Information Percolation, Momentum, and Reversal*

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Abstract

We propose a rational model to explain time-series momentum. The key ingredient is word-of-mouth communication, which we introduce in a noisy rational expectations framework. Word-of-mouth communication accelerates information revelation through prices and generates momentum. Social interactions allow investors with heterogeneous trading strategies—contrarian and momentum traders—to coexist in the marketplace. As a result, momentum is not completely eliminated, although a significant proportion of investors trade on it. We also show that word-of-mouth communication spreads rumors and generates price run-ups and reversals. Our theoretical predictions are in line with several empirical findings.

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1 Introduction

One of the most pervasive facts in finance is price momentum. It is documented everywhere, both across and within countries (Rouwenhorst, 1998) and asset classes (Asness, Moskowitz, and Pedersen, 2013). It appears in the cross-section of returns, where it refers to securities' *relative performance* (Jegadeesh and Titman, 1993), but also in the time-series of returns, where it refers to a security's *own performance* (Moskowitz, Ooi, and Pedersen, 2012).[1]

Price momentum challenges rational explanations—a rational theory for momentum must provide a mechanism whereby momentum arises and persists as a profitable anomaly, *although investors can decide to trade on it*. Behavioral theories therefore prevail.[2] But the wide prevalence of momentum suggests that a same behavioral trait cannot explain this phenomenon across different markets. Furthermore, the weak link between momentum and various measures of investor sentiment (Moskowitz et al., 2012) indicates that models based on sentiment have yet to identify the main source driving momentum.

This paper provides a rational explanation of time-series momentum. Our building block is a noisy rational expectations economy (Grossman and Stiglitz, 1980) in which a large population of risk-averse agents trade a risky asset over several trading rounds. In this standard setup, investors trade based on private information but also based on information that is publicly broadcast through equilibrium prices. Our contribution is to introduce an additional channel of information acquisition: we assume that private information diffuses among the population of investors through word-of-mouth communication. Investors therefore trade in centralized markets, but also search for each other's private information—trading is centralized, but information exchange is decentralized.

We model communication among agents through the information percolation theory (Duffie and Manso, 2007), according to which agents exchange information in random, bilateral private meetings. When embedded into a centralized trading model, information percolation has two effects. First, as agents accumulate information through random meetings, the average precision of information in the economy increases at an accelerated—exponential—rate. Second, through these random meetings, agents acquire heterogeneous amounts of information. The percolation mechanism therefore dictates both how the average precision evolves over time and how individual precisions are distributed across agents.

The increase in average precision simultaneously improves the quality of investors' in-

[1]Cross-sectional and time-series momentum are related, but distinct empirical anomalies. Notably, Moskowitz et al. (2012) show that time-series momentum is not fully explained by cross-sectional momentum.

[2]According to these theories, momentum and reversal are explained by various behavioral biases. For instance, momentum traders, conservative investors, and attribution bias generate momentum, while newswatchers, representativeness heuristic, and overconfidence generate reversals. See Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999).

formation and reduces the risk of holding the stock. These two effects—information and risk—jointly operate to produce momentum. To understand this, suppose the price is below fundamentals. Because the average agent receives signals above the price, she concludes that the stock is undervalued and exerts buying pressure. The price therefore increases towards fundamentals. Meanwhile, as information becomes more precise over time, agents face less risk for holding the asset. This gradually decreases the risk premium, causing the price to increase on average. Overall, both the information and the risk effects jointly increase the price, thereby creating momentum in stock returns.

The heterogeneity in individual precisions generates different trading strategies. Agents who have little information rely more on public information broadcast through prices. Because they observe momentum in prices, they find it optimal to trade on it. Instead, agents who gather large amounts of information through random meetings build a strong knowledge of the market and thus are in a good position to tell whether price changes reflect information or noise. Consequently, they find it optimal to be contrarians and bet against the market. Hence, while everyone—any agent, including an econometrician—observes momentum, not everyone is a momentum trader and momentum thus appears as a profitable empirical anomaly. Furthermore, information percolation constantly reshuffles the distribution of individual precisions across the population and thus has implications for momentum timing. For instance, some agents start by trading on momentum (riding the bubble) but bet against it later to collect their gains.

An extension of our model shows that information percolation can also generate reversals. The idea is that word-of-mouth communication is a natural propagator of rumors (Shiller, 2000). When private information contains a rumor, this rumor circulates among investors—who are aware of its existence but cannot observe it—creating a disconnect between the stock price and the fundamental. Ultimately, the rumor subsides and produces a price reversal.

Our model can explain several empirical findings. For instance, in our model the cost of capital is decreasing in the precision of information. This prediction finds support in the accounting literature (Lambert, Leuz, and Verrecchia, 2011). Furthermore, information percolation creates a hump-shaped relationship between momentum returns and firm size (Hong, Lim, and Stein, 2000). We also show that momentum in our model exhibits time-series properties similar to those documented in Moskowitz et al. (2012) and Jegadeesh (1990). A challenging finding for risk-based explanations is that momentum is a hedge for extreme events (Moskowitz et al. 2012; Bandarchuk and Hilscher 2013), an additional feature that our model can explain. Finally, in our model momentum is positively related to trading volume (Lee and Swaminathan, 2000) and to disagreement (Zhang 2006; Verardo 2009).

We further analyze to what extent our results rely on our benchmark assumptions. We first show that momentum arises even for small magnitudes of noise trading. That is, noise trading is a necessary ingredient for equilibrium existence (Grossman and Stiglitz, 1980), but not a driver of our results. Furthermore, extending our model, we introduce a large, unconstrained, risk-neutral arbitrageur who could conceivably eliminate momentum. We find that this is not the case—the arbitrageur must also take into account the fact that her trades move prices adversely. Finally, while we derive our results in a model with a finite horizon, we show that they carry over to a fully dynamic setup. In particular, momentum obtains whether the asset pays a single liquidating dividend or an infinite stream of dividends.

We believe that private exchange of information is linked to momentum for several reasons. First, private information is an important driver of stock price variations (French and Roll, 1986) and provides an incentive for investors to implement heterogeneous trading strategies. Public news, instead, do not predict prices (Roll, 1988), nor do they explain price changes (Chan, Fong, Kho, and Stulz, 1996), nor do they generate trading heterogeneity.[3] Second, word-of-mouth communication is an innate channel of information processing, "*a central part of economic life*" (Stein, 2008), and plays an important role in stock market fluctuations (Shiller, 2000) and in investors' decisions (Shiller and Pound, 1989).[4] Further, evidence indeed suggests that word-of-mouth communication is related to momentum.[5]

Leading rational theories of momentum are based on growth-options models (Berk, Green, and Naik 1999; Johnson 2002; Sagi and Seasholes 2007). Our theory abstracts from firm decisions and directly builds on information transmission as a driver of investors' decisions and thereby of stock returns. Previous rational-expectations models (Holden and Subrahmanyam 2002; Cespa and Vives 2012) suggest that an increase in information precision generates momentum. In our model, this effect arises through word-of-mouth communication, which further generates heterogeneity in information precision across agents. Finally, among well-established explanations for momentum, Hong and Stein (1999) is the one most closely related to this paper. In contrast to Hong and Stein (1999), agents in our model are rational, make profits on average, and learn from prices—while these features would eliminate momentum in Hong and Stein (1999), they create momentum and momentum trading in our model. Moreover, we do not postulate momentum trading exogenously, but let investors optimally decide whether to trade on momentum or not.[6]

[3] See also Mitchell and Mulherin (1994), Gropp and Kadareja (2007), Tetlock (2010), and Koudijs (2010).

[4] See also Grinblatt and Keloharju (2001), Hong, Kubik, and Stein (2004), Feng and Seasholes (2004), Ivkovic and Weisbenner (2005), Brown, Ivkovic, Smith, and Weisbenner (2008), Shive (2010), and Cohen, Frazzini, and Malloy (2008).

[5] Momentum profits are decreasing in analyst coverage, supporting the notion that momentum is caused by slow information diffusion (Hong et al., 2000). See also Hou and Moskowitz (2005) and Verardo (2009).

[6] The theoretical literature on momentum is large. Other papers related to momentum, but unrelated to social interactions and information diffusion, include Albuquerque and Miao (2014), Vayanos and Woolley (2010), Makarov and Rytchkov (2009), Biais, Bossaerts, and Spatt (2010), and Wang (1993).

2 Information Percolation in Centralized Markets

To model word-of-mouth communication among investors, we use the information percolation mechanism of Duffie, Malamud, and Manso (2009). Specifically, we offer a model of *centralized* trading (a noisy rational expectations equilibrium) and *decentralized* information gathering (information percolation).

2.1 Information Setup

We build an economy with four trading dates, indexed by $t = 0, 1, 2, 3$, and a final liquidation date, $t = 4$.[7] The economy is populated by a continuum of investors indexed by $i \in [0, 1]$. There is a risky security with payoff $\widetilde{U}$ realized at the liquidation date. The payoff of this security is *unobservable* and follows a normal distribution with zero mean and precision H.[8]

Immediately prior to each trading session, each investor i obtains a private signal about the asset payoff, $\widetilde{z}_t^i$:

$$\widetilde{z}_t^i = \widetilde{U} + \widetilde{\epsilon}_t^i$$

where $\widetilde{\epsilon}_t^i$ is distributed normally and independently of $\widetilde{U}$, has zero mean, precision S, and is independent of $\widetilde{\epsilon}_j^k$ if $k \neq i$ or $j \neq t$. The precision of private signals is constant over time and is the same across investors.

We now introduce a mechanism whereby signal precisions increase over time and become heterogeneous across agents. From date $t = 0$ onward, we assume that private information diffuses across the population of agents through word-of-mouth communication. Between time 0 and time 3, we assume that agents meet each other randomly. Meetings take place at Poisson arrival times with intensity λ—the *only* parameter we add to an otherwise standard equilibrium model.

When agents meet, they exchange their initial signal and other signals that they received during previous meetings (if any). Agents are infinitesimally small and therefore are indifferent between telling the truth or lie—if they attempt to lie, they will not be able to move prices, and therefore will not benefit from their lies. For this reason, we assume that they tell the truth. Moreover, because signals are normally distributed, an agent's private information is completely summarized by two numbers: her total number of signals and her posterior expectation of the fundamental. These two numbers are what agents actually exchange when they meet and "talk."

[7] The number of four trading dates is the minimum necessary structure to show our effects; the model can be easily extended to an arbitrary number of N trading dates.

[8] We refer to the precision of a random variable as the inverse of its variance.

To illustrate how information percolation works, pick two agents—say D and J—out of the crowd. D and J start with one signal. Suppose the first time they meet someone, they meet each other. They exchange their signals truthfully and therefore end up with 2 signals after the meeting. Suppose further that D meets someone else, say E who also has two signals (i.e., E also met someone before). Since D and E are part of an infinite crowd of agents, the person that E has met cannot be J, it must be someone else—i.e., meetings do not overlap.[9] Hence, after the meeting, D and E both part with four signals each. Signals keep on adding up randomly in the exact same way for every agent in the economy.

