbb{I}_R$, i can be connected to each $I \in \mathbb{I}_R$ through multiple intermediation chains of different lengths. Given the interbank network and each realization of investment opportunities, each borrower of i that is on at least one of i's shortest paths to the set of banks with realized investment opportunities receives at least one unit from i:

$$\text{(4)} \qquad \forall\, \mathbb{I}_R,\ \forall\, j \in B(i;G) \text{ if } \exists I \in \mathbb{I}_R \text{ s.t. } j \in SP(i,I) \Rightarrow i \text{ lends } j \text{ at least one unit.}$$

This rule ensures that in the interim period, if i's fund is (directly or indirectly) lent to $I \in \mathbb{I}_R$, it is intermediated through i's shortest path to I; that is, minimum intermediation rents are paid. The intuition is that when bank i can lend to a bank with an investment opportunity through multiple intermediaton paths, at $t = 1$, it chooses the option that provides it with the highest possible rate. What the lender is not able to do in the interim period is add a new lending contract. After the investment opportunities are realized, if i wants to be able to borrow from j, link e_{ji} needs to exist in G. Moreover, only lending contracts along the shortest paths are realized at $t = 1$.

j must lend the unit along one of the $SP(i, \mathbb{I}_R)$ paths on which it lies. Within $SP(i, \mathbb{I}_R)$, j has discretion to allocate i's unit so that j satisfies the minimum size constraint over all its realized lendings. The unit j has raised from outsiders receives equal treatment. Starting from leaf banks, at every bank, units are lent according to the rule specified above to satisfy the minimum size constraint. Any excess unit is divided equally among all the corresponding shortest paths. The process is done recursively starting from the leaf nodes until either all the units are allocated to investment opportunities, or no credit line exists along which a unit can be lent.[48]

Finally, the face value of the debt is contingent on the network G as well as the realization of $\mathbb{I}_R$, which means it is contingent on all the realized lendings. It is set such

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to pay the lender a side payment above and beyond the face value of debt enforceable by the contract. Note that in the period during which actual lending happens, no extra funding is available to make an early side payment. As a result, ruling out side payments is also a reasonable assumption.

[48]This detail can be specified differently without altering the results as long as 4 is satisfied. The reason is that contracts can be written on what happens at date $t = 1$. At $t = 0$, banks correctly forecast the expected rates they will be pledged, as well as their expected probability of default given any set of rules and adjust their connections accordingly. This particular choice helps explain the deviations. Moreover, introducing diversification allows me to abstract away from mis alignment of lender and borrowers in their portfolio choice due to diversification motives.

that in expectation (over realizations of random returns $\{\tilde{R}_k\}_{k\in\mathbb{I}_R}$), each bank i receives $\mathcal{L}(i; G, \mathbb{N}_F, \mathbb{I}_R)$.[49]

Recall that the set of lending commitments along with the banks themselves constitute G. This particular choice of G warrants some explanation, because it does not refer to the *realized* financial network. This representation captures a reduced form for the dynamic game played among banks, such as the one described in Moore (2011). In the full dynamic game of Moore (2011) in each period, some banks receive funding and some receive investment opportunities, and lending happens each period.[50] In the subsequent periods, borrower banks find it optimal not to immediately pay back their debt and instead lend their levered-up resources to other banks that have an investment opportunity. The simplification I have made abstracts away from these detailed dynamics. Abstracting away from dynamics of network formation allows me to instead focus on characterizing the properties of the equilibrium network.

At $t = 1$, given the equilibrium network G and each realization of funding and investment opportunities ($\mathbb{N}_F$ and $\mathbb{I}_R$), the contracts determine the number of units lent along each potential lending agreement, as well as the face value of debt corresponding to this realized lending, as described above. Let $m_{ij} = m(i, j; G, \mathbb{N}_F, \mathbb{I}_R)$ denote the size of the loan from bank i to j, and let $D_{ji} = D(j, i; G, \mathbb{N}_F, \mathbb{I}_R)$ denote the face value corresponding to this loan. Moreover, let $D_i^h = D(i; G, \mathbb{N}_F, \mathbb{I}_R)$ be the face value of debt from i to households.

The first proposition provides bounds on the flow of funds at date $t = 1$ given the realization of investment opportunities. The following definition is useful for understanding the proposition.

Definition 4. *A "cut" is a partition of the nodes of a graph into two disjoint subsets that are joined by at least one edge.*

The "cut-set" of the cut is the set of edges whose end points are in different subsets of the partition. Edges are said to be "crossing" the cut if they are in its cut-set.

In a flow network, an "s-t cut" is a cut that requires the source and the sink to be in different subsets, and its cut-set only consists of edges going from the source's side to the sink's side.

[49] I have chosen this structure to avoid any additional market incompleteness except that each bank can only choose a (limited) set of counterparties and then has to execute all of its trades through these established counterparties. This structure is consistent with the costly establishment of relationship lending (information, trust, etc.), as well as the observation that hedge funds, even large ones, typically maintained only one or two prime brokerage relationships and did not frequently switch. (https://www.wellsfargo.com/downloads/pdf/com/securities/hedge-fund-risk.pdf).

[50] Moore (2011) assumes a bank cannot invest in its own project.

In a weighted graph, the "size" of a cut is the sum of the weights of the edges crossing the cut.

Now construct the following auxiliary graph $\hat{G}$ from G, given the realization of $\mathbb{I}_R$: remove all edges among I banks. Moreover, remove $\mathbb{I} \setminus \mathbb{I}_R$ and all the remaining edges incident on them from G. Define the weight of edge e_{ij} to be m_{ij}. Finally, reverse the direction of all edges. I say i_1 is i_2's parent if $e_{i_1 i_2}$ exists in $\hat{G}$.

Proposition 2. *For every subset $\hat{\mathbb{I}}_R \subset \mathbb{I}_R$, let $\hat{\mathbb{I}}_R$ be the source(s) and let (different subsets of) leaf bank(s) be the sink(s). Consider each s-t cut $C(\hat{\mathbb{I}}_R)$ with the following property: if b is on the source side of the cut, all parents of b are also on the source side. Let $Size(C)$ denote the size of the cut; that is, $Size(C) = \sum_{e_{ij} \in C} m_{ij}$; and $X_S(C)$ denote the number of banks on the sink side of the cut. Moreover, let $C_o(\hat{\mathbb{I}}_R)$ be the cut with the above property that only has $\hat{\mathbb{I}}_R$ on the source side of the cut. Finally, let $Count(C)$ be the number of edges in the cut set, $Count(C) = \sum_{e_{ij} \in C} 1$.*

$$\begin{cases} Size(C) \leq X_S(C) & \forall \hat{\mathbb{I}}_R \; \forall C(\hat{\mathbb{I}}_R) \\ Count(C_o) \leq Size(C_o) & \forall \hat{\mathbb{I}}_R \end{cases}$$

where the first inequality hold with equality when C i such that only leaf nodes are on the sink side.

The main intuition is that each bank in $\mathbb{I}_R$ is entitled to at least one unit from each of its lenders, which gives the lower bound. These lenders will then draw their credit lines from their own lenders, and so on. As a result, the amount of money that flows into each set of banks cannot be more than the amount of money that their lenders (direct and indirect) have. Note that these bounds do not necessarily uniquely determine each m_{ij}.[51]

At $t = 2$, given any realization of project returns $\{R_k\}_{k \in \mathbb{I}_R}$, the borrower may or may not be able to pay lenders back in full. Let $d_{ji} = d(j, i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ and $d_j^h = d(j; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ denote the per-unit repayment bank j makes to lender bank i and households, respectively. As a convention, $s^h = D_j^h = d_j^h = 0$ if j has not borrowed from households. By construction, $d_{ji} \in [0, D_{ji}]$. Finally, let $L(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ and $A(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ denote the total liabilities and assets of bank i at date 2 when all

[51] Recall the rules I introduced earlier in the section to deal with the allocation of extra funds. The reason is exactly to resolve the above indeterminacy in a way that is not consequential to results but allows me to abstract away from bank choices when the flow of funds is not uniquely determined.

the uncertainty is resolved:

$$L_i = L(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \sum_{j\in\mathbb{N}} m_{ji} d_{ij} + s_i d_i^h$$
$$A_i = A(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \mathbb{1}[i \in \mathbb{I}_R]\Big(\tilde{R}_i\big(s_i + \sum_{j\in\mathbb{N}}(m_{ji} - m_{ij})\big)\Big) + \sum_{j\in\mathbb{N}} m_{ij} d_{ji},$$

where $\mathbb{1}[i \in \mathbb{I}_R]$ is the indicator function that takes value one if i has access to an investment opportunity. Consequently, the per-unit (partial) repayment from j to i in each state of the world can be written as

$$d_{ji}(j, i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \max\big\{0, \min\{D_{ji}, D_{ji}\frac{A_j}{L_j}\}\big\}, \tag{5}$$

and a similar expression holds for d_j^h. The above expression simply means that if a borrower does not have sufficient funds to repay its lenders, each lender will be paid back pro-rata, and there is limited liability.[52]

The next proposition states that given any network G, realizations $\mathbb{N}_F$, $\mathbb{I}_R$, and $\{R_k\}_{k\in\mathbb{I}_R}$), and face values of debt $\{D_{ij}\}_{i,j\in\mathbb{N}}$ and $\{D_i^h\}_{i\in\mathbb{N}_F}$ set at date $t = 1$, the above system of equations has a unique solution.

Proposition 3. *Given any set of funding and investment opportunities, a potential lending network G, and face values of realized lending contracts, the system of equations (5) has a unique solution.*

The above proposition is similar to the payment equilibrium of Acemoglu et al. (2013) and clearing vector of Eisenberg and Noe (2001). The proof of the proposition, as well as all other proofs of the paper, are provided in the appendix.