Meetings are idiosyncratic and therefore produce a rich heterogeneity of information across agents—while agents start off holding one signal, they end up holding heterogeneous numbers of signals as soon as they meet each other. This percolation-driven heterogeneity is summarized by a cross-sectional distribution of the number of signals, π_t. Formally, say that between time $t-1$ and t each agent i collects a number ω_t^i of signals, including the one received at time t, and *excluding* the signals received up to time $t-1$. That is, for the rest of this paper, when we mention the number of signals that an agent has at time t, we refer to the number of *additional* signals, thus excluding the ones collected up to time $t-1$.[10]

Since agents are initially endowed with a single signal, the initial distribution of signals has 100% probability mass at $n = 1$ and 0% probability mass at $n > 1$. As information diffuses (at dates $t > 0$), the distribution π_t takes positive values over $\mathbb{N}^*$. For example, an agent who did not meet anyone between $t-1$ and t receives only one additional signal at t and thus is of type $n = 1$. An agent who collected ten signals between $t-1$ and t receives one more signal at t and thus is of type $n = 11$, and so on. Following Duffie et al. (2009), the cross-sectional distribution of the number of signals satisfies the following Boltzmann equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_t(n) = \lambda\pi_t * \pi_t - \lambda\pi_t = \lambda\sum_{m=1}^{n-1}\pi_t(n-m)\mu_t(m) - \lambda\pi_t(n) \tag{1}$$

where $*$ denotes the discrete convolution product and μ represents the cross-sectional distribution of the *total* number of signals, which we define in Appendix A.1. The summation term on the right hand side in (1) represents the rate at which new agents of a given type are

[9]In other words, there is a zero probability that the set of agents that D has met before time t overlaps with the set of agents that J has met before time t. This eliminates the concern that we are introducing *persuasion bias* in the terms of Demarzo, Vayanos, and Zwiebel (2003): an agent might share her signals to another agents who passes those signals at subsequent meetings to other agents and maybe the same signals will come back to the first agent—without her knowledge. The infinite mass of agents prevents this double accounting of signals to happen, since the probability for an agent to meet in the future precisely those agents who got her signals is zero. Thus, for every pair (i, j) of agents, their signal sets are always disjoint.

[10]Notice that both the distribution over the total number of signals and the distribution over additional signals may be equivalently used; we choose to use distribution of additional signals because it helps us better separate and understand the effects of information percolation on the equilibrium price and trading strategies.

created. The second term in (1) captures the rate at which agents leave a given type (e.g., when an agent of type 10 meets someone, she is no longer of type 10).

This setup has the advantage of leading to a closed-form solution for the cross-sectional distribution π_t of the number of signals. It is given in the following proposition, the proof of which is provided in Appendix A.1.[11]

Proposition 1. *The probability density function over the additional number of signals collected by each agent between $t-1$ and t, with $t \geq 1$, is given by*

$$\begin{aligned}
\pi_1(n) &= e^{-n\lambda}\left(e^{\lambda}-1\right)^{n-1} \\
\pi_2(n) &= \sum_{i=0}^{n-1}\sum_{j=0}^{i}\left[\binom{i-1}{j-1}\mathbf{1}_{\{i+j=n-1\}}e^{-i\lambda-(j+1)\lambda}\left(e^{\lambda}-1\right)^{i}\right] \\
\pi_3(n) &= \sum_{i=0}^{n-1}\sum_{j=0}^{i}\sum_{k=0}^{j}\left[\binom{i-1}{j-1}\binom{j-1}{k-1}\mathbf{1}_{\{i+j+k=n-1\}}e^{-(i+j)\lambda-(k+1)\lambda}\left(e^{\lambda}-1\right)^{i}\right]
\end{aligned}$$

Figure 1 illustrates the evolution of the cross-sectional distribution $\pi_t(n)$ of signals for a given value of the meeting intensity λ, at time 1 (upper panel) and time 2 (lower panel). The distribution dynamics reflect an increase in both average precision and precision heterogeneity over time. First, the mass of the distribution gradually shifts towards larger number of signals. As a result, the average number of signals, and therefore the average precision, increases over time. Second, while the distribution is initially concentrated at 1—each agent starts off with one signal—it rapidly spreads to reflect the growing heterogeneity in precision across the population. This heterogeneity itself varies through time, as Figure 1 shows.[12]

One potential concern is that agents get from their peers signals that have already been used in previous trading sessions. The price therefore partially reflects this information—the signal is partially "stale". By the law of large numbers, however, the price only aggregates information about the fundamental $\widetilde{U}$ and idiosyncratic noise $\widetilde{\epsilon}_t^i$ in individual signals wipes out in the aggregate. As a result, agents consider signals that they receive genuinely new, even though their information content decreases over time.

[11] Although we present here closed-form solutions for the distribution of signals, we can also compute this distribution numerically through inverse Fourier transform in a very efficient way. We provide the details on the numerical procedure in Appendix A.1.2.

[12] Other aspects are worth noting. First, the probability mass at $n = 1$ stays constant for all periods (it equals $e^{-\lambda}$, roughly 37% in this case); these are investors who do not meet anyone during the last period and consequently end up with only the usual signal received at the trading date. Second, at time 2 there is zero probability mass for $n = 2$. The reason is that no investor could gather 2 additional signals between dates 1 and 2, since that would require meeting an investor with only one signal during that period, and no such investor exists.

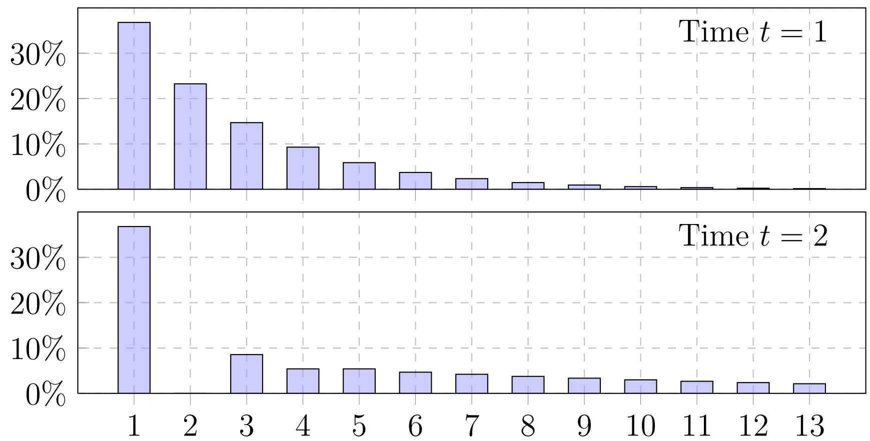

Figure 1: Evolution of Cross-Sectional Densities

The Figure depicts the evolution of the probability density function of the number of additional signals through time when agents are endowed with one signal at each period. Each graph depicts π at time $t = 1$ and $t = 2$, respectively, with $\lambda = 1$.

2.2 Equilibrium Prices

We embed the information percolation mechanism of the previous section within a rational-expectations model *à la* Grossman and Stiglitz (1980). In this model of trading, information diffuses both through prices and word-of-mouth communication.

2.2.1 Setup

Investors have exponential utility with common coefficient of absolute risk aversion $1/\gamma$. The asset payoff is realized and consumption takes place at time $t = 4$, while trading takes place at times $t = 0, 1, 2, 3$. Each investor i is endowed at time $t = 0$ with a quantity of the risky asset represented by X^i. At each trading date, investor i chooses a position in the risky asset, $\widetilde{D}^i_t$, to maximize expected utility of terminal wealth, denoted by $\widetilde{W}^i_4$:

$$\max_{\widetilde{D}^i_t} \mathbb{E}\left[e^{-\frac{1}{\gamma}\widetilde{W}^i_4} \middle| \widetilde{F}^i_t \right]$$

$$\text{subject to} \quad \widetilde{W}^i_4 = X^i \widetilde{P}_0 + \sum_{j=0}^{2} \left[\widetilde{D}^i_j \left(\widetilde{P}_{j+1} - \widetilde{P}_j \right) \right] + \widetilde{D}^i_3 \left(\widetilde{U} - \widetilde{P}_3 \right)$$

where $\widetilde{F}^i_t$ represents the information set of investor i at time t. This information set comprises: (i) private signals received at each date and collected through the information percolation mechanism described in Section 2, and (ii) prices (endogenously determined in equilibrium and denoted by $\widetilde{P}_t$) as public signals.[13]

[13]Our model bears similarities with Brennan and Cao (1997), with the main difference that we embed an information diffusion mechanism. We focus on a single asset economy, featuring several trading dates and

The aggregate per capita supply of the risky asset at time $t = 0$, $\widetilde{X}_0 = \int_0^1 X^i di$, is normally and independently distributed with zero mean and precision Φ. New liquidity traders enter the market in trading sessions $t = 1, 2, 3$. The incremental net supply of liquidity traders, $\widetilde{X}_t$, is normally distributed with zero mean and precision Φ.

The noisy supply prevents asset prices from fully revealing the final payoff $\widetilde{U}$. We postpone a discussion of the noise trading assumption to Section 4.3. We adopt a random walk specification for the noisy supply (or, equivalently, we assume that increments in noisy supply are *iid*).[14] Under this specification, any pattern in the correlation of returns depends *only* on the time distribution of private information. We can, therefore, isolate the link between the diffusion of information and the serial correlation of returns.[15] Under this specification, our results are not artificially driven by the persistence of noise traders' demand.

The solution method for finding a linear, partially revealing rational expectations equilibrium is standard and is relegated to Appendix A.2. We describe the equilibrium below.

2.2.2 Equilibrium

We first introduce notation and terminology for further use. At each date t, agent i receives ω_t^i new signals. From Gaussian theory, it is sufficient to keep track of her average incremental signal, a single signal with precision $S\omega_t^i$, which we denote by $\widetilde{Z}_t^i$:

$$\widetilde{Z}_t^i = \widetilde{U} + \widetilde{\varepsilon}_t^i, \quad \text{where } \widetilde{\varepsilon}_t^i \sim N\left(0, \frac{1}{S\omega_t^i}\right)$$

For convenience, we group all the information concerning the random variables in the present setup in Table 1.

Variable	Symbol	Mean	Precision	Comments
Fundamental	$\widetilde{U}$	0	H	
Per capita supply	$\widetilde{X}_0 = \int_0^1 X^i di$	0	Φ	at date $t = 0$
Liquidity traders	$\widetilde{X}_t$	0	Φ	at date $t = 1, 2, 3$
Private signals	$\widetilde{Z}_0^i = \widetilde{U} + \widetilde{\varepsilon}_0^i$	(for $\widetilde{\varepsilon}_0^i$) 0	S	at date $t = 0$
Private signals	$\widetilde{Z}_t^i = \widetilde{U} + \widetilde{\varepsilon}_t^i$	(for $\widetilde{\varepsilon}_t^i$) 0	$S\omega_t^i$	at date $t = 1, 2, 3$

Table 1: Random variables in the present model

An important statistic in this economy is the cross-sectional average of the number of

a final liquidation date, in order to keep the setup comparable with leading momentum theories, such as Daniel et al. (1998) and Hong and Stein (1999).

[14] *i.i.d.* incremental changes in noisy supply are likely to happen when time between consecutive trading dates is small.

[15] Other specifications, such as an an AR(1) noise trading process, give qualitatively similar results.

additional signals at time t, $\Omega_t \equiv \sum_{\omega_t \in \mathbb{N}} \pi_t(\omega_t)\, \omega_t$, where $\pi_t(\omega_t)$ has been defined in Section 2. At time 0, investors start with a single signal and therefore $\omega_0^i = \Omega_0 = 1$, $\forall i \in [0, 1]$.

We finally define some variables from Theorem 1 below, needed for the description of the equilibrium solution. The conditional precision of agent i about the final payoff $\widetilde{U}$, given all available information (private and public signals), is denoted by K_t^i and defined in (4). The cross-sectional average of conditional precisions over the entire population of agents is denoted by K_t and defined in (3).

Theorem 1, the proof of which is given in Appendix A.2, describes the risky asset prices at each date in a noisy rational expectations equilibrium with information percolation.

Theorem 1. *There exists a partially revealing rational expectations equilibrium in the 4 trading session economy in which the risky asset price, $\widetilde{P}_t$, for $t = 0, .., 3$, is given by:*

$$\widetilde{P}_t = \frac{K_t - H}{K_t} \widetilde{U} - \sum_{j=0}^{t} \frac{1 + \gamma^2 S \Omega_j \Phi}{\gamma K_t} \widetilde{X}_j \tag{2}$$

where

$$K_t \equiv \sum_{\omega \in \mathbb{N}} K_t^i(\omega) \pi_t(\omega) = H + \sum_{j=0}^{t} S\Omega_j + \sum_{j=0}^{t} \gamma^2 \Phi S^2 \Omega_j^2 \tag{3}$$

$$K_t^i \equiv \mathrm{Var}^{-1}\left[\widetilde{U} \middle| \widetilde{F}_t^i\right] = H + \sum_{j=0}^{t} S\omega_j^i + \sum_{j=0}^{t} \gamma^2 \Phi S^2 \Omega_j^2. \tag{4}$$

Equation (2) shows that the asset price is a linear function of the final payoff and supply shocks, as is customary in the noisy rational-expectations literature. Furthermore, without information percolation ($\lambda = 0$), we recover a standard rational-expectations equilibrium: average precision is constant ($\Omega_t = 1, \quad \forall t$) and there is no precision heterogeneity across agents ($K_t^i = K_t, \quad \forall i, t$). With information percolation ($\lambda > 0$) instead, average precision increases over time and affects the price dynamics. We now analyze the implication of this effect for the serial correlation of asset returns.

3 Information Percolation and Momentum

In our rational-expectations equilibrium model, the information flow through prices becomes gradually more precise over time. As a result, expected returns and risk premia gradually decrease over time, generating momentum in stock returns. In this section we explain how word-of-mouth communication generates this effect and discuss evidence supporting the link between information precision and risk premia.

To measure momentum, we follow Banerjee, Kaniel, and Kremer (2009) and regress next-period returns on last-period returns:

$$E[\Delta\widetilde{P}_t|\Delta\widetilde{P}_{t-1}] = \frac{\mathrm{cov}(\Delta\widetilde{P}_t, \Delta\widetilde{P}_{t-1})}{\mathrm{var}(\Delta\widetilde{P}_{t-1})}\Delta\widetilde{P}_{t-1} \equiv \rho_t \Delta\widetilde{P}_{t-1}. \tag{5}$$

The coefficient ρ_t, the formula of which is given in Appendix A.3, dictates whether returns exhibit momentum ($\rho > 0$) or reversal ($\rho < 0$).

3.1 Serial Correlation without Information Percolation

To understand how word-of-mouth communication affects returns, we first consider an economy without social interactions and obtain the result highlighted in the proposition below, the proof of which is available in Appendix A.3.

Proposition 2. *In the absence of word-of-mouth communication among investors, returns exhibit reversal.*

Reversal is a standard result in noisy rational-expectations models; it originates from inventory considerations (Grossman and Miller, 1988). Suppose we observe a price increase today; either the stock is undervalued and informed investors exerted buying pressure, or noise traders exerted noninformational buying pressure. In the former case, informed investors keep buying the asset until its price exactly reflects its fundamental value and returns therefore exhibit momentum—this is the information effect. In the latter case, informed investors act as market makers and accommodate the increase in noninformational demand. Because they are risk averse, they require a higher risk premium for holding the asset, which increases the asset return for the next period, thus causing returns to exhibit reversal—this is the risk-aversion effect. In standard models, the latter effect dominates and thus returns exhibit reversals.

3.2 Serial Correlation with Information Percolation

We now show how information percolation affects the tradeoff between information and risk. Information percolation increases average precision over time, which reinforces the first (information) effect: prices converge faster towards fundamental. Furthermore, an increase in average precision reverts the second (risk-aversion) effect: less uncertainty decreases the risk premium over time. Both effects jointly operate to generate momentum.

To see this, notice that prices in (2) have two parts. First, suppose that we know the fundamental value $\widetilde{U}$ of the asset (say it is $\widetilde{U} = 1$); we can then compute how prices converge to the fundamental on average

$$E[\widetilde{P}_t|\widetilde{U}] = \left(1 - \frac{H}{K_t}\right)\widetilde{U} \equiv T_t\widetilde{U}.$$

The coefficient T reflects the fundamental trend in prices. When there is no private information, $T = 0$ and prices only reflect noise; when information is perfect, $T = 1$ and prices have fully converged towards fundamental on average. When information is imperfect $T \in [0, 1]$: as market learning improves, precision increases and the trend grows towards 1, causing prices to converge towards fundamentals. We plot the fundamental trend T in the upper panel of Figure 2.

The second part of prices represents a risk premium for holding the asset. The risk premium acts as a discount on the price that increases (conditional) expected future returns to compensate investors for bearing noise trading risk. Suppose that we observe the sequence $\{\widetilde{X}_j\}_{j=0}^t$ of noise trading shocks; we can then compute how the risk premium θ evolves on average:

$$E[\widetilde{P}_t|\{\widetilde{X}_j\}_{j=0}^t] = -\frac{1}{\gamma K_t}\sum_{j=0}^{t}(1 + \gamma^2 S\Omega_j\Phi)\widetilde{X}_j \equiv -\sum_{j=0}^{t}\theta_{t,j}\widetilde{X}_j$$

Say each supply increment is $\widetilde{X}_t = 1$. Then, the risk premium at time t equals $\theta_t = \sum_{j=0}^t \theta_{t,j}$. We plot the risk premium for this particular case ($\widetilde{X}_t = 1$) in the lower panel of Figure 2.

The black lines in Figure 2 confirms our previous discussion regarding the case without information percolation ($\lambda = 0$). In particular, price convergence towards fundamental is slow—the information effect is weak—and the risk premium increases over time, causing returns to exhibit reversal.

Introducing information percolation allows the market to become increasingly precise and to learn faster. As a result, prices converge faster towards fundamentals and the information effect becomes more pronounced, as illustrated by the dashed and dotted lines in the upper panel. An increase in precision in turn alleviates risk-averse investors' inventory concerns for holding the asset. The risk premium therefore goes down and overturns the risk-aversion effect, further contributing to momentum.

To provide further insight into the above discussion, consider the following example. Suppose the price is below fundamentals. On average, agents (especially the better-informed ones) receive signals above the price. As precision increases, an increasing fraction of investors becomes aware that the stock is undervalued and exerts buying pressure, causing the price to converge faster towards fundamentals. At the same time, buying the stock becomes less and less risky due to the overall learning improvement and the risk premium goes down. The information effect therefore offsets the usual risk-aversion effect, which generates reversal in

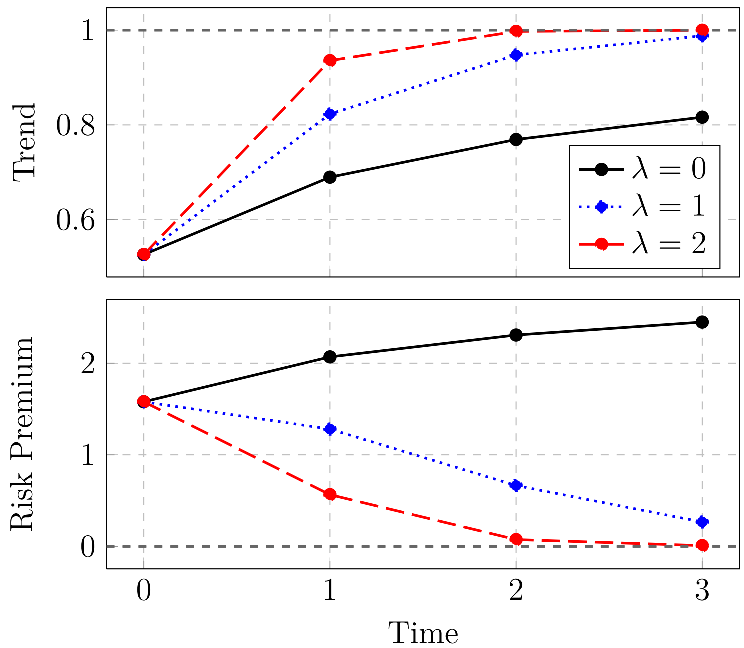

Figure 2: Risk Premium and Fundamental Trend in Prices

The Figure depicts the fundamental trend in prices in the upper panel and the risk premium in the lower panel, both as a function of time. Both are plotted for a meeting intensity of $\lambda = 0$ (the black solid line), $\lambda = 1$ (the blue dotted line), and $\lambda = 2$ (the red dashed line). The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

standard rational-expectations setups (Proposition 2).

Our model suggests that fluctuations in the risk premium are driven by the gradual increase in information precision. That is, an increase in precision decreases the risk premium for holding the asset. This mechanism is consistent with empirical findings. In particular, there is a consensus in the accounting literature that increasing the precision of information reflected in prices decreases the cost of capital (Lambert et al., 2011):[16] when more information is impounded into prices, expected returns are lower.

Empirically, several sources (other than information precision) drive risk premium fluctuations. Among the most well-known sources of risk premium fluctuations are the dividend-price ratio (Campbell and Shiller, 1988), dividend yields (Hodrick, 1992), the Fama and French (1996) factors, the aggregate dividend payout ratio (Lamont, 1998), or the consumption-aggregate wealth ratio (Lettau, 2001). These variables may lead to a "clustering" of expected and realized returns, thereby producing momentum. It is, however, well-established that these variables do not explain momentum (e.g., Lewellen and Nagel 2006; Nagel and Singleton 2011). The source of risk premium variation we identify—information precision—is seemingly unrelated to these variables.

[16] See also Botosan, Plumlee, and Xie (2004), Francis, LaFond, Olsson, and Schipper (2005), Lambert et al. (2011), and Amir and Levi (2014).

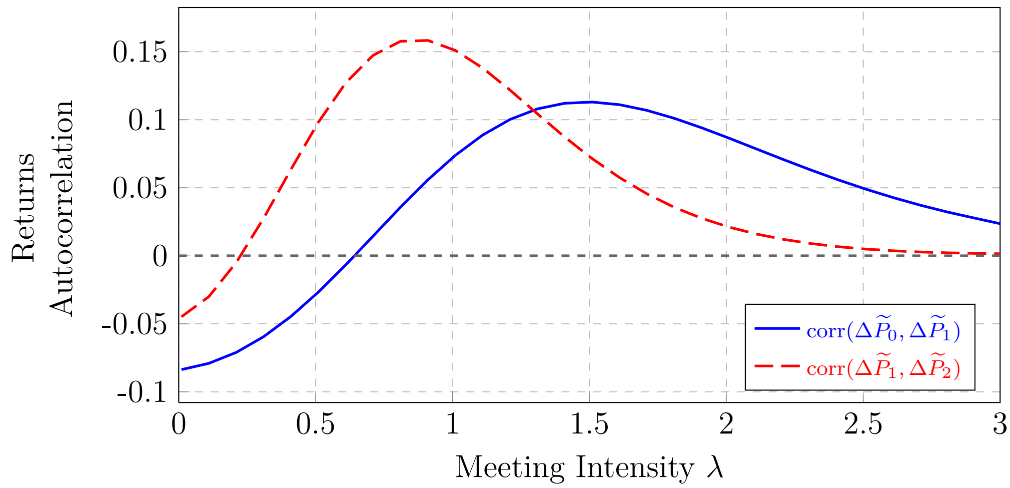

Figure 3: Information Percolation and Serial Correlation in Returns
The Figure represents the serial correlation of returns as function of the meeting intensity λ. Serial correlation is represented at time $t = 1$ (the solid blue line) and at time $t = 2$(the dashed red line). The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

We now study how the magnitude of momentum depends on the meeting intensity. To do so, we plot the serial correlation for the first- (the solid blue line) and the second-period returns (the dashed red line) as a function of the meeting intensity in Figure 3. The correlation between two consecutive returns has the same sign as that of the coefficient ρ_t in (5) and is more convenient to interpret.