Given the solution to the system of (partial) debt repayments at $t = 2$, specified by (5), using backward induction, the face value of each debt contract at date $t = 1$ is set such that in expectation, each bank i receives its share of surplus according to $\mathcal{L}(i; G, \mathbb{N}_F, \mathbb{I}_R)$. This completes the specification of contracts.

[52] Note that this definition implies that all debt is pari passu. Junior household debt can be interpreted as capital and be used to study the effect of capital requirements.

4.2 Bank Optimization Problem

Let $S(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_i\}_{i \in \mathbb{I}_R})$ denote the ex-post profit of bank i, which can be written as

$$S(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) = A(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) - L(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}).$$

Let $\mathbb{1}[i \text{ survives}; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}]$ be the indicator function that is equal to one if bank i survives at $t = 2$ and zero otherwise. With this notation, banker i's optimization problem at $t = 0$ can be written as:

$$\text{(6)} \qquad \max_{\{e_{im}, e_{mi}\}_{m \in \mathbb{N}, m \neq i}} \hat{\mathcal{V}}_i\big(\{e_{im}, e_{mi}\}; G, \mathbb{N}_F\big) = \mathbb{E}\Big[S(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) + \mathbb{1}[j \text{ survives}; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}] V_i\Big] \quad \text{subject to (1).}$$

where the expectation is taken over both realization of investment opportunity (at date $t = 1$), which determines $\mathbb{I}_R$, and realization of project returns (at date $t = 2$), $\{R_k\}_{k \in \mathbb{I}_R}$. The choices of other banks are reflected in G. As I explained in section 3.1, the notion of equilibrium here is not pairwise stability of Jackson and Wolinsky (1996), which only allows for unilateral (breaking links) or bilateral (adding a link) deviations. Rather, group stability is used, which allows for joint deviations.

Note that V_i is a constant, so one can write the expectation of the indicator function for bank i survival as a probability function. Let $P(i; G, \mathbb{N}_F)$ denote the probability that bank i survives given the funding realizations and financial network G formed at $t = 0$:

$$P(i; G, \mathbb{N}_F) = \mathbb{E}\Big[\mathbb{1}[i \text{ survives}; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}]\Big].$$

With this notation, (6) is simplified to:

$$\max_{\{e_{im}, e_{mi}\}_{m \in \mathbb{N}, m \neq i}} \mathbb{E}\Big[S(i; G, \mathbb{N}_F, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})\Big] + P(i; G, \mathbb{N}_F) V_i \quad \text{subject to (1).}$$

The intuition for the bank's optimization problem is the following: consider the first term in the above objective function. Because the banking sector is non-competitive and each player gets part of the surplus, each bank would like to use the structure of its connections to extract more rents. Each bank balances the costs and benefits of exposure to more

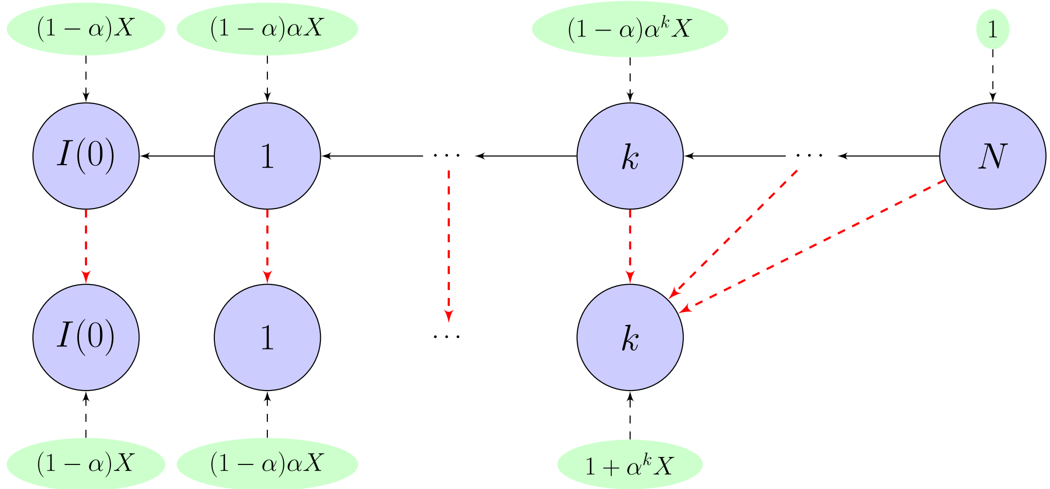


Figure 7: α-Rule for Surplus Division. X is the expected net surplus per unit of investment.

risk via intermediation and chooses the set of lending and borrowing relationships that maximizes its total expected profit.

Note that iid investment return across different investor banks $I \in \mathbb{I}_R$ implies that for any given aggregate scale of investment, total expected net surplus is independent of the distribution of investment among banks. As a result, as long as the identity of intermediators does not change the scale of the project, the rent-seeking activity translates into a change in the division of surplus in favor of intermediators, without any aggregate welfare implications. However, this surplus redistribution is not the only effect of a change in the identity of the intermediator. Intuitively, all banks along the path of intermediation are exposed to the risk of failure if the investment fails, so a change in the set of banks that do the intermediation also changes the cost of default.[53] As a result, the identity and characteristics of the intermediaries does not merely have a redistribution effect.

4.3 Lending Structure and Division of Surplus

In this section, I specify a highly tractable rule for surplus division, α-*rule*, which I use throughout the paper. The results go through with any fixed surplus division $\mathcal{L}$ that satisfies the properties of section 4, as will be clear in the proofs.[54][55]

[53] For the intermediator itself, as well as potentially for other banks through diversification. The latter point is discussed in section 5.1.

[54] Two such alternative division rules are discussed in the appendix.

[55] Perfect rigidity is not necessary. As long as the intermediator cannot forgo all of its rents, the possibility of inefficiency exists although the conditions become more extreme. The core-periphery structure of the financial system is also preserved.

Consider an intermediation chain of infinite length, and one unit of funding intermediated along the chain. The share of net surplus received by each bank along the chain, starting from the final borrower, falls at rate α, so that the initial lender (who is infinitely far away) receives a negligible share of net surplus and only breaks even. In other words, the initial lender only receives the cost of initial investment. Because the sum of the shares should add up to one, the final borrower receives share $(1-\alpha)$, the immediate intermediator receives $(1-\alpha)\alpha$, and the intermediator at distance d receives $(1-\alpha)\alpha^d$. Now suppose the initial lender is at distance k (instead of being infinitely far away). It receives the cumulative share of all *hypothetical* intermediators at distance k and further, so it gets α^k share of net surplus plus the cost of initial investment. This particular division of surplus implies the lender bears all the cost of intermediation.[56] Figure 7 summarizes α-rule.

5 Results

In this section, I provide results for the general number of banks. I will first abstract away from diversification in order to highlight the main mechanism of the model. This abstraction requires a general version of assumption 2, explained below, which works for general asymmetric networks with general intermediation chains.[57] In section 5.1, I allow for diversification and show how it interacts with the main mechanism of the model, and why the same qualitative results still go through.

Assumption 3. **[Assumption 2 Revisited]** *If a bank i owes funds to multiple banks, all of its funding is randomly assigned to exactly one of them such that in expectation, each borrower receives the amount determined by $\mathcal{L}$.*[58] *An I bank that receives an investment opportunity invests all of its funds in its own project.*

The first result addresses the length of intermediation chains. The model predicts an

[56] I should emphasize that choice of this rule is merely to simplify the exposition. α-rule has a particularly desirable characteristic that greatly simplifies the calculations: each borrower does not care about the chain traversed by each unit of funding it receives, so the face value of unit of debt payable to each lender can be calculated in a straight-forward manner regardless of the source of each unit. In other words, D_{ij} only depends on the distance of i to the final borrower.

[57] Note that I do not need to assume diversification away completely; in particular, the first part of assumption 3 is not necessary. All I need to assume is that if an I bank has an investment opportunity, it invests all of its funds in its own project. However, such assumption treats $I \to I$ credit lines differently from $NI \to I$ credit lines, which might not be plausible. As a result, I choose to make the following more restrictive version of the assumptions which deals similarly with all the credit lines.

[58] The face value of debt contracts from i to both its borrowers and lenders adjusts accordingly. Because this assumption maintains the same level of expected surplus, the face values can adjust such that in expectation, each party receives the same share.

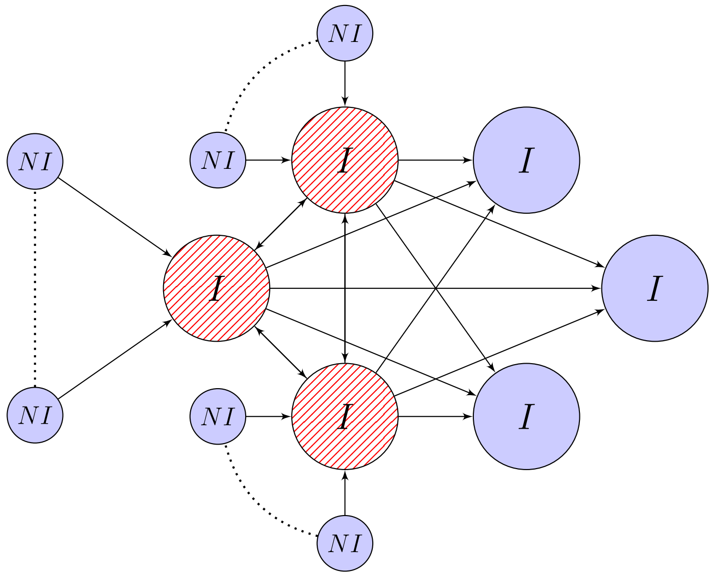


Figure 8: Equilibrium interbank lending structure with sufficiently many NI banks. The hatched red banks are banks $I \in S$ in theorem 1.

endogenous maximum length for any intermediation chain.