The relation between momentum and the meeting intensity is hump-shaped. The increasing part of this relation illustrates the momentum mechanism previously discussed. Without information percolation and for low meeting intensities, risk-aversion dominates and returns exhibit reversal. Taking the meeting intensity as a proxy for the trading frequency, this result suggests that our model can explain reversal at short horizons (Lehmann, 1990; Nagel, 2012). For higher values of the meeting intensity, average precision rises and both the information and the risk-aversion effects jointly operate to produce momentum of increasing magnitude.

While information percolation generates momentum, its magnitude eventually declines for large values of the meeting intensity. As word-of-mouth communication intensifies, average precision spikes and prices initially reveal a large chunk of information. As a consequence, prices almost immediately converge towards the fundamental—this effect is apparent in the lower panel of Figure 2 (dashed red line)—and price fluctuations mainly reflect uncorrelated noise trading increments. In the limit when the meeting intensity is infinite, all information is revealed instantly and prices are martingales.

Overall, our model predicts a hump-shaped relation between momentum and the information diffusion speed, consistent with the empirical finding of Hong et al. (2000). Using firm size as a proxy for information diffusion speed, they document a hump-shaped pattern between firm size and the magnitude of momentum, similar to the one shown in Figure 3 (we elaborate on this aspect in Section 5.1).

4 Momentum Trading

The purpose of this section is to analyze how information percolation affects investors' optimal trading strategy. An important feature of our model is that *every* investor observes momentum from prices. We show that *some* investors profitably trade on momentum. This result raises the following question: if momentum is a profitable anomaly and investors trade on it, why does it persist? The key ingredient to address this question is the heterogeneity of information endowments that information percolation generates. Because agents have heterogeneous precisions they implement heterogeneous trading strategies—some are trend-followers, others are contrarians and momentum is therefore not completely eliminated by momentum trading.

4.1 Momentum as a Profitable Empirical Anomaly

In this section, we show that technical analysis (i.e., momentum trading) works in our model and that any investor can make profits by just following the trend. To do so, we take the view of an econometrician (who has no information other than past and current prices) and study whether a simple momentum strategy is profitable. We compute the expected cumulative profits of a *naive momentum strategy*[17] that starts at time 1 and consists in buying one unit of the stock at time t if prices went up ($\Delta\widetilde{P}_t > 0$) or selling one unit of the stock if prices went down ($\Delta\widetilde{P}_t < 0$):

$$\text{Momentum Profits } = \mathbb{E}\left[\sum_{t=1}^{3}\left(\mathbf{1}_{\Delta\widetilde{P}_t>0} - \mathbf{1}_{\Delta\widetilde{P}_t<0}\right)\Delta\widetilde{P}_{t+1}\right]$$

We plot econometrician's expected profits in Figure 4. For low meeting intensities, the econometrician's strategy performs poorly: returns exhibit reversal and a naive momentum strategy is suboptimal. As information percolation accelerates, profits become positive and their magnitude strongly relates to the magnitude of momentum (as depicted in Figure 3). Taking the econometrician's view therefore shows that momentum appears as a profitable

[17] In our model, agents choose optimal portfolios that are more sophisticated than this naive momentum strategy (regardless of their information endowment), because they know the structure of the economy as described in Theorem 1. They are therefore free to choose whether they are trend-chasers or contrarians. The point that we want to make here is that momentum appears to the *econometrician* (who knows less than the agents) as a profitable empirical anomaly.

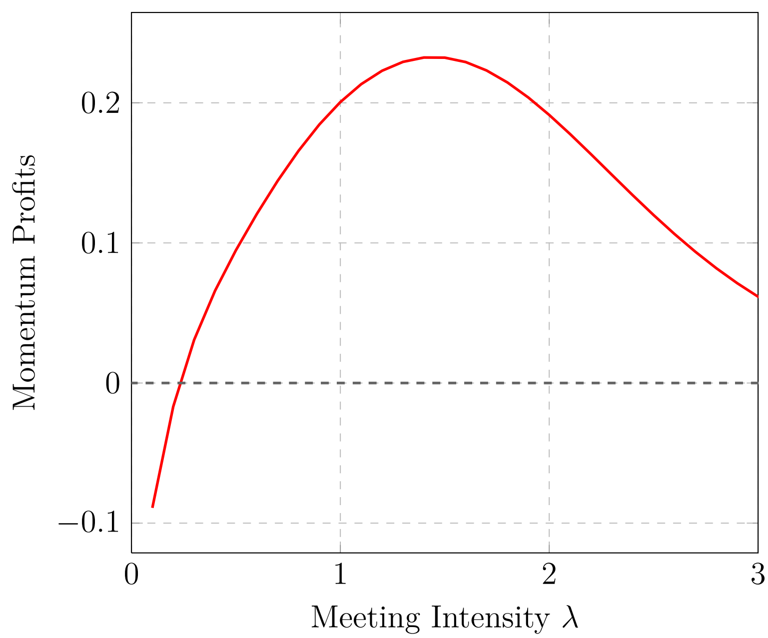

Figure 4: Profits of a Momentum Strategy as a Function of λ
The line represents the profits on a naive momentum strategy, performed by an econometrician, as a function of the meeting intensity λ. The econometrician only observe prices and buys one unit of the asset if prices went up or sells one unit of the asset if prices went down. The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

empirical anomaly and that any agent in this economy can take advantage of it by using simple "technical trading" rules.

4.2 Momentum and Contrarian Trading

We now investigate which agents optimally trade on momentum in our model. We first introduce additional variables, needed for the description of agents' optimal trading strategy.

The conditional expectation of agent i about the final payoff $\widetilde{U}$ is denoted by $\widetilde{\mu}_t^i$ and defined in (6). We define two types of informational advantages in Eq. (7) below. First, a_t^i represents the *marginal* informational (dis)advantage arising from private signals gathered from time $t-1$ to t. Second, A_t^i represents the *cumulative* informational (dis)advantage arising from private signals gathered from time 0 to t. These informational (dis)advantages are relative measures, comparing the individual precision of investor i with the average precision across the population of investors (marginal and cumulative). Finally, we normalize public signals (prices) under the form (8).

Theorem 2, the proof of which is given in Appendix A.2, describes investors' asset demands at each date in our model.

Theorem 2. *In the partially revealing rational expectations equilibrium of the 4 trading* *session economy, individual asset demands,* $\widetilde{D}_t^i$*, for* $t = 0, .., 3$*, are given by:*

$$\widetilde{D}_t^i = \gamma \left[\sum_{j=0}^{t} \left(S\omega_j^i \widetilde{Z}_j^i - S\Omega_j \widetilde{Q}_j \right) - A_t^i \widetilde{P}_t \right]$$

where

$$\widetilde{\mu}_t^i \equiv \mathbb{E}\left[\widetilde{U} \middle| \widetilde{F}_t^i \right] = \frac{1}{K_t^i} \sum_{j=0}^{t} \left[S\omega_j^i \widetilde{Z}_j^i + \gamma^2 S^2 \Phi \Omega_j \widetilde{Q}_j \right] \tag{6}$$

$$a_t^i \equiv S\omega_t^i - S\Omega_t, \;\; A_t^i \equiv K_t^i - K_t \tag{7}$$

$$\widetilde{Q}_t \equiv \widetilde{U} - \frac{1}{\gamma S \Omega_t} \widetilde{X}_t \tag{8}$$

The optimal trading strategy of the individual investor i *at time* $t = 1, 2, 3$*, denoted by* $\Delta\widetilde{D}_t^i \equiv \widetilde{D}_t^i - \widetilde{D}_{t-1}^i$ *takes the following form:*

$$\gamma \left[a_t^i \left(\widetilde{Z}_t^i - \widetilde{P}_{t-1} \right) + S\Omega_t \left(\widetilde{Z}_t^i - \widetilde{P}_{t-1} \right) - \frac{K_t}{1 + \gamma^2 S \Phi \Omega_t} \left(\widetilde{P}_t - \widetilde{P}_{t-1} \right) - A_t^i \left(\widetilde{P}_t - \widetilde{P}_{t-1} \right) \right] \tag{9}$$

4.2.1 Optimal Trading Strategies without Information Percolation

We first study the optimal trading strategies of Theorem 2 in an economy without information percolation ($\lambda = 0$). In this case we have $\omega_t^i = \Omega_t = 1$, $a_t^i = A_t^i = 0$, and $K_t^i = K_t$, for any $i \in [0, 1]$ and for $t = 0, 1, 2, 3$. As a result, the trading strategy in (9) only has two terms:

$$\Delta D_t^i = \gamma \left[S \left(\widetilde{Z}_t^i - \widetilde{P}_{t-1} \right) - \frac{K_t}{1 + \gamma^2 S \Phi} \left(\widetilde{P}_t - \widetilde{P}_{t-1} \right) \right]. \tag{10}$$

The first term in Eq. (10) shows how investor i trades based on *private* information: if this term is positive, her private signal indicates that the stock is undervalued and she therefore increases her position in the asset. The more precise her signal is, the more aggressively she buys. The second term shows how investor i trades based on *public* information: if this term is positive, the price has increased and there is a chance that noise traders' demand has increased; hence the investor decreases her position to accommodate the supply.

Whether an agent focusses on private or public information determines how she trades. To measure the trading behavior of investor i, we follow Brennan and Cao (1997) and take the view of the econometrician. Because the econometrician does not observe supply shocks nor private signals, she computes the expected trade of investor i conditional on the (observable) price change at time t, $\Delta \widetilde{P}_t$:

$$\begin{aligned}\mathbb{E}\left[\Delta \widetilde{D}_t^i \middle| \Delta \widetilde{P}_t\right] &= \gamma S \mathbb{E}\left[\widetilde{Z}_t^i - \widetilde{Q}_t \middle| \Delta \widetilde{P}_t\right] \\ &= \gamma S \mathbb{E}\left[\widetilde{U} + \widetilde{\varepsilon}_t^i - \widetilde{U} + \frac{1}{\gamma S}\widetilde{X}_t \middle| \Delta \widetilde{P}_t\right] \\ &= \mathbb{E}\left[\widetilde{X}_t \middle| \Delta \widetilde{P}_t\right] \\ &= m_t \Delta \widetilde{P}_t\end{aligned} \tag{11}$$

The sign of the coefficient m_t determines the investment style. When this coefficient is positive, an agent tends to buy after price increases and sell after price decreases. The econometrician therefore concludes that the agent is a momentum trader. Conversely, when m_t is negative, the econometrician concludes that the agent is a contrarian. This leads to the following result.

Proposition 3. *In an economy without information percolation, all informed investors adopt contrarian strategies, i.e., their trades are negatively correlated with the current price change.*

Proof. Following from (2) and (11), the conditional expected trade can be written as

$$\mathbb{E}\left[\Delta \widetilde{D}_t^i \middle| \Delta \widetilde{P}_t\right] = \underbrace{-\frac{1+\gamma^2 S \Phi}{\gamma K_t} \frac{\operatorname{var}\left(\widetilde{X}_t\right)}{\operatorname{var}\left(\Delta \widetilde{P}_t\right)}}_{m_t} \Delta \widetilde{P}_t$$

It is straightforward to see that $m_t < 0$. Thus, all investors are contrarians. $\square$

Proposition 3 has the following implication: suppose one can generate momentum with homogenous information endowments, then no agent would trade on it. This result is unsatisfactory in two respects. First, a theory for momentum without momentum trading is incomplete, because it cannot explain why momentum persists as a profitable anomaly. Second, this result is at odds with the empirical finding of Moskowitz et al. (2012) that *"speculators' position load positively on time series momentum, while hedgers' positions load negatively on it"*. Although momentum may arise in a standard rational expectations model (as we show in Section 6.1), we must yet explain why some traders engage in momentum trading and others in contrarian trading. Below we show how information percolation helps generate this prediction.