Lemma 1. *There is no intermediation chain of length more than l_{max}, such that $\alpha^{l_{max}} X \geq (1-p)V_{NI}$ and $\alpha^{l_{max}+1} X < (1-p)V_{NI}$.*

This lemma is intuitive: the share of each bank along the chain falls as the length of the chain grows, whereas the expected cost of default is constant. Under assumption 2, each bank j fails if the project at the single I bank to which j has (directly or indirectly) lent fails, so the expected cost of default is $(1-p)V_{NI}$. The trade-off with a benefit that decreases by distance and a constant cost determines the endogenous maximum length of the intermediation chain.

The next theorem presents the main result of the paper.

Theorem 1. *Assume $k_{NI} > k_I$, and $\kappa > M$ for a properly chosen constant M. A family of equilibria exists with the following structure: choose a subset S of the I banks, referred to as "core." S is a complete digraph. Each NI bank lends to exactly one $I \in S$, such that at least k_I NI banks lend to each $I \in S$. Every $I \in S$ lends to every other I bank, and every $I \notin S$ does not lend to any bank. This family of equilibria is inefficient.*

Moreover, let $s = |S|$ be the size of the set of intermediating I banks. Then there exist a sequence of strictly increasing constants $\{M_s\}_{s=1,\cdots,k_I}$, where $M = M_{k_I}$, and the financial structure with core size s as described above is an equilibrium iff $\kappa > M_s$.

The above family of equilibria is depicted in Figure 8. The main idea of the general proof is the following: if the project is profitable enough, I banks will be able to cover their cost of default using the intermediated rents. Moreover, they will be able to connect to every other I bank if they have enough NI lenders. On the other hand, each NI bank would want to get as high a return as possible (be as close as possible to an I bank), as well as receive positive returns as often as possible (be connected directly or indirectly to as many I banks as possible). Because sufficiently many NI banks exist, there are configurations in which each I bank is able to be connected to every other I bank (a *well-connected I bank*). As a result, any subset of well-connected I banks can act as intermediators in a stable structure.[59]

Clearly, there are multiple equilibria, but all of them share the same properties. The degree of inefficiency varies among the equilibria, and equilibria with smaller S (i.e., fewer I banks as intermediators) are less inefficient. The second part of the theorem implies that financial structures with smaller core sizes are equilibria for a wider range of parameters. This result is intuitive as well: if the core is smaller, each I bank in the core can receive funding from more NI banks and absorb more intermediation rents, which in turn cover a higher expected cost of default.

The next theorem provides an efficiency benchmark:

Theorem 2. *Assume $k_{NI} > k_I$. A solution to the social planner's problem is an NI bank that borrows from every other NI bank and lends to every I bank.*

This efficient structure is depicted in Figure 1b. Note that the NI bank at the core (hatched red NI bank) can be interpreted as a central clearing house in that all of the lending goes through this particular bank. Recall that diversification is assumed away in this section, so what makes the existence of the central clearing party (CCP) optimal is not the gains to diversification. Rather, the CCP is an entity that is able to channel the funding to all the investment opportunities optimally without being exposed to *excessive* risk.[60]

[59]There is one point worth mentioning here. When the starting network has multiple layers (a necessary condition, with α-rule, is $\alpha^{l_{max}} \geq 3$), it might be that the deviating coalition are not able to impose feasibility by their deviation. I assume that they deviate if there is a feasible restructuring of the rest of the network which is consistent with the deviation. An alternative rule would be that the deviation is valid if random dropping of links in the remaining network, in order to impose feasibility, makes each member of the coalition strictly better off (in expectation). Both rules give very similar results.

[60]Note that the efficient structure is not unique. Introducing diversification imposes more structure on the efficient network. See Acemoglu et al. (2013) for a similar discussion. I will discuss this issue in more detail in section 5.1.

To better understand the structure of equilibria, thinking about the sub-structures that can exist in an equilibrium is useful. For simplicity I focus on the range of parameters for which the project is highly profitable, i.e. X is sufficiently large.[61] Moreover, I make the following assumption:

Assumption 4. *Consider a realization of $\mathbb{I}_R$. If bank b has access to multiple $I \in \mathbb{I}_R$ through intermediation chains of different lengths, it can use the shortest chain to bargain its share in other chains up to what he gets in the shortest one. b's (direct and indirect) borrowers in each longer chain divide the remaining share pro-rata.*

Consider the following simple structure. $NI_0 \to NI_1$, $NI_1 \to I_1$, and $NI_1 \to NI_2 \to I_2$. When both I banks have investment opportunities, NI_1 has direct access to one and indirect access to the other. The above assumption says that NI_1 can bargain up its share in the chain $NI_1 \to NI_2 \to I_2$ to α. I_2 and NI_2 divide the remaining $(1-\alpha)$ share with proportions $\frac{1}{1+\alpha}$ and $\frac{\alpha}{1+\alpha}$, respectively.

The above assumption has an important implication for behavior of banks. It implies that all else equal between two intermediators, i cannot be worse off if the intermediator to which it lends is connected to extra I banks, even if through longer chains. The following lemma formalizes this intuition.

Lemma 2. [**Dominance**] *Consider two banks j_1 and j_2. Let $SPL_i = \{l^i_1, l^i_2 \cdots, l^i_{z_i}\}$ be the set whose elements are lengths of shortest paths of j_i to $\mathbb{I}$. Assume elements of each set are sorted in increasing order. Also, without loss of generality, assume j_1 has more shortest paths to $\mathbb{I}_R$, $z_1 > z_2$. A leaf bank b prefers to lend to j_1 if*

$$\forall\, k \le z_2 : \; l^1_k \le l^2_k$$

independent of l^1_k for $k > z_2$.

The next lemma restrict the structures that can arise in any possible equilibrium. For the purpose of this lemma, consider the rules for division of surplus, $\mathcal{L}$, in which removing an intermediator strictly increases the share of every bank along the intermediation chain. The following variation of α-rule can be used.

[61] The only benefit from assuming X is sufficiently large is that I do not need to worry about participation constraints being violated. A similar argument goes through for smaller X's but I have to check for many cases and it is tedious. Moreover, by construction, a small X restricts the set of viable structures. Focusing on larger X's allows for a wider range of feasible structures, and characterizing the equilibria is more interesting.

For pure technical reasons, I also assume k_{NI} is divisible by k_I when necessary.

Define (α, ϵ)-rule the following way: first, as in α-rule, the share of surplus falls at rate α going from the final borrower to the initial lender. Second, let the final borrower be at position 1, and let the initial lender be at position n of an intermediation chain of length n. The bank at position i receives $(n-i)\epsilon$ less share, relative to what he gets in α-rule, and this share is transferred to the final lender. Take ϵ arbitrary small, $\epsilon \in o(\frac{1}{n^2}(1-\alpha)\alpha^n)$. As a result, the bank in position $i \neq n$ in the chain receives $(1-\alpha)\alpha^{i-1} - (n-i)\epsilon$ while the bank at position n (initial lender) receives $\alpha^{n-1} + \frac{(n-1)(n-2)}{2}\epsilon$. Note that since $\epsilon \in o(\frac{1}{n^2}(1-\alpha)\alpha^n)$, (α, ϵ)-rule is arbitrarily close to α-rule.

Lemma 3.

(a) In any equilibrium, there is no bank that is not directly lending to every I bank but has excess funding pledged to it for every realization of $\mathbb{I}_R$.

(b) In equilibrium, there is no cycle among the NI banks.

The proof of the first part of the lemma involves joint deviations of NI and appropriate Is. Proof of the second part is more involved and is done inductively. Consider a bank j in the cycle. Intuitively, the proof relies on a joint deviation of j's lender and borrower in the cycle, with its lenders and borrowers out of the cycle. This deviation "dis-intermediates" j to absorb its rents. The following corollary is a direct result of lemma 3.

Lemma 4. *In any equilibrium in which at least one NI bank intermediates, at least one NI bank that intermediates only lends to I banks.*

5.1 Diversification

Here I relax assumption 3 to allow banks to hold diversified portfolios, and study the equilibrium structures. I find that the same structure of equilibria emerges, albeit with a twist. I focus on an economy with two I banks, $k_I = 2$, and an unrestricted number of NI banks. Restricting the number of I banks keeps the problem tractable while incorporating the main intuition associated with diversification.

Consider the range of parameters for which without diversification, each I bank is willing to intermediate even one unit of funding.[62] Consider the 2-I core-periphery structure that is an equilibrium without diversification; that is, a generalization of network structure 4c in Figure 4. Assume each I_i has credit lines from Y_i of NI banks, where $Y_1 + Y_2 = k_{NI}$.

[62] That is, intermediation rents associated with one unit of funding covers the expected cost of default. The same argument generalizes to the case in which an I bank requires more intermediation rents to be exposed to counterparty risk.

Assets	Liabilities
$\frac{Y_1+Y_2}{2}\tilde{R}$	$Y_1 D_{11}$
$\frac{Y_1-Y_2}{2}D_{21}$	

(a) Net Lender (I_1)

Assets	Liabilities
$\frac{Y_1+Y_2}{2}\tilde{R}$	$Y_2 D_{22}$
	$\frac{Y_1-Y_2}{2}D_{21}$

(b) Net Borrower (I_1)

Figure 9: Balance sheet of banks I_1 and I_2 when banks net out their payments. There are two I banks and k_{NI} NI banks. Y_i NI banks lend to I_i such that $Y_1 > Y_2$, so I_1 is the net lender and I_2 is the net borrower. In equilibrium, $D_{21} = R$ and $D_{22} = (1 + \alpha X)/p$.

I will focus on the only relevant state of the world for diversification, which is when both I_1 and I_2 have investment opportunities.[63] As described in section 4, I_i lends $\frac{Y_i}{2}$ to I_j. Let D_{ii} denote the face value of debt promised by I_i to each of its NI lenders. Moreover, let D_{ij} denote the face value of the debt payable to I_j by I_i.

I assume banks net out their payments at date $t = 2$. As a result, when $\frac{Y_i}{2}D_{ji} > \frac{Y_j}{2}D_{ij}$, j owes i the difference, namely, $\frac{Y_i}{2}D_{ji} - \frac{Y_j}{2}D_{ij}$.[64] So I_j is the *net borrower* and I_i is the *net lender*.

Without loss of generality, let $i = 1$ and $j = 2$ in the above discussion, so that I_1 is the net lender. Assumption 4 is extremely useful in determining D_{12} and D_{21}. Note each I_i has access to two investment opportunities: its own investment, which provides it with all the return (out of which he has to pay his lenders); as well as I_j investment opportunity. By assumption 4, each I_i receives all the return from investment for each unit it lends to I_j.[65] This argument pins down both inter-I face values to be exactly R, $D_{12} = D_{21} = R$. So I_1 being the net lender implies $Y_1 > Y_2$. Consequently, at $t = 2$, bank I_2 owes I_1 a net payment of $\frac{Y_1-Y_2}{2}R$.

The balance sheets of I_1 and I_2 are depicted in Figure 9. The critical observation is that survival of the net borrower solely depends on its own investment, while for the net lender, it also depends on whether the net borrower pays back. As a result, when both I banks invest, the net borrower survives exactly with probability p, whereas net lender's survival probability depends on other parameters of the model as well as the structure of the network, and is determined in equilibrium.

[63] Recall that both investment opportunities realize with probability q^2.

[64] Although both banks lend to each other, and face values of debt are determined in equilibrium.

[65] Note that I_j accepts as long as it has funding pledged to it directly by NI banks and the share of that investment covers its expected cost of default.

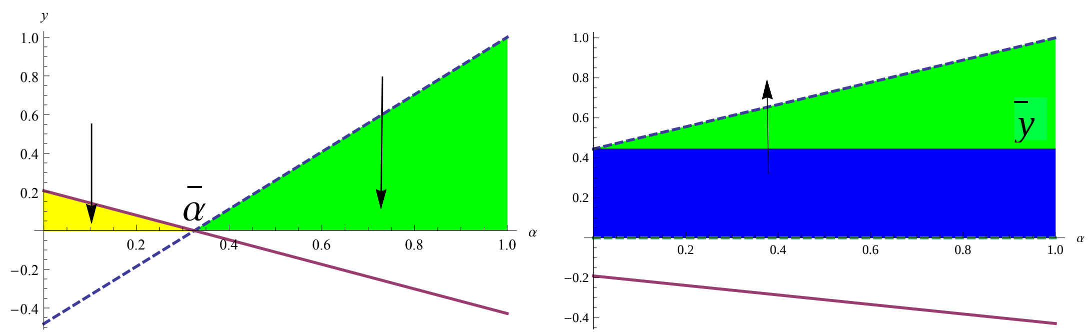
Failure probability of I_2 determines the face value payable to its NI peripheries to be $D_{22} = \frac{1+\alpha X}{p}$. As a result, the only remaining equilibrium object is D_{11}. D_{11} depends on the share of surplus that goes to a direct lender, the endogenous probability of (partial) repayment by I_1, as well as Y_1 and Y_2.

I show that depending on the value of R, there can be two cases, as depicted in Figure 10. In each plot, the horizontal axis is α, the share of surplus that goes to a direct lender in a chain of length two, and the vertical axis is the ratio of the number of peripheries of the net borrower to the net lender, $y = \frac{Y_2}{Y_1}$. Note that $0 \leq y \leq 1$ and $0 \leq \alpha \leq 1$, so only the unit square in the first quadrant is relevant. Within this area, below the solid red line (yellow region), the liabilities of I_1 are low, so having more peripheries increases the gain to diversification, and I_1 survives with probability $1-(1-p)^2$. The reverse situation happens below the dashed blue line (green region). Here the liabilities are so high that I_1 fails unless all of his assets pay, so having many direct lenders increases his liabilities and leads to a higher probability of default, and I_1 survives only with probability p^2 . In the intermediate region, above both lines, I_1 survives exactly when its investment survives and fails exactly when its investment fails; that is, with probability p. On the horizontal axis, $y = 0$, I_1 fails with probability p as I will explain shortly.

Note that in the right panel, 10b, the boundaries of the regions (the dashed and the solid line) do not cross for any $0 \leq \alpha \leq 1$, so the green region, where I_1 survives unless both investments fail, disappears. Recall that $R > \frac{1}{p}$ for the project to be positive NPV. The intuition is that if the project is positive NPV but the upside is not sufficiently high, I_1 fails if its own project, i.e. its larger asset, does not pay off. In other words, there are different combinations of (p, R) with the same NPV, that is, constant pR. I_1 prefers the combinations with higher R because it provides I_1 with sufficient resources to be able to pay its lenders, even if only I_1's smaller asset pays back. In this case $\bar{\alpha} < 0$.

In the left panel, 10a, $\bar{\alpha} > 0$. When $0 \leq \alpha < \bar{\alpha}$, I_1 bank prefers to have many peripheries to lie below the red line, which would imply an unbalanced core-periphery structure, while for $\bar{\alpha} < \alpha \leq 1$ it prefers to have similar number of peripheries as I_2 has, which will be a more balanced core-periphery structure.

In order to determine the equilibrium outcome I need to consider the incentives of the NI banks as well. Interestingly, these incentives are not necessarily aligned with that of the I banks. The rationale is that the relevant range of the parameters for NI peripheries, to prefer one structure to the other, is determined only by $\alpha = 0$, i.e. on the vertical axis. Here is the idea: The reason I_1 fails more often in certain regions compared to others, with the same successful assets, is that its liabilities are higher, i.e. α is high. However, NI



(a) $R > \frac{2}{p(2-p)}$ (b) $R < \frac{2}{p(2-p)}$

Figure 10: Possible Equilibira with two I banks and k_{NI} NI banks and diversification. The x-axis is the share of expected net surplus that goes to the lender in a direct lending, α, and the y-axis is the ratio of the number of NI peripheries of I_2 to I_1, y. The arrows show the direction of the deviation of the NI banks.

banks have to pay the households only one unit in expectation, regardless of what α is. As a result, α is not relevant in determining failure probability of the NI banks.

Imagine two different economies; L and H, with two different levels of α; $\alpha_L = 0$ and $\alpha_H > \bar{\alpha}$.[66] Denote the NI banks in economy L and H, NI_L and NI_H, respectively. First consider economy L and assume Y_1 and Y_2 are such that y lies below the solid red line. For this level of y, if at least one of the assets held by I_1 pays back (probability $(1-(1-p)^2)$, NI_L peripheries of I_1 are payed back in full. They pay all of what they get to households[67], and they survive with probability $(1-(1-p)^2$, the same probability as I_1 survives.

Now consider economy H. Here I_1 survives only if both of its assets pay back, that is, if both investments are successful, because its liabilities are too high. This happens with probability p^2. However, when I_1 fails it makes partial payments if either of his assets pay back. As a result, for every state of the world, what each NI_H bank gets in the H economy, is at least as high as what each NI_L bank gets in the L economy. As NI_L and NI_H banks have the same expected liabilities, NI_H cannot fail more often than NI_L. This implies that for each (p, R, V_I, V_{NI}), and each level of y, the probability of default for an NI periphery of I_1, for any α, is the same as probability of default of an NI with $\alpha = 0$.

Given the above discussion I can now characterize the equilibria. First consider 10b. Here the realized return of the project, R, is so low that even at $\alpha = 0$, regardless of level of y, I_1 fails if its larger asset, namely, its own investment, does not pay back. However,

[66]This example is purely for illustration, so ignore the fact that NI_L's participation constraint is violated at $\alpha = 0$.

[67]Because $\alpha = 0$.

depending on the level of y and α, I_1 may need its second asset to also pay back in order to survive. Specifically, if y is high I_1 survives only if both assets pay back.

Finally, for any value of α if y is below a certain threshold $\bar{y}$[68], determined at $\alpha = 0$ and independent of α, NI peripheries of I_1 jointly deviate with I_2 and lend to I_2 instead of I_1 because they fail less often. Such deviation pushes y up and above $\bar{y}$. Any $y > \bar{y}$ is an equilibrium because NI peripheries of I_1 has no incentive to deviate to I_2, because they fail with the same probability in both places. So between the $y = \bar{y}$ and the dashed blue line, there are two sources of inefficiency: first, I_1 is exposed to the risk of default of I_2. Second, I_1 is not diversified in the best possible way.