4.2.2 Optimal Trading Strategies with Information Percolation

In this section we analyze the effect of information percolation on the optimal trading strategies of Theorem 2. Information percolation generates heterogeneous precisions across investors, as we show in Section 2. As a result, some agents now have a marginal or a cumulative informational (dis)advantage (a and A, respectively) with respect to the average agent in the economy. The first and the last term in Eq. (9) will therefore cause investors' optimal trading strategy to differ through the diversity of informational advantages.

To understand how information percolation operates on informational advantages, consider first an investor who currently meets someone with many signals—this investor receives "hot information" and therefore has a large *marginal* informational advantage. Because this hot information significantly increases her current precision, she wants to trade aggressively on it. If this hot information reveals that the stock is undervalued, she increases her position in the stock through the first and second terms in Eq. (9). These terms therefore cause an agent to be a momentum trader.[18]

Now consider an investor who has met many agents in the past—this investor has a large *cumulative* informational advantage. Because this investor has built a strong knowledge of the market, she is able to better infer supply shocks from the price signal and thus trades more aggressively on public information. If she observes a stock price increase, she decreases her position in the stock to accommodate noise traders' demand through the third and last terms in Eq. (9). These terms therefore cause an agent to be a contrarian.

To illustrate the tradeoff between momentum and contrarian trading, we plot the heterogeneity of "investment styles", m_1, at time $t = 1$ for different values of the meeting intensity λ in Figure 5 (studying investment styles at time $t = 1$ is convenient because marginal and cumulative informational (dis)advantages coincide). Using the probability density function defined in Section 2, we then consider two investor types: (i) the 5% percentile *least informed investor* (dotted red line) and (ii) the 95% percentile *best informed investor* (dashed blue line).[19] The shaded area between the 5% percentile and 95% percentile therefore captures 90% of the investor population.

Shutting down information percolation ($\lambda = 0$), there is no trading heterogeneity and all informed investors are contrarians, the result of Proposition 3. When λ is positive, however, investors adopt heterogeneous trading strategies: Figure 5 shows that, as λ increases, there is an expanding spectrum of trading strategies. Importantly, for higher meeting intensities, some investors have sufficiently large marginal informational advantages to trade aggressively on their private information—these investors become momentum traders (dark gray area).

This result demonstrates that informed agents do not only trade against noise traders, but also trade against each other. To see this, notice that noise traders act as momentum traders.

[18]It can be easily shown that $\widetilde{Z}_t^i - \widetilde{P}_{t-1}$ is positively correlated with $\Delta \widetilde{P}_t$.

[19]As Proposition 3 shows, the average informed investor is contrarian. The reason is that noise traders are momentum traders and the average informed investor must take the other side of the market. We focus the analysis on informed agents who trade optimally on their information.

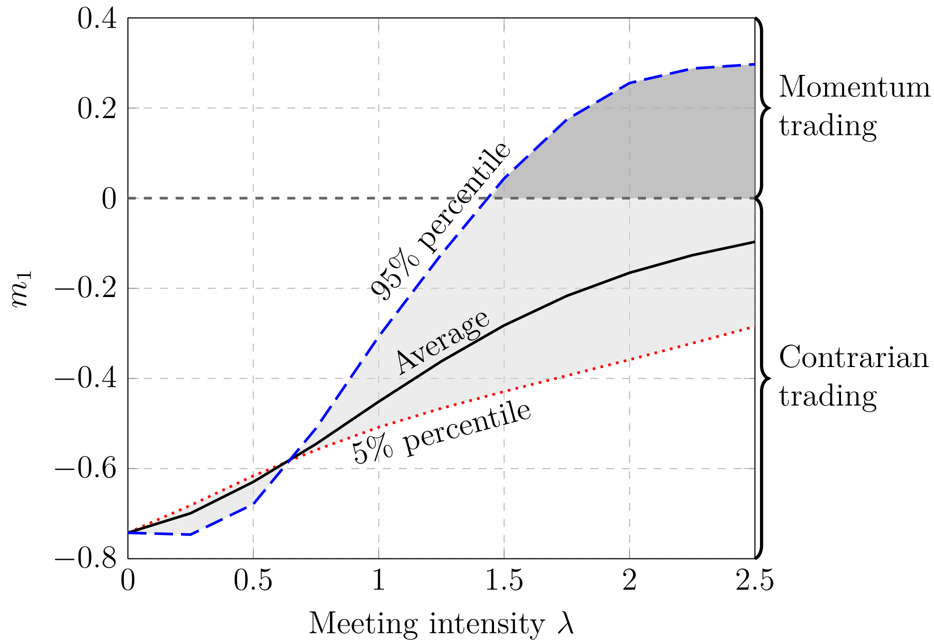

Figure 5: Information Percolation and Heterogeneous Trading Strategies at $t = 1$
The Figure depicts the coefficient of the price difference $\Delta\widetilde{P}_t$ in the expectation $\mathbb{E}\left[\Delta\widetilde{D}_1^i \middle| \Delta\widetilde{P}_1\right]$ at time $t = 1$, i.e., m_1. A positive coefficient means momentum trading, whereas a negative coefficient means contrarian trading. There are three lines. The black solid line corresponds to the average informed investor, the red dotted line corresponds to the 5% percentile less informed investor, and the blue dashed line corresponds to the 95% percentile better informed investor. Thus, the gray area represents 90% of the population of investors. The sign of the "investment style coefficient" m_1 defines momentum or contrarian trading.

Informed investors must in turn act as contrarians for markets to clear. Without information percolation, this observation implies that *all* informed investors must be contrarians (Proposition 3). When agents have heterogeneous precisions, however, some investors, besides noise traders, can also be momentum traders. Information percolation therefore provides a richer description of trading.

Overall, information percolation determines endogenously who is a momentum trader and who is a contrarian. In our model momentum traders are agents who either possess hot information or have little knowledge of the market and attempt to free ride on others' information. Instead contrarians are agents who either have a strong knowledge of the market or received little fresh information.

This conclusion is consistent with empirical findings. On the one hand, speculators—who presumably possess "hot information"—benefit from momentum as their fundamental information gets incorporated into prices (Moskowitz et al., 2012).[20] Similarly, mutual fund flows (Lou

[20]See also Kelley and Tetlock (2013b): retail short selling negatively predicts returns, fully consistent with the hypothesis that retail short sellers are well-informed. Also, retail market orders indeed convey, and benefit

2009; Akbas, Armstrong, Sorescu, and Subrahmanyam, 2014)—who presumably have little knowledge of the market and delegate their portfolio—chase past performance and further exacerbate market anomalies. On the other hand, specialists (Hendershott and Seasholes, 2007) and commercial investors, or hedgers, (Moskowitz et al., 2012)—who presumably have a broad experience on how the market operates—are contrarians and liquidity providers.

4.2.3 Information Percolation and Momentum Timing

We now study how information percolation affects trading strategies dynamically. We plot investment styles at time $t = 2$ in Figure 6. At time $t = 2$, marginal and cumulative advantages need not coincide—some investors can now have a marginal advantage but not a cumulative one, and vice-versa.

Consider first an investor who was relatively less informed at time 1 (panel (a)). When the meeting intensity is high, there is a high chance that she gets a large marginal advantage at time 2. Consequently most investors who had a low informational advantage at time 1—who were contrarians—adopt trend following strategies at time 2. On the contrary, panel (c) shows that an investor who was well informed at time 1—who was a momentum trader—becomes a contrarian at time 2. The reason is that a large marginal advantage at time $t = 1$ converts to a large cumulative informational advantage at time $t = 2$. Hence, it is likely that at time $t = 2$ her cumulative advantage overwhelms her marginal advantage. Finally, panel (b) shows that the average agent—who is necessarily contrarian at time $t = 1$ due to market clearing—may become a momentum trader at time $t = 2$ if she receives hot information. Market clearing, however, requires that on average she remains a contrarian.

In conclusion, the timing of information suggests that there is an optimal "momentum timing" strategy. Investors who receive "hot information" early on have a timing advantage and can bet on momentum when it is most profitable to do so. On average, they choose to "ride the bubble" first and bet against it near the end of the run-up. Instead, investors who received information later on are likely to be late momentum traders.

4.3 Noise Trading and Momentum

In the previous section, we have analyzed the trading strategies of informed investors. But informed investors do not only trade among themselves—if an agent knew she was trading with another informed agent, she would not trade. Noise is therefore an essential ingredient (Black, 1986). There are many reasons that can generate noise trading in financial markets—labor income (human capital), political uncertainty, natural disasters or predictable life cycle needs,

from, fundamental information (Kelley and Tetlock, 2013a).

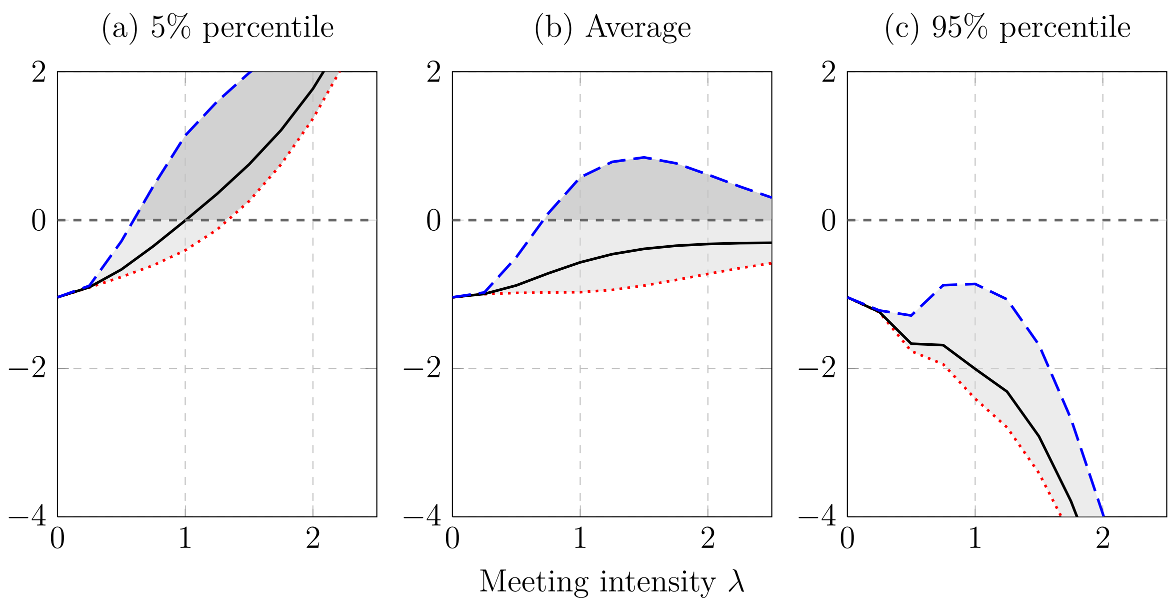

Figure 6: Information Percolation and Heterogeneous Trading Strategies at $t = 2$ The Figure depicts the same coefficient as in Figure 5, at time 2, i.e., m_2. The lines are the same as in Figure 5. Panel (a) is for the trader who at time 1 was the 5% less informed investor, panel (b) is for the trader who at time 1 was the average investor, and panel (c) is for the trader who at time 1 was the 95% better informed investor.

but also less predictable events such as job promotions or unemployment, deaths or disabilities, or myriad other causes (Glosten and Milgrom, 1985). Due to these risks, there will always be noninformational trades.[21]

In this section, we elaborate on the role of noninformational trades for our results. In particular, we investigate how much noise trading is necessary to generate momentum.