Now consider 10a. For $\alpha < \bar{\alpha}$, every NI lenders of I_2 prefers to instead lend to I_1 and save on the expected cost of default. I_1 likes that too. So every NI periphery of I_2 deviates to I_1 as long as I_2 has one periphery. If I_2 loses its last periphery, when both I banks have an investment opportunity, even if I_1 lends to I_2 and I_2 invests, I_2 does not receive a share of his own investment's net surplus, because I_1 absorbs all the returns. However, I_2 still incurs the expected cost of default. As a result, participation constraint of I_2 is violated and $I_1 \to I_2$ will not happen when both banks have the investment opportunity. Consequently, I_1's probability of default would rise to p, and I_2's last periphery would be indifferent between deviating or not, which by definition of equilibrium implies it does not deviate.[69]

On the other hand, when $\alpha > \bar{\alpha}$, I_1 fails more often below the dashed blue line while NI lenders to I_1 still fail less often. As a result, NI peripheries of I_2 want to deviate and lend to I_1. Interestingly, I_1 does agree to this deviation although it increases its probability of default. The reason is that the return it gets from investing this extra unit, more than covers the incremental cost of default, $p(1-p)V_I$. Depending on the number of NIs and the relative value of V_{NI} and V_I, such a deviation can be efficient or inefficient. In particular, if the number of deviating NIs is relatively large and $\frac{V_{NI}}{V_I}$ is not too low, this deviation is welfare enhancing since it provides better diversification for many NIs at the cost of extra failure for the one I.

Finally, one should consider $y = 0$, where only I_1 lends to I_2, separately. It turns out that as long as intermediation rents are sufficiently high, $y = 0$ is also an equilibrium. The intuition is that NIs would not benefit from any joint deviation with I_2 unless I_1 agrees to the deviation and adds the $e_{I_2 I_1}$ credit line, which would require I_1 to lose at least one of

[68] $\bar{y}$ is where the dashed blue line crosses the vertical axis.

[69] The fact that I_2 remains with a single NI periphery is simply because I assumed intermediation rents are high enough so that intermediating a single unit of funding covers I's extra cost of default. If intermediating c units is necessary to keep I_2 intermediating, then it will end up with c peripheries.

its peripheries to I_2, and I_1 does not agree to be part of such deviation. The next theorem formalizes the above intuition.

Theorem 3. *Let y denote the ratio of the number of NI peripheries of net borrower to net lender I bank. When $R > \frac{2}{p(2-p)}$, there are two core-periphery equilibria with I banks at the core: $y = 0$ with I_1 at the core, and $y = \frac{1}{k_{NI}-1}$ with both I_1 and I_2 at the core.*

When $R < \frac{2}{p(2-p)}$, the single-core equilibrium is still an equilibrium. There are multiple two-core equilibria, one for each $y > \bar{y}$, where $\bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$.

The above argument shows that adding diversification does not alter the main mechanism of the paper. Moreover, it enables me to study the interesting question of under-insurance in the context of the model.

Consider the $y = 0$ equilibrium. Imagine I_1 is able to offer the following deal to I_2 when both have investment opportunities: I_1 will lend half of its funds to I_2 in order to fully diversify, and it pays I_2 exactly enough to cover I_2's expected cost of default, $(1-p)V_I$. Such an offer increases I_1 and all of NI's probability of survival from p to $1-(1-p)^2$, whereas it imposes some extra cost of default (that of I_2) on the economy. One can show that if $k_{NI} > \frac{V_{NI}}{V_I} 1 - p$, the above strategy is socially efficient. However, I_1 would not make such an offer even if it could, because its individual gain to diversification, $p(1-p)V_I$, is lower than the price that it has to pay, $(1-p)V_I$. This means that I_1 does not internalize the positive externality of it buying insurance on its lenders. In other words, the price of insurance is too high for I_1, which leads to voluntary under-insurance. The above intuition is very similar to the one pointed out by Zawadowski (2013) and Acemoglu et al. (2013).

5.2 Discussion

Rent-seeking is an important friction in numerous economic environments. In the current model, banks seek rents by acting as intermediators. By borrowing from more banks and lending to more banks, the intermediator does earn more rents, but it also channels funds to where investment opportunities are, so intermediation does enhance the total surplus. The question that naturally arises is whether any welfare costs are associated with this rent-seeking behavior. I have shown that the answer depends on the characteristics of the intermediator: if bank b contributes to the scale of investment by channeling funding from outside the financial system, then intermediating additional funds within the system does not expose it (and the banking system as a whole) to any extra risk, so b is an appropriate intermediator. On the contrary, if b lends only if it intermediates funds from within the system, the implication is that b's lending decision is privately justified only via intermediation rents. As these rents exist in the system even without b,[70] b's role as an intermediary does not enhance the surplus and only increases the loss in the event that the investment fails. From the social planner's perspective, such a bank b should not be an intermediary. In equilibrium, competition implies that only borrowers that offer the highest rates are able to attract lenders, and they are exactly the banks that will be intermediating the funds as well. Because banks that can potentially invest are able to offer the highest rates, in equilibrium, they emerge as intermediators although they might not be contributing to the scale of the project by providing additional funding. Existence of such banks along the intermediation chain, in any role other than final investor, exposes the system to additional risk.

In the context of the model, efficiency calls for isolating the risk as much as possible without hurting the optimal scale of investment, whereas stability is driven by banks' incentives to earn the highest possible returns by cutting the intermediation chains as much as possible. Inefficiency arises when these two forces do not align, and it is in the form of excessive exposure to counterparty risk.

Note that net expected return to investment can be large if p is large; that is, when probability of project failure is low. As such, not only the intermediation spreads are high but also the expected cost of default is fairly low, although V_I can be very large. Consequently, it is highly likely that the more inefficient equilibria arise; that is, the equilibria with many I banks at the core. In this scenario, even though the expected cost of default for each bank is small, the ex-post realized losses can be arbitrarily large. This interpretation rationalizes the high degree of interconnectedness among large financial institutions during the run-up to the financial crisis of 2008, as well as the enormous losses once the financial sector collapsed. One interpretation of a large banks is a bank with high V. It is reasonable to associate a high V_I to potential investor banks, in which case, if the equilibrium of the model is inefficient, the efficiency loss is large. Moreover, recall that if a bank i does not raise funding from households, it should cover its cost of default from intermediation rents if it expands from pure investment to also doing intermediation. To do so, i should "steal" rents from banks that never get to invest, while keeping them above their participation constraint. If many banks exist that are able to attract funding from households, i can get direct connections to many of them and generate a lot of surplus through intermediation. This would be even easier if these NI banks are small (i.e., low V_{NI} and so low expected cost of default) and have access to a lot of funding, so that satisfying their borrowing constraint is easier. As a result, increasing the number of NI banks amplifies the inefficiency,

[70] If the same scale of investment is supported, which is the case here.

because it allows for existence of larger cores, namely, cores that involve more I banks.

6 Policy Implications

Multiple policies targeting the structure of financial networks can be studied in the context of the model. First, the model provides a new rationale for introduction of a Central Clearing Party (CCP). Designating a non-investing bank as the CCP and enforcing all the lendings to go through CCP has welfare gains by reducing loss in the event of default, because it prevents excessive bilateral exposure among banks with investment opportunities. Note that this effect is independent of any diversification gains a CCP may provide, and is different from roles identified by Duffie and Zhu (2011) and Bond (2004). Introduction of the CCP allows efficient allocation of funding through a specific intermediary, which in turn prevents intermediation by banks that would choose to expose themselves to excessive probability of default to absorb the intermediation rents. The model predicts that such a structure, although socially efficient, is not an equilibrium when intermediation rents are sufficiently high, so intervention is necessary to implement this financial structure. Adding diversification does not contradict this mechanism.

Second, part of Title VII of the Dodd-Frank Wall Street Reform and Consumer Protection Act was a proposed cap on the number of counterparties and swaps, which was later eliminated from the finalized rules.[71] Gofman (2012) calibrates a model of bank bilateral-exposures using federal funds data to empirically study the welfare effects of this policy, and suggests such policy a can entail potentially large welfare losses because banks would not be able to effectively channel funding to profitable investment opportunities.

The current paper provides sharp theoretical predictions about such a policy: in the context of the model, financial structures exist that would allow the optimal scale of investment without entailing an excessive risk of failure, but such efficient allocation of funding requires intermediaries with many connections. In other words, the model does not suggest the scale of intermediation should be reduced. However, banks that take on the intermediation role should either contribute to the scale of investment themselves, in which case intermediation does not expose them to an extra risk of failure, or purely specialize in investment so that their failure does not impose any losses other than anything related to the risky investment. In the context of the model, imposing such a limit increases the length of intermediation chains, and shifts the composition of the family of the core-periphery equilibria toward the

[71] See CFTC/SEC (2012) and Stroock Special Bulletin (http://www.stroock.com/SiteFiles/Pub1201.pdf) for more detail.

structures which larger cores. In other words, given the proposed limit the core should be sufficiently large such that all the projects can be funded. Thi shift increases the scale of inefficiency.

7 Conclusion

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a core-periphery network – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.

This paper has introduced a model which I believe can be used to study several interesting questions. As a starting point, we can use this model to incorporate other channels which may have been important in exacerbating the systemic risk. In particular, I have abstracted away from liquidity risk, which also played an important role in the financial crisis (Gorton and Metrick (2012)). How financial institution restructure the interbank network in the face of failure of some banks is an important avenue for future research.[72]

There are at least two ways to introduce liquidity risk into the model. First, note that I have assumed lenders must have sufficient funds to satisfy all borrower claims. As a result the contagion in my model spreads from borrowers to lenders. An interesting extension would be to allow lenders to promise liquidity to several borrowers, with lenders defaulting on their contingent promises if several borrowers demanded liquidity at once. This extension would enrich the model and open the possibility of contagion from lenders to borrowers.[73]

Alternatively, I could allow banks to hold liquidity for precautionary reasons in order to invest at an optimal scale. Since there is no room for liquidity holding in the current model, the socially efficient level of investment is to invest all the funds. Introducing precautionary liquidity motive would provide a framework to study the optimal scale of investment and its interaction with the network structure.

[72]On similar vein, I would like to understand how allowing financial institutions to also have concurrent exposures through secured and unsecured debt, as well as over-the counter derivative, as is the empirical fact in the US financial sector, changes conclusions of my model.

[73]I am grateful to Stefano Giglio for pointing this intuition.

More broadly, one can think of specialization by intermediaries in the context of the model. Whether banks should specialize and if so in which activities has long been an issue of debate among economists. The current model cannot answer this question because it takes the existence of different types of banks (and their numbers) as given. Assessing how efficiency considerations change, were specialization allowed to be a bank's choice variable, is an avenue that I plan to study in future research.

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8 Appendix

8.1 Proofs

will present all the proofs using α-rule as it greatly simplifies the exposition of the proof. It is easy to verify that all the proofs go through with any rule for surplus division which satisfies the properties of section 4.1. The general proofs are available upon request.

I will first show that the our structures in 11 are the only possible equilibria of the economy with four banks and then prove proposition 1.

Lemma 5. *Network structures depicted in Figure 4 are the only possible equilibria with four banks.*

Proof. Any structure in which an NI does not lend to any other bank is trivially not an equilibrium. Aside from those, all the feasible structures with four banks are depicted in 11. Each structure consists of the four banks and credit lines among them depicted in black.

Finally, the deviations which rule out the four structures on the right (11c, 11d, 11g and 11h) are depicted as red or crossed out edges. For instance in 11h, NI_1 has two units pledged to him but is only lending to a single I bank. NI_1 and I_2 strictly prefer to jointly deviate together. NI_1 saves on the intermediation rent payed to I_1 when only I_2 has an investment opportunity, while post deviation I_2 gets to invest 50% of time when both I_1 and I_2 get the investment opportunity and prior to deviation I_2 would not invest.[74] $e_{I_1 I_2}$ is removed since nothing is ever lent over that credit line and we move from 11h to 11a.

[74] If the bargaining rule is such that both final lender and initial borrower save on intermediation rents when an intermediator is removed the second part of argument is redundant as I_2 also saves on intermediation rents when only he gets the investment opportunity and lending goes through I_1. However, in α-rule borrower does not care for the source of funds so the second part of argument is necessary.

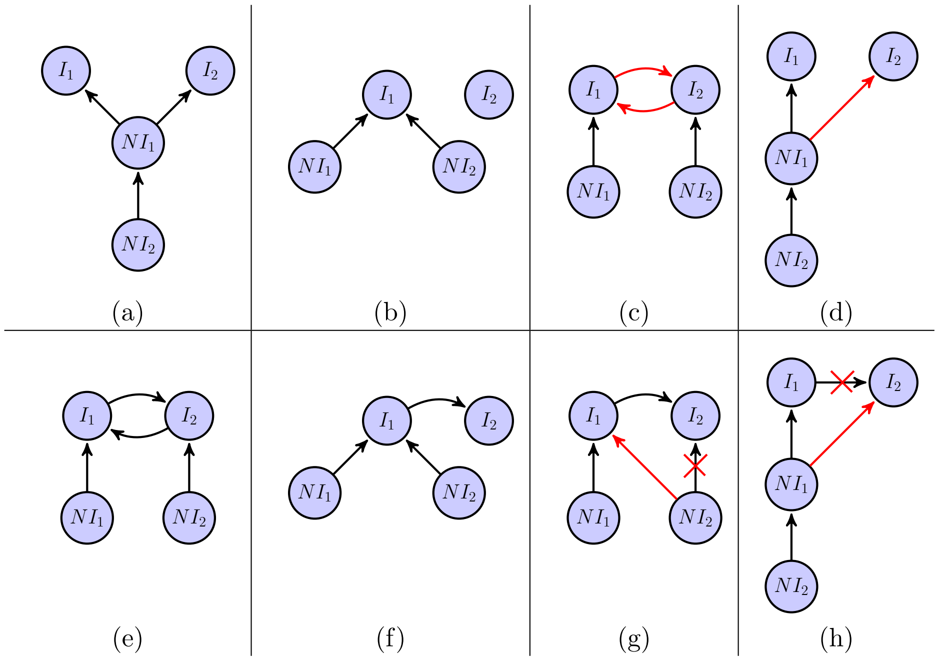

Figure 11: Feasible lending structures for an economy with two I and two NI banks. The black edges are in the feasible structure. The red and crossed-out edges are the deviations which rule out each particular structure as an equilibrium.

More over, any structure in which more than a single I bank never gets funding of a subset of NI banks cannot be an equilibrium either. In particular, here the relevant structure is 11c.Before and after the red deviation each I bank gets:

$$
\begin{aligned}
\mathcal{V}_I^{before} &= (1-q)V_I + q((1-\alpha)X + pV_I) \\
\mathcal{V}_I^{after} &= (1-q)_I^V + q^2((1-\alpha)X + pV_I) + q(1-q)(2(1-\alpha)X + pV_I) \\
&\quad + q(1-q)(\alpha(1-\alpha)X + pV_I)
\end{aligned}
$$

It is easy to verify that the latter expression is always larger than the former. The other deviations can be justified with similar arguments. □

Proposition 1. Assume the economy is in 11c. The face values of debt are set as explained in section 3.1. In expectation, an I bank and NI_2 get the following, respectively:

$$
\begin{aligned}
\hat{\mathcal{V}}_I^a &= (1-q)^2 V_I + q^2[p(V_I + R - D)] + q(1-q)[p(V_I + 2(R-D))] + (1-q)q[p(V_I + D - D_1)] \\
\hat{\mathcal{V}}_{NI_2}^a &= (1-q)^2 V_{NI} + q^2[p(V_{NI} + D) - 1] + q(1-q)[p(V_{NI} + D) - 1] + q(1-q)[p(V_{NI} + D_1) - 1]
\end{aligned}
$$

Now consider 11a:

$$\hat{\mathcal{V}}_I^c = (1-q)^2 V_I + q^2\big[\frac{1}{2}V_I + \frac{1}{2}[p(V_I + 2(R-D))]\big] + q(1-q)[p(V_I + 2(R-D))] + (1-q)qV_I$$
$$\hat{\mathcal{V}}_{NI_2}^c = (1-q)^2 V_{NI} + q^2[p(V_{NI} + D_1)) - 1] + 2(1-q)q[p(V_{NI} + D_1) - 1]$$

where I have substituted D for D_2. Note that $D - D_1 = \alpha(1-\alpha)X$ and it represents the intermediation spread. Substitute D and D_1 and compare what either bank gets in 11a and 11c to see that NI_2 always prefers to deviate to 11c while I bank would deviate if:

$$\frac{\alpha(1-\alpha)}{(1-p)V_I} > 1 + \frac{q}{2(1-q)}.$$

Let $\kappa = \frac{\alpha(1-\alpha)}{(1-p)V_I}$ and $\bar{\kappa} = 1 + \frac{q}{2(1-q)}$ and take the joint deviation of the two I banks along with NI_2 to see that 11a is not an equilibrium if $\kappa > \bar{\kappa}$.

Now is 11c an equilibrium when $\kappa < \bar{\kappa}$? Counter intuitively, the answer is yes. Although both I bank prefer to deviate back to 11a, they need both NI banks to join the deviation and no NI bank agrees to be a leaf who is always intermediated, when in the current structure he gets to lend anytime there is an investment opportunity and with positive probability he gets un-intermediated rent. 11c ceases to be an equilibrium when intermediation rents do not cover the cost of default anymore and each I_i would prefer to unilaterally break $e_{I_iI_j}$ link. This happens when $\kappa < 1$. Finally, is 11c an equilibrium if $\kappa > \bar{\kappa}$? Yes since none of the I bank can improve on what either NI bank gets in this structure, so there is no way to convince NI banks to join any deviation.

Now assume the economy is in 11d. I_1 and each NI bank receive:

$$\hat{\mathcal{V}}_{I_1}^d = (1-q)^2 V_I + q^2[p(V_I + 2(R-D))] + q(1-q)[p(V_I + 2(R-D))]$$
$$+ (1-q)q[p(V_I + 2(D-D_1))]$$
$$\hat{\mathcal{V}}_{NI}^d = (1-q)^2 V_{NI} + (q^2 + q(1-q)[p(V_{NI} + D_1)) - 1] + (1-q)q[p(V_{NI} + D_1) - 1]$$

In 11b I_1 and each NI get:

$$\hat{\mathcal{V}}_{I_1}^b = (1-q)V_I + q[p(V_I + 2(R-D))]$$
$$\hat{\mathcal{V}}_{NI}^b = (1-q)V_{NI} + q[p(V_{NI} + D) - 1]$$

Although NI does not want to deviate from 11b to 11d but I_1 will unilaterally deviate and break $e_{I_1I_2}$ link if that increases his expected profit, which happens if $\kappa < \frac{1}{2}$.

Finally, consider 11b. Two type of deviations is perceivable: either the two I banks jointly deviate and add $e_{I_1 I_2}$, which happens when $\kappa > \frac{1}{2}$.[75] Second, one might think that even if the above deviation cannot happen i.e. $\kappa < \frac{1}{2}$, the two NI banks can jointly deviate with I_2 to go to 11a. This latter deviation requires $\alpha > \frac{1}{2-q}$ and $\kappa > \underline{\kappa}$. However, we know that $\alpha < \kappa < \frac{1}{2}$ and $\frac{1}{2-q} > \frac{1}{2}$, so this deviation is never feasible. □

Proposition 2. For every cut C, parents of node b in $\hat{G}$ are exactly the banks to whom b is lending to G. By construction of C, these parents are all included on the source side of C. So and node who is on the sink side of C only lends to banks on the source side. The total amount of funding which flows into any set of nodes cannot be more that total funding raised by their direct and indirect lenders. The total flow is by construction Size(C) and total funding raised at direct and direct lenders is $X_S(C)$, which is the number of banks on the sink side of C. So Size(C) $< X_S(C)$. When only leaf nodes are on the sink side, every edge in the cut set on a shortest path, and each leaf node has exactly one unit of funding, so the inequality holds with equality.