We focus on momentum at time $t = 1$ and observe that the minimum meeting intensity, $\underline{\lambda}$, above which there is momentum satisfies

$$\underline{\lambda} = \log\left(1 + \frac{H}{S + S^2\gamma^2\Phi}\right).$$

This threshold is decreasing in the precision of noise trading, which implies that less noise helps generate momentum. In other words, if liquidity provision is not too important, better information translates into momentum.

For a given meeting intensity, decreasing the magnitude of noise has two effects. On the one hand, less noise increases the magnitude of momentum by lowering the threshold $\underline{\lambda}$. On the other hand, less noise decreases momentum by allowing more information to be revealed through prices. To understand which effect dominates, we fix the meeting intensity

[21] Noise can also result from the allocational role of financial markets (Grossman, 1995).

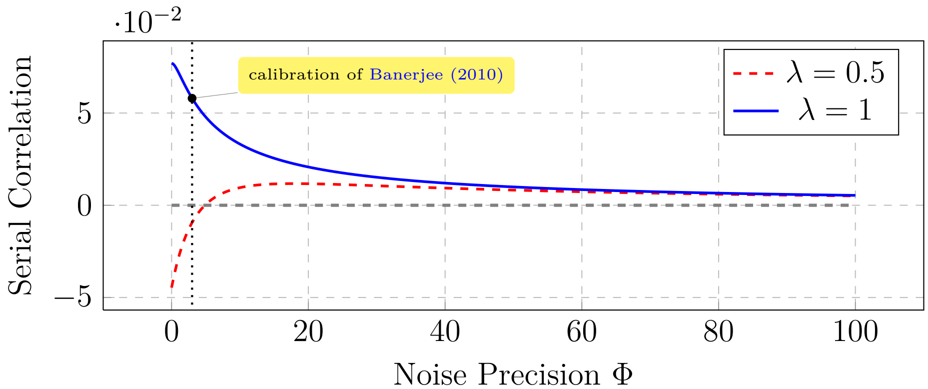

Figure 7: Momentum Magnitude as a Function of Noise

The Figure depicts the magnitude of returns serial correlation as a function of the noise precision Φ. Each line corresponds to meeting intensity of $\lambda = 0.5$ (the red solid line), and $\lambda = 1$ (the blue dashed line). The calibration is $H = S$ and $\gamma = \frac{1}{3}$.

and plot the serial correlation of returns as a function of noise precision in Figure 7. Each line corresponds to a different meeting intensity.

For low meeting intensities, increasing noise precision—reducing noise—allows momentum to arise (the red line)—in this case, the first effect dominates. As we further increase the noise precision, the second effect eventually dominates and decreases the magnitude of momentum. For higher meeting intensities, the second effect always dominates and momentum decreases with noise (the dashed blue line). Overall, we need noise to generate momentum, but the noise magnitude need not be important. In particular, we can generate momentum with an arbitrarily small supply shock, provided that the meeting intensity is sufficiently large—even small supply shocks create reversals and this effect must be undone through the percolation mechanism we described.

Although momentum arises even for small magnitudes of the noisy supply, we would like to know how much momentum our model generates with a noise precision that is empirically reasonable. To do so, we borrow a calibration from the literature. Banerjee (2010) calibrates a rational-expectations model on S&P 500 data to match *monthly returns* on the Fama French three factors over the period from January 1983 to December 2008 and finds the precision of supply shocks to be roughly three times higher than the precision of dividend shocks. In our case, this would imply $\Phi = 3$. Using this precision value and a meeting intensity of one meeting per month, we are now looking at the point that is highlighted in a yellow box in Figure 7. This point shows that our model can generate momentum of roughly 5% magnitude with relatively little noise and a low meeting intensity.

4.4 Uncontrained, Uninformed, Risk-Neutral Arbitrageur

In our model, noise traders create a source of risk. Because informed investors are risk averse, they do not fully extract momentum in stock returns. A risk-neutral, unconstrained arbitrageur—even uninformed—could therefore enter the market and collect the remaining gains (i.e., eliminate momentum). To eliminate momentum, this arbitrageur must necessarily be large enough so that her trades impact prices. The arbitrageur therefore faces a tradeoff between trading aggressively on momentum and moderating her price impact. As a result, she optimally decides not to eliminate momentum completely. The main conclusion is that momentum remains quantitatively significant, even in the presence of a risk-neutral, unconstrained, uninformed arbitrageur.

We introduce a large arbitrageur who has an *endogenous* price impact $\lambda_t > 0$ for trading $\widetilde{x}_t$ shares of the stock at time t; accordingly, we conjecture that the stock price now writes

$$\widetilde{P}_t = \varphi_t \widetilde{U} + \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j - \gamma_t \widetilde{X}_t + \lambda_t \widetilde{x}_t \left(\{\widetilde{Q}_j\}_{j=0}^t \right)$$

where φ, ξ, and γ are coefficients to be solved in equilibrium. Because the arbitrageur is uninformed (she only observes prices), her strategy is observable and the price signal now writes[22]

$$\widetilde{Q}_t = \frac{1}{\varphi_t} \left(\widetilde{P}_t - \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j - \lambda_t \widetilde{x}_t \left(\{\widetilde{Q}_j\}_{j=0}^t \right) \right).$$

Finally, the risk-neutral, unconstrained arbitrageur maximizes expected profits:

$$\max_{\widetilde{x}_t} E \left[X \widetilde{P}_0 + \sum_{j=0}^{2} \left[\widetilde{x}_j \left(\widetilde{P}_{j+1} - \widetilde{P}_j \right) \right] + \widetilde{x}_3 \left(\widetilde{U} - \widetilde{P}_3 \right) \middle| \{\widetilde{Q}\}_{j=0}^t \right].$$

The presence of the arbitrageur leads to complicated, but closed-form solutions for prices and optimal demands; for convenience, Theorem 3, the proof of which is given in Appendix A.4, reports their shortened expressions.

Theorem 3. *There exists a partially revealing rational expectations equilibrium in the 4 trading session economy in which the price signal, $\widetilde{Q}_t$, for $t = 0, .., 3$, satisfies*

$$\widetilde{Q}_t = \widetilde{U} - \frac{1}{rS\Omega_t} \widetilde{X}_t$$

[22] If the arbitrageur deviates from this strategy, informed traders could not detect the deviation and would attribute it to noise traders, which gives the arbitrageur an incentive to distort the price signal $\widetilde{Q}_t$. Because this possibility entails significant complications that are immaterial to the point we want to make, we simply discard it.

and in which the arbitrageur's demand $\widetilde{x}_t$ satisfies

$$\widetilde{x}_t = \frac{1}{2\lambda_t}\left(E\left[\widetilde{P}_{t+1}|\{\widetilde{Q}_j\}_{j=0}^t\right] - \varphi_t\widetilde{Q}_t - \sum_{j=0}^{t-1}\xi_{j,t}\widetilde{Q}_j\right). \tag{12}$$

The price coefficients satisfy

$$\varphi_t = \frac{A_t - rS\sum_{j=0}^{t-1}\Omega_j}{D_t}, \quad \xi_{j,t} = \frac{B_{j,t}+rS\Omega_j}{D_t}, \quad \gamma_t = \frac{C_t+1}{D_t}, \quad \lambda_t = \frac{1}{D_t}$$

where A, B_j, C, and D correspond to the coefficients of the aggregate demand of informed traders

$$\int_{\omega_t^i}\widetilde{D}_t^i = A_t\widetilde{U} + \sum_{j=0}^{t-1}B_{j,t}\widetilde{Q}_j - C_t\widetilde{X}_t - D_t\widetilde{P}_t.$$

The arbitrageur trades on momentum but does not fully eliminate it. To see this, we rewrite her demand in (12) as

$$\lambda_t\widetilde{x}_t = E\left[\widetilde{P}_{t+1} - \widetilde{P}_t|\{\widetilde{Q}_j\}_{j=0}^t\right]. \tag{13}$$

The expression on the right-hand side of (13) is the measure of momentum we have used in the previous sections.[23] Hence, the magnitude of momentum in the model is directly pinned down by the left-hand side of (13). The arbitrageur is a momentum trader when returns exhibit momentum and a contrarian when returns exhibit reversal. Conversely, returns exhibit momentum (or reversal) only if she trades on it.[24]

The arbitrageur can only make profits if she allows momentum to persist. She therefore optimally forgoes profits. To illustrate this, we compute the unconditional profits Π she expects to make between time t and $t+1$. In particular, simple computations show that

$$\Pi = E\left[\widetilde{x}_t\left(\widetilde{P}_{t+1} - \widetilde{P}_t\right)\right] = \frac{1}{\lambda_t}E\left[\left(E\left[\widetilde{P}_{t+1} - \widetilde{P}_t|\{\widetilde{Q}_j\}_{j=0}^t\right]\right)^2\right] = \lambda_t E\left[\widetilde{x}_t^2\right].$$

We then compare these profits to those of the econometrician of Section 4.1. The econometrician—who is not strategic—actually trades according to (13), but ignores price impact ($\lambda_t \equiv 1$). The profits that the arbitrageur optimally forgoes to keep momentum in the model are therefore

[23] Our measure of momentum is based on a coarser set of information. In the present context, it satisfies

$$E\left[\widetilde{P}_{t+1} - \widetilde{P}_t|\widetilde{P}_t - \widetilde{P}_{t-1}\right] = \lambda_t E\left[\widetilde{x}_t|\widetilde{P}_t - \widetilde{P}_{t-1}\right].$$

[24] If the arbitrageur had no price impact (i.e., $\lambda_t = 0$), her traders would be infinite and she would eliminate momentum in the limit.

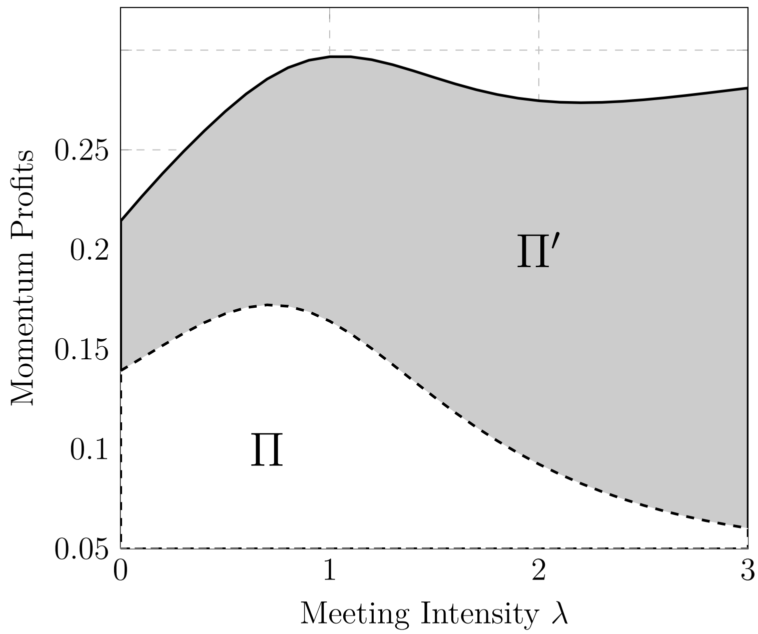

Figure 8: Profits of the Arbitrageur as a Function of λ

The solid line represents the total profits on momentum in the model, as a function of the meeting intensity λ. The dashed line represents the profits made by the arbitrageur. The shaded area represents the profits that arbitrageur forgoes. The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

given by

$$\Pi' = E\left[\left(E\left[\widetilde{P}_{t+1} - \widetilde{P}_t | \{\widetilde{Q}_j\}_{j=0}^t\right]\right)^2\right] - \Pi = \left(1 - \frac{1}{\lambda_t}\right) E\left[\left(E\left[\widetilde{P}_{t+1} - \widetilde{P}_t | \{\widetilde{Q}_j\}_{j=0}^t\right]\right)^2\right].$$

We plot the profits she makes and the profits she forgoes at time $t = 1$ in Figure 8.