For the second inequality, note that every edge with one end in $Iin\mathbb{I}_R$ and the other in $\mathbb{NI}$ is on the shortest path of some NI to $\mathbb{I}_R$,[76] so there is at least one unit lent over such edge in G. By construction the sum of flows of funding on such edges is Size(C_o) which I just argued is at least as large as the number o such edges. □

Proposition 3. The proof of this proposition is very similar to that of Acemoglu et al. (2013), proposition 1. The proof proceeds in multiple steps. First one can define the total liabilities of bank i to bank j by multiplying the per-unit payment by number of units lent and then define the share of each bank j in bank i liabilities. Then I define an appropriate mapping function $\Phi(.)$ which maps the min of partial and full payments to itself. It is straight forward to show that this mapping is a contraction which maps a convex and compact subset of Euclidean space to tself. As a result by Brouwer fixed point theorem, this contraction mapping has a fixed point which is the set of feasible interbank face values of debt and their relevant partial payments. For detail of generic uniqueness see Acemoglu et al. (2013). □

[75] Deviating to 11c is also possible but the former deviation is viable whenever the latter is, so there is no need to consider the latter.

[76] It is certainly on the shortest path of the NI at the end point, and maybe on the shortest path of others.

Lemma 1. Consider a bank b who lends along a longest chain of length l_{max} with probability non zero.[77] There is no diversifiation so if the ultimate borrower I fails every bank who has lent to him through any chain fails. As a result when bank NI lends directly or to indirectly to a bank I then he fails with probability $(1-p)$ regardless of the length of the intermediation chain. However, when he lends through his longest chain of length l_{max} in expectation he gets $\alpha^{l_{max}}X$. As a result l_{max} is the largest number for which b's participation constraint is not violated, which means $\alpha^{l_{max}}X \geq (1-p)V_{NI}$ and $\alpha^{l_{max}+1}X < (1-p)V_{NI}$. □

Theorem 1. I will show that there is no feasible deviation for the relevant set of parameters. So let $S(G)$ and s denote the core and the size of the core, respectively. So there are c I banks in $S(G)$ and $k_I - s$ out of the core. First consider the unilateral deviation of $I_1 \in S(G)$. First note that if I lends to one other I he would lend to as many I's that he can, since everything is linear; and similarly if he drops a lending he drops every lending. So I_1's relevant unilateral deviation is to drop all of his links to I banks and stop intermediating. That is the case if intermediation rents that I_1 captures is not sufficient to cover his cost of default. With a core of size s, the division of peripheries which maximizes the profit of the worst-off member of the core is the equal division of NI peripheries, so that each $I \in S(G)$ gets $\frac{k_{NI}}{s}$ lending to him. So I_1 deviates if $\frac{k_{NI}}{s}\alpha(1-\alpha)X < (1-p)V_I$ which determines a lower bound on M_s: $M_s \geq \frac{s}{k_{NI}}$.

Second, consider other possible deviations: Each I who is in the core has maximum possible lending relationships so I's at the core can not jointly add anything. Third, can a combination of I's in the core with the less NI peripheries and NIs themselves form a profitable deviation? No since in the current structure, every NI gets an expected return of αX with probability q and $\alpha^2 X$) with probability $(1-q)(1-(1-q)^{k_I-1})$, and every single lending generates positive expected net profits (net of cost of default), so this is the maximum possible expected profit any bank can get without having any funds pledged from the interbank network. Given that no NI bank agrees to be of distance 2 to the closest I bank, There is no such deviation.

Fourth, can a combination of I's outside the core and NI's form a profitable deviation? With the exact same argument as the last paragraph there is no such feasible deviation because it is not possible to make NIs better off than what they are without making some NI worse off.

[77] Note that b can lend over shorter paths to other banks I as well.

Finally, can NI's jointly deviate without any I's? Again the answer is no, for the following reason: the first reason is that any structure out of this family implies one of the following two cases: either there is some NI at distance 2 to his closest I bank without any improvement in probability of being involved in the investment opportunity which will be rejected by that NI; or some I bank who does receive funding from NI(s) but does not lend to all other I's. Such I can not offer a rate as high as $I \in S(G)$ in the proposed equilibrium structure offers, so this is not a viable deviation either. □

Theorem 2. First note that in this structure the minimum size constraint as well as the participation constraint of every bank is satisfied.[78] Regardsless of which bank receives the investment opportunity, all the funding will be channeled to some investment opportunity. Moreover, since every NI bank is lending to all I banks through the same tree, in other words removing I banks and all edges incident on them results in a single connected component, maximal concentrated risk is achieved. Said differently, when multiple I banks receive investment opportunity one and only one of the invests, which given the no diversification assumption 3 it is welfare enhancing since it concentrates risk as much as possible and saves on expected cost of default of some I's. Finally, for any realization of investment opportunities, aside form the single I bank who does the investment, every other bank with a realized lending and/or borrowing relationship provides funding for the investment, so removing him from the set of active lenders decreases the scale of investment by one while also decreasing the expected cost of default by $(1-p)V_{NI}$. By condition 1 the former is larger, so this removal will be welfare destroying. □

Lemma 2. j_1 is connected to at least z_2 of $I \in \mathbb{I}$, through "pointwise" weakly shorter paths, as defined in the lemma. Call this set $\mathbb{I}_{j_2}^{z_2}$. When any $I \in \mathbb{I}_{j_2}^{z_2}$ is in $\mathbb{I}_R$, the expected rate that j_1 (and consequently any lender to j_1) receives on their (indirect) lending is independent from distance of any $I \notin \mathbb{I}_{j_2}^{z_2}$ but $I \in \mathbb{I}_R$ to whom j_1 is connected. As a result the expected return that j_1 (and his lenders) receive conditional on realization of an investment opportunity at $I \in \mathbb{I}_{j_2}^{z_2}$ is larger that what j_2 (and his lenders) receive when what of the I banks j_2 is connected to is in $\mathbb{I}_R$. The above two events happen with exactly same probability (equal to at least one out of z_2 binomial random variables being one). Conditional the former event not happening j_1 still earns positive rents when $I \in \mathbb{I}$

[78] See footnote 25.

$\mathbb{I}_{j_2}^{z_2}$ is in $\mathbb{I}_R$ which more than covers his expected cost of default[79], while j_1 earns no rents. So in expectation over all realizations of investment opportunities, j_1 and his lenders are better off than j_2 and his lenders, respectively. □

Lemma 3. The first part of the statement is trivial. If an NI bank has funding pledged to him by households or from the interbank market and NI has excess funding for every realization of $\mathbb{I}_R$ it means he can add a lending relationship without violating the minimum size constraint. There are three cases: First, $\exists I$ such that NI is not directly lending to I. Then both I and NI strictly prefer to add $NI \to I$ lending. Second, NI either directly or indirectly lends to every I, but through intermediation chains of different lengths. Let $\hat{I}$ be the I who has the longest shortest path among $SP(NI, \mathbb{I})$. Then $\hat{I}$ share is bargained down with positive probability when he borrows from NI, and NI has to pay intermediation rents with positive probability when lending to $\hat{I}$, so they both strictly prefer to add $NI \to I$. The same argument applies if the lengths are the same but the structure of intermediaries is not so that one I gets a smaller share of NI's funding in expectation.

Third, NI is directly or indirectly lending to every I bank but all the intermediation chains are exactly the same length and all I banks are exactly symmetric relative to NI, such that they cannot get a larger share of NI funding by adding a direct link.[80] Without confining $\mathcal{L}$ as mentioned in the text, with such structures no I bank has an incentive to add a link. However, if every banks' share along an intermediation chain improves if an intermediary is removed, then every I to whom NI is not lending directly also has an incentive to circumvent the intermediators even if he gets the same share of NI funding as before for every realization of $\mathbb{I}_R$. This completes the proof of the first part.

The proof of the second part is inductive. I first prove that a 2-cycle ($NI_1 \to NI_2$ and $NI_2 \to NI_1$) is not part of any equilibrium and then I will show the inductive step of a profitable deviation from a k-cycle to a $(k-1)$ cycle. The main idea is that using funding to support a cycle among NI banks is not the most profitable use of funding and given the rest of the cycle, one member of the cycle can always jointly deviate with banks outside the cycle to increase the profits of the deviating coalition.

Let i and j be two NI banks in a 2-cycle. Let $\mathbb{I}_{i \setminus j}$ ($\mathbb{I}_{j \setminus i}$) be the set of $\mathbb{I}$ banks to whom i (j) lends directly or indirectly, but not through j (i). Also, let $C(i;G)$ ($C(j;G)$) be the set of i (j) direct creditors, and I will suppress argument G. With some abuse of notation,

[79]Because I assume participation constraint must be satisfied for each realization of lending.

[80]Imagine a *directed balanced binary tree* rooted at NI with 2 intermediators each lending to 2 I banks, and NI borrowing from 3 other NI banks.

let $B(j)$ denote the direct borrowers on the shortest path of j to $\mathbb{I}_{i\backslash j}$. The deviation is by i, $C(j)$ (or $C(C(j))$,...) and a subset of borrowers of i and or j on the shortest path to $\mathbb{I}_{j\backslash i} \cup \mathbb{I}_{j\backslash i}$ as explained below. The is that by dropping e_{ij}, i can exactly mimic what j was doing and also circumvent j's rents and redistribute it among everyone so that everyone in the coalition is strictly better off.