For low meeting intensities, Figure 8 shows that the arbitrageur extracts half of the momentum rents, consistent with the behavior of a monopolist. As the meeting intensity increases, however, the momentum profits she forgoes significantly increase. The reason is that she now trades against agents who are better informed on average; accordingly, she has a larger price impact and therefore trades less aggressively on momentum. We conclude that it is difficult to arbitrage away momentum in a market characterized by fast diffusion of information among investors.

We finally show that momentum remains quantitatively significant. We plot momentum at time $t = 1$ and $t = 2$ in Figure 9. Comparing the magnitude of momentum in Figure 3 and 9, we observe that the arbitrageur eliminates about half of the momentum in stock returns. Momentum also requires a higher meeting intensity to arise.

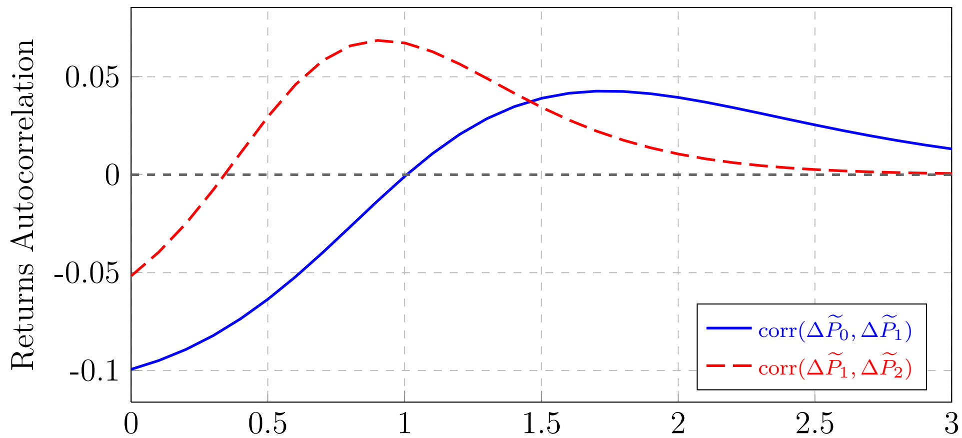

Figure 9: Serial Correlation in Returns with a Risk-Neutral Arbitrageur
The first and the second panel represent the serial correlation of returns and the fraction of momentum traders, respectively. Each is represented at time $t = 1$ (the solid blue line) and at time $t = 2$(the dashed red line). The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

5 Testable Theoretical Predictions

In this section we show that information percolation produces a set of empirically-relevant predictions. In particular, the empirical literature finds that momentum differs for stocks with certain characteristics,[25] momentum exhibits a non-linear term structure pattern,[26] and momentum is positively related to other economic variables such as past volatility, past trading volume, and disagreement.[27] Further, stock returns exhibit strong reversals at horizons shorter or equal to one month.[28] While some of these findings challenge existing theories of momentum, many of them are in line with our explanation. We analyze each one in turn.

5.1 Cross-sectional and time-series properties of momentum

Empirical findings suggest that the magnitude of momentum varies across different types of firms. In particular, Hong et al. (2000) use firm size as a proxy for the speed of information diffusion and document a hump-shaped relation between firm size and the magnitude of momentum.

As Figure 3 illustrates, our model predicts a hump-shaped relation between momentum and the meeting intensity λ. For a low meeting intensity the model generates reversals.[29] Then, as the meeting intensity increases stock returns exhibit momentum. Finally, the magnitude

[25] Hong et al. (2000), Hou, Peng, and Xiong (2006), Daniel and Titman (2000).
[26] Hong, Hong, and Ungureanu (2010), Moskowitz et al. (2012).
[27] Bandarchuk and Hilscher (2013), Lee and Swaminathan (2000), Zhang (2006), Verardo (2009).
[28] Jegadeesh (1990), Lehmann (1990).
[29] Hong et al. (2000) indeed suggest that these reversals are driven by liquidity shocks. See French and Roll (1986) for further evidence on this.

of momentum decays for larger meeting intensities, as prices converge almost instantly to the fundamental. Taking λ as a proxy for firm size, a hypothetical version of our model with multiple firms would therefore generate a cross-sectional pattern of momentum similar to that documented by Hong et al. (2000). In that respect, the non-linear dynamics of percolation lead to a hump-shaped pattern of momentum that coincides with that found empirically in the cross-section of stock returns.[30]

The magnitude of momentum varies across different lookback periods used to build the momentum strategy. Specifically, Moskowitz et al. (2012) find that the magnitude of momentum is large for short lookback periods (one to six months) and usually decays as the lookback period increases, with weak or no evidence of momentum for periods longer than 12 months.[31]

Information percolation bears similar time-series implications. A feature of our model is that the average precision of information increases over time, as shown in Proposition 1. Thus, different speeds of information diffusion can generate different patterns of momentum magnitude with respect to the length of the lookback period. To show this, we perform the following exercise. We consider a version of our model with more than four trading periods and compute momentum using results from Theorem 1, which can be easily extended to N trading periods.

In Figure 10, we plot the serial correlation of returns for looback periods ranging from one to eleven months and for a meeting intensity of $\lambda = 1$. As the speed of information diffusion increases, the model can generate momentum at short horizons. Momentum then decays as the length of the lookback period increases. For lookback periods larger than four months, momentum vanishes.[32]

At frequencies less than one month, stock returns exhibit strong reversals, as shown by Lehmann (1990) and Jegadeesh (1990). These short-term reversals are usually interpreted as rewards for liquidity provision (Nagel 2012; Cheng, Hameed, Subrahmanyam, and Titman 2013). Figure 3 suggests that our model can generate short-term reversals. To see this, recall that the amount of information that agents accumulate in our model depends on the time elapsed between trading rounds—the longer the time between trading dates the more information agents accumulate through random meetings. Consequently, for a given meeting

[30] Hong et al. (2010) find empirical evidence for the link between the serial correlation of stock returns and the diffusion of information. They also document a non-linearity in the term-structure of the serial correlation, in line with our model's predictions.

[31] Not all the asset classes display exactly the same decaying pattern. Whereas for commodities, equities, and currencies it seems to be decaying, other asset classes feature a U-shaped pattern.

[32] We do not attempt to match the magnitude of momentum, nor the exact length of the decaying pattern. Our purpose is to highlight a theoretical mechanism that can generate similar patterns. Empirical work is yet needed to estimate plausible values of λ and other parameters to better match the data.

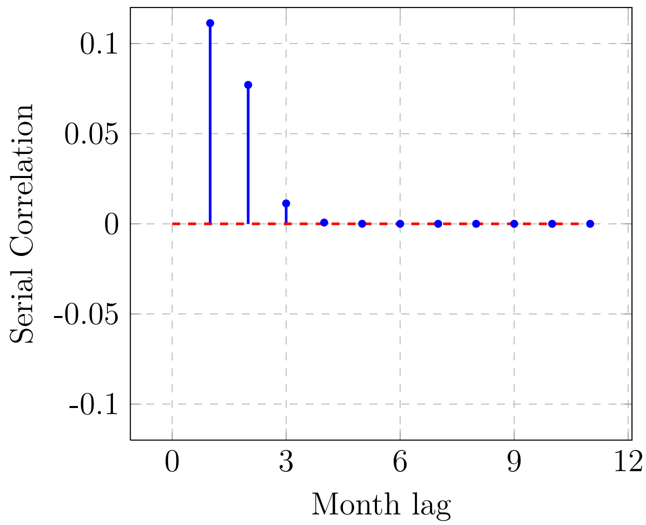

Figure 10: Term Structure of Momentum.
The bars represent values of the serial correlation of returns, when the lookback period varies from one to eleven months. The calibration is $H = S = \Phi = 1$, $\lambda = 1$, and $\gamma = \frac{1}{3}$.

intensity, our model predicts that the sign of serial correlation varies for different trading frequencies. At high trading frequencies, agents have little time to talk between trading rounds, noise trading dominates the effect of information percolation and most informed investors engage in contrarian strategies (Nagel, 2012). At lower trading frequencies instead, the information percolation mechanism bites and generates momentum, as shown in Section 3. Depending on the trading frequency considered, our model can therefore generate short-term reversals or medium-term momentum.

5.2 Past extreme returns, trading volume, and momentum

Stocks that have extreme past returns tend to have higher momentum profits.[33] This finding suggests that momentum is a hedge for extreme events, an additional challenge posed to risk-based explanations of momentum. We show that information percolation generates this positive link between past volatility and momentum. In our model, both the magnitude of momentum and the variance of stock returns depend on average precision of information. As the average precision increases, it jointly increases the magnitude of momentum and the variance of returns, which creates a positive relation between the two.

We plot the variance of stock returns, $\mathrm{Var}(\tilde{P}_1 - \tilde{P}_0)$, for different values of the meeting intensity λ in Figure 11. The past return variance is increasing in the speed of information diffusion. Intuitively, a larger diffusion speed makes prices more responsive to information and

[33] See, e.g., Jegadeesh and Titman (1993), Fama and French (1996), Moskowitz et al. (2012), and Bandarchuk and Hilscher (2013).

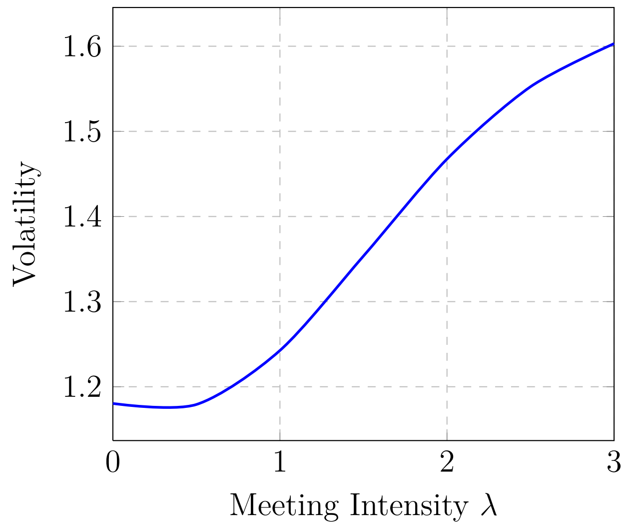

Figure 11: Volatility of stock returns at time 1 as a Function of λ
The line represents the volatility of stock returns at time 1, $\text{Var}(\tilde{P}_1 - \tilde{P}_0)$, as a function of the meeting intensity λ. The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

thus more volatile (Andrei, 2013). Our model therefore predicts that high past return variance predicts higher momentum, as in Moskowitz et al. (2012) and Bandarchuk and Hilscher (2013). In our model, however, the relation between past volatility and momentum is not causal—both terms are actually driven by the speed of information diffusion λ.

We now turn to the relation between past trading volume and momentum. Lee and Swaminathan (2000) show that higher past trading volume predicts higher momentum. In our model trading volume is driven by two forces. First, agents have heterogeneous information and thus trade because they disagree on the expected value of the fundamental. Second, information precision is heterogeneous across agents. This information asymmetry induces additional trades. As a result, information percolation generates higher levels of trading volume as compared to that in an economy without information percolation but with the same average precision, as shown in Figure 12.

Figure 12 further shows that trading volume increases with the speed of information diffusion, which implies a positive relation between past trading volume and momentum.[34] The reason is that traders are now better informed and thus trade more aggressively. Since a higher speed of information diffusion also generates momentum, it follows that high past trading volume predicts higher momentum, consistent with Lee and Swaminathan (2000). Furthermore, just like the relation between volatility and momentum, the relation between

[34]The relation is positive only for low to moderate values of λ. If the meeting intensity is extremely large, trading volume further increases, whereas momentum diminishes. This can be related to the *"Momentum Life Cycle"* hypothesis (Lee and Swaminathan, 2000), whereby high volume stocks are *late stage* momentum stocks and therefore exhibit less momentum.