If $\mathbb{I}_{i\backslash j} = \emptyset$ ($\mathbb{I}_{j\backslash i} = \emptyset$) then no funding is ever lent over e_{ji} (e_{ij}), so e_{ji} (e_{ij}) is redundant and there is no cycle in the first place.

If none of the above is the case, without loss of generality assume $\mathbb{I}_{i\backslash j}$ and $\mathbb{I}_{j\backslash i}$ are non-overlapping. consider the joint deviation of i, $C(j)$ and $\mathbb{I}_{j\backslash i}$. $\mathbb{I}_{j\backslash i}$ are strictly better off because there is no change in what they were getting from $C(j)$. They were however being bargained down with positive probability when i's funding was intermediated to them, which does not happen post deviation.[81] i is strictly better off because he earn intermediation rents j was earning before. $C(j)$ is most subtle. $C(j)$ will get strictly more when only a subset of $\mathbb{I}_{i\backslash j}$ receive the investment opportunity, unless if the former was already lending through shortest paths to the latter. However, if that is the case, at each $k \in C(j)$, the funding raised from households was certainly used to sponsor this (one of these) latter shortest(s) in the event where all banks in both $\mathbb{I}_{i\backslash j}$ and $\mathbb{I}_{j\backslash i}$ receive an investment opportunity. So k's funding raise from households is not essential for sponsoring j's paths to $\mathbb{I}_{j\backslash i}$. As a result $C(j)$ themselves are not necessary for the deviation, and it is sufficient if their lenders $C(C(j))$ jointly deviate with i.[82] Now $C(C(j))$ does not save when $\mathbb{I}_{i\backslash j}$, but when $\mathbb{I}_{j\backslash i}$ because they used to be intermediated an extra level (through j and $C(j)$; now only through i). The last possible case is that $C(C(j))$ lose too much deviating to i from $C(j)$. If this holds for both i and j it means $C(i)$ and $C(j)$ can jointly deviate to an appropriate borrower of either i or j (since i and j own funds were used to sponsor the loop), and lend to exact same subset of $\mathbb{I}$. [83] In other words given that j's funding is used to sponsor $j \to i$, if $C(j)$ is already lending through shorter paths to every $\mathbb{I}_{i\backslash j}$ They could have originally disintermediated j by designating one of $B(j)$ banks as the intermediator and having him mimic j while cutting every chain at least by one. Since borrowers also strictly benefit from borrowing through shorter chains, this deviation is viable and breaks the circle.

Now consider a n-cycle, where $NI_k \to NI_{k+1}$, $NI_{n+1} = NI_1$. Let $C(j)$ and $B(j)$ be

[81] Note that if i and j are very asymmetric, the correct one should be chosen to deviate, which is the dominant one in the sense of lemma 2. If neither end of cycle dominate the other, either one can be chosen.

[82] Or recursively for $C(C(C(j)))$.

[83] Then one edge of the cycle is omitted such that the cycle is not cut off from the network and remains connected to the same appropriate borrower. This is the best feasible outcome for i and j.

defined analogous to above for each NI_j. The deviation is by NI_{k-1}, NI_{k+1}, $C(k)$ and $B(k)$. Clearly, any node on a circle who is not lending out of the circle is disintermediated immediately. □

Lemma 4. I will use proof by contradiction. Assume the claim does not hold. So for every NI either he does not lend to any other bank or he lends to another NI bank. The first one is ruled out by the first part of lemma 3. So each NI bank has at least one out going edge which does not go to on I, so there are at least k_{NI} edges with both ends in $\mathbb{NI}$, which is of size k_{NI}. It is a well known result from graph theory which says any minimally connected graph (i.e. no cycles) with V nodes has exactly $V-1$ edges, while here the structure among NI's has k_{NI} nodes and k_{NI} edges. As a result it cannot be minimally connected and a cycle exists among NI's, which violates the second part of 3. □

Theorem 3. As explained in the text, the structure of equilibrium and the face value of debt from I_1 to his NI peripheries are jointly determined in equilibrium, based on which of the following regions the total liabilities of the net lender I_1 lies in:

$$\begin{cases} Y_1 D_{11} < \frac{Y_1-Y_2}{2}R & I_1 \text{ survives with probability } 1-(1-p)^2 \\ \frac{Y_1-Y_2}{2}R \le Y_1 D_{11} < \frac{Y_1+Y_2}{2}R & I_1 \text{ survives with probability } p \\ \frac{Y_1+Y_2}{2}R \le Y_1 D_{11} & I_1 \text{ survives with probability } p^2 \end{cases}$$

First note that liabilities can be high for two reasons: either α is high so that a large share of surplus goes to the lenders, or default probability of borrower is high. In the first region above liabilities are so high that unless both assets pay, I_1 fails. In the middle region I_1 fails if his asset investment fails and survives otherwise, and in the last region I_1 survives unless both assets fail. In the first two regions there will be partial payments. Let $\hat{D} = D_{22} = \frac{1+\alpha X}{p}$, which is the face value of debt which corresponds to the case where a bank fails exactly when his own investment fails.

Region One ($Y_1 D_{11} < \frac{Y_1-Y_2}{2}R$).

$$p^2 D_{11} + p(1-p)\frac{Y_1+Y_2}{2Y_1}R + (1-p)p\frac{Y_1-Y_2}{2Y_1}R = \alpha X + 1$$
$$D_{11} = \frac{1}{p}(\hat{D} - (1-p)R)$$

In order for the total liabilities with the above face value to be in region one it must be that

$$\frac{Y_2}{Y_1} < \frac{2}{pR}\hat{D} - \frac{2-p}{p}$$

Region Two ($\frac{Y_1-Y_2}{2}R \leq Y_1 D_{11} < \frac{Y_1+Y_2}{2}R$).

$$\begin{aligned} &pD_{11} + p(1-p) + (1-p)p\frac{Y_1 - Y_2}{2Y_1}R = \alpha X + 1 \\ &D_{11} = \hat{D} - (1-p)\frac{R}{2}(1 - \frac{Y_2}{Y_1}) \end{aligned}$$

In order for the total liabilities with the above face value to be in region two it must be that

$$\frac{Y_2}{Y_1} < \frac{2}{pR}\hat{D} - \frac{2-p}{p} \tag{7}$$

$$\frac{Y_2}{Y_1} < 1 - \frac{2}{R(2-p)}\hat{D} \tag{8}$$

Region Three ($\frac{Y_1+Y_2}{2}R \leq Y_1 D_{11}$).

$$\begin{aligned} &(1-(1-p)^2)D_{11} + p(1-p) = \alpha X + 1 \\ &D_{11} = \frac{1}{2-p}\hat{D} \end{aligned}$$

In order for the total liabilities with the above face value to be in region two it must be that

$$\frac{Y_2}{Y_1} < 1 - \frac{2}{R(2-p)}\hat{D}$$

Let $y = \frac{Y_2}{Y_1} \leq \frac{1}{2}$ denote the ration of the NI peripheries of I_2 to I_1. The inequality holds because I_1 is assumed to have more peripheries. The two inequalities defined in 7 characterize the three regions in which I_1 fails with different probabilities; where each region characterizes the set of (α, y) for which the probability of I_1 failure is the same.

The two lines cross each other and zero, if they do so, at $(\bar{\alpha}, 0)$ such that

$$1 = \frac{2}{R(2-p)}\frac{1+\bar{\alpha}X}{p}$$

However, the two lines will not cross zero (and each other) at any $\alpha \geq 0$ if even at $\alpha = 0$ I_1's own investment must survive for him to survive. This happens if

$$\frac{2}{pR}\frac{1}{p} - \frac{2-p}{p} > 0$$

which implies

$$R < \frac{2}{p(2-p)} \tag{9}$$

So the equilibria in the two case defined by 9 should be studied separately. For now ignore the constraint that α should be such that intermediation rates are high enough so that either one or both of the I banks agree to intermediate, i.e. ignore the participation constraint.[84]

First consider the case where 9 holds. As a result, the two lines defined in 7 never cross and the lower line is irrelevant. The relevant metric for decision of NI peripheries is probability of default of I_1 at $\alpha = 0$. So in this case when $y > \bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$, NI peripheries of either I bank are indifferent between moving around since they have no room to improve on their default probability. However, when $y < \bar{y}$, NI peripheries of I_1 deviate to I_2 until $y \geq \bar{y}$.

When 9 does not hold, the two lines defines in 7 cross at $\alpha = \bar{\alpha}$. At $\alpha = 0$, if $y < \hat{y} = 1 - \frac{2}{Rp(2-p)}$, then NI (and I_1) fail only with probability $(1-p)^2$. As a result, regardless of value of α, NI peripheries of I_2 deviate to I_1, except the last periphery. This deviation is profitable for NIs because it decreases their probability of default. For $\alpha < \bar{\alpha}$ it is profitable for I both because he captures more intermediation rents and his default probability decreases. For $\alpha > \bar{\alpha}$, I_1s intermediation rents increases but his probability of default also increases, but the first force over weights the second as $\alpha(1-\alpha)X > (1-p)V_I > p(1-p)V_I$. $\square$

[84]Note that I have assumed participation constraint must be satisfied case by case. When only one bank get the investment opportunity diversification does not come in, so this argument does not affect the range of α for which either one or both Is are willing to intermediate. The final equilibria are the ones which are consistent with both sets of conditions