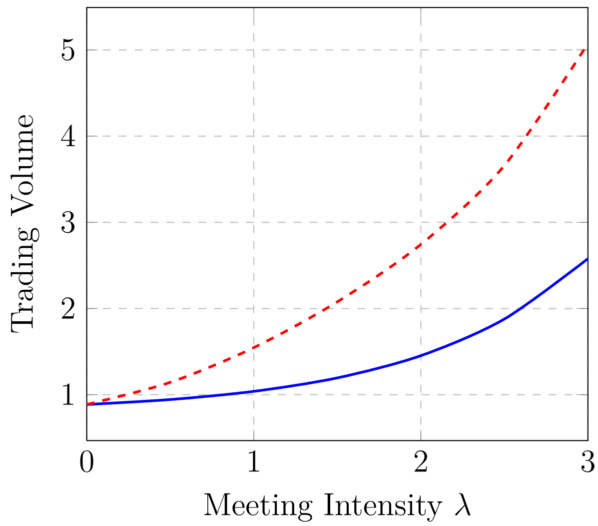

Figure 12: Trading volume at time 1 as a Function of λ
The blue line represents the trading volume at time 1 in an economy without information percolation, in which the average precision is assumed to grow exogenously over time. The red dashed line represents the trading volume at time 1 in an economy with information percolation. To make the comparison meaningful, average precisions are equal in both economies. The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

past trading volume and momentum is not causal in our model. Rather, both variables are driven by the speed of information diffusion λ.

5.3 Disagreement and momentum

There is an empirical link between heterogeneity of beliefs (disagreement) and momentum. Verardo (2009) shows that momentum profits are significantly larger for portfolios subject to higher disagreement, whereas Zhang (2006) shows that forecast dispersion (as a proxy for information uncertainty) leads to slower diffusion of information and therefore lower momentum.

We conduct the same analysis as we did for trading volume in the previous section and find similar results, as Figure 13 illustrates. First, comparing an economy with and without information percolation but with the same average precision, disagreement is higher in the former economy. The heterogeneity in information precision generates this effect. Second, disagreement is a hump-shaped function of the speed of information diffusion (for large values of λ, most investors become well informed and thus agree on the value of the fundamental). Both the momentum (Figure 3) and disagreement patterns are hump-shaped, but the range of meeting intensities over which they become decreasing differs. For reasonable values of meeting intensities, however, momentum and disagreement are positively related.

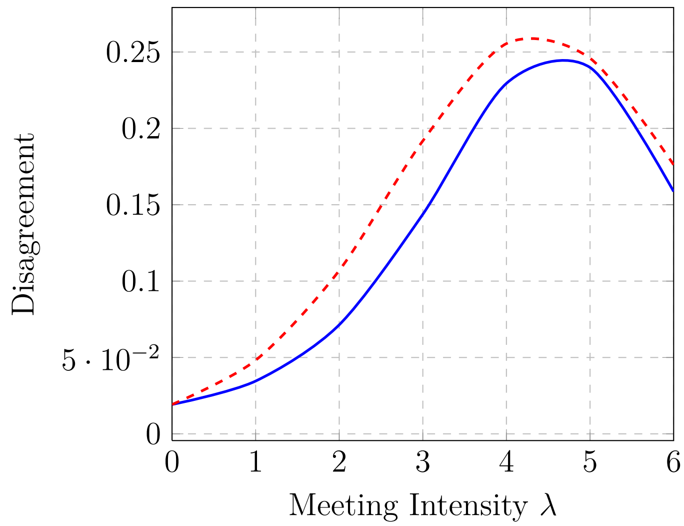

Figure 13: Disagreement at time 1 as a Function of λ

The blue line represents the disagreement at time 1 in an economy without information percolation. The red dashed line represents the disagreement at time 1 in an economy with information percolation. The average precision of information is the same in both economies. The calibration is $H = S = \Phi = 1$ and $\gamma = \frac{1}{3}$.

6 Discussion of Alternative Models

Our modeling assumptions raise two additional questions that we address in this section. First, how is this model different from one in which the average market precision increases exogenously, thereby generating momentum (Holden and Subrahmanyam, 2002)? Second, is momentum an artifact of the single liquidation date of our model?

We show that in a model in which the precision of information increases exogenously, all traders are contrarians. Such a model does not generate momentum trading and therefore cannot explain why momentum is not completely eliminated by momentum trading. Then, we address potential concerns regarding the finite horizon of our model. We build a setup in which the asset pays an infinite stream of dividends, as opposed to a single liquidating dividend. In this setup, we demonstrate that information percolation generates or amplifies momentum. We conclude that our results are not entirely driven by the fact that the world is run once and gradually comes to an end.

6.1 Alternative Model with Exogenous Increase in Precision

We assume away word-of-mouth communication among investors and assume instead that investors' signal precision S_t improves arbitrarily over time ($S_t > S_{t-1}$). Applying the measure of momentum of Banerjee et al. (2009) to this particular setup, we obtain the following result.

Proposition 4. *In an economy with exogenous increase in information precision, momentum arises at time $t = 1$ if and only if agent's precision shows sufficient improvement over time.*

Proof. Substituting $S\Omega_t \equiv S_t$ in (47) and simplifying shows that returns exhibit momentum at time $t = 1$ if

$$S_1 - S_0 > \frac{H}{1 + \gamma^2 \Phi S_0} > 0.$$

The right-hand side is strictly positive. □

To generate momentum, the necessary improvement in agents' precision is increasing in the precision H of the fundamental and risk aversion $1/\gamma$ and decreasing in the precision of the supply increments Φ. Furthermore, if agents' past precision of private information is large (S_0 large), an even larger precision of private information today (S_1) is needed to produce momentum. To illustrate this, suppose agent's precision increases geometrically over time, i.e., $S_t = a^t$. Using our calibration, we need at least $a \approx 2$ to generate momentum—the precision of the signal at time $t = 1$ must be at least twice that of the signal at time $t = 0$.

This observation allows us to highlight the first contribution of our model with respect to a model with exogenous increase in average precision: while a substantial improvement in agent's precision may be difficult to justify exogenously, this phenomenon naturally arises when investors interact through word-of-mouth communication. Our model therefore provides a micro-foundation for this gradual increase in agents' precision.

The second contribution of our model becomes immediately apparent when considering agents' trading strategy. Proposition 5, directly related to Proposition 3, makes this point.

Proposition 5. *In an economy with exogenous increase in information precision, all investors implement contrarian trading strategies at time $t = 1$.*

Proof. In this setting, agents have no informational advantage ($A_t^i \equiv a_t^i \equiv 0$), since they all have the same precision. As a result, the trading measure of section 4 simplifies to

$$\operatorname{cov}(\Delta \widetilde{D}_t^i, \Delta \widetilde{P}_t) \equiv -\frac{1 + \gamma^2 S_1 \Phi}{\gamma \Phi K_0} < 0$$

This expression is negative for all admissible parameter values. □

An exogenous increase in average market precision can generate momentum with contrarian investors only. But what makes momentum a puzzle is precisely that some agents trade on it and yet that it persists. Without heterogeneity in individual precisions, an exogenous increase in average precision therefore cannot provide a satisfactory answer to the momentum puzzle.

6.2 An Infinite Horizon Model

Our theory for momentum relies on an economy with a single liquidation date. In this section, we show that our results carry over to a fully dynamic setup. In particular, we build a stationary version of the model and show that, even in this case, information percolation generates or amplifies momentum.

We present a simplified version of the model and relegate all technical details to Appendix A.5. We consider an economy that goes on forever and in which one risky asset (stock) pays a stochastic dividend D_t per share. As in the finite version of the model, new liquidity traders enter the market in every trading session. To keep things simple, let us assume that the dividend process D_t and the supply process X_t follow random walks:

$$D_t = D_{t-1} + \varepsilon_t^d \tag{14}$$

$$X_t = X_{t-1} + \varepsilon_t^x \tag{15}$$

We will discuss more general processes at the end of this section.

All investors observe the past and current realizations of dividends and of the stock prices. Additionally, each investor observes a signal about the dividend innovation 3-steps ahead:

$$\tilde{z}_t^i = \varepsilon_{t+3}^d + \tilde{\epsilon}_t^i$$

As in the baseline model, investors meet and share private information over time. A fundamental difference, however, is that investors do not talk about a single liquidation value, but about several dividends revealed at different times in the future. That is, not only do they share information about the dividend 3-steps ahead, but they also share information about the dividend 2-steps ahead, and so on.[35]

Unlike the baseline model, we consider an overlapping generation of agents, as in Bacchetta and Wincoop (2006), Watanabe (2008), Banerjee (2010), and Andrei (2013). This assumption considerably simplifies the analysis by ruling out dynamic hedging demands.[36] The solution method, which follows Andrei (2013), proceeds by specifying an equilibrium

[35] Note that the model can be extended to a general case in which investors receive information about the dividend T-steps ahead at the expense of analytical complexity and without altering the main intuition presented here.

[36] In the infinite-horizon case the portfolio maximization problem is substantially more complicated. The fixed point problem cannot be reduced to a finite dimensional one, but Bacchetta and Wincoop (2006) and Andrei (2013) show how to approximate the problem to a desired accuracy level by truncating the state space. The (numerical) results for the infinite horizon model are very close to those obtained in the overlapping generations model. See also Albuquerque and Miao (2014).

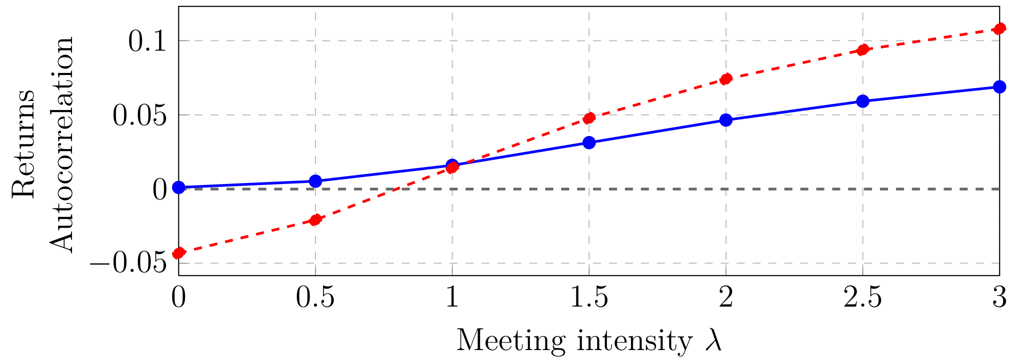

Figure 14: Serial Correlation of Returns in the Stationary Model
The figure depicts the serial correlation of returns, $\text{corr}(P_{t+1} - P_t, P_{t+2} - P_{t+1})$, for different levels of the meeting intensity λ. There are two cases: (*i*) the dividend and supply processes are random walks (solid blue line) and (*ii*) the dividend and supply processes are mean-reverting with AR(1) parameter 0.9. The calibration for the rest of the parameters ensures the existence of an equilibrium in the stationary model: $R = 1.1$, $H = 1$, $S = 10$, $\Phi = 1/100$, and $\gamma = 3$, although most of the calibrations we have tried give the same qualitative results.

price that is a linear function of model innovations:

$$P_t = \alpha D_t + \beta X_{t-3} + (a_3\ a_2\ a_1)\epsilon_t^d + (b_3\ b_2\ b_1)\epsilon_t^x$$

where $\epsilon_t^d \equiv (\varepsilon_{t+1}^d\ \varepsilon_{t+2}^d\ \varepsilon_{t+3}^d)^\top$ are the 3 future unobservable dividend innovations and $\epsilon_t^x \equiv (\varepsilon_{t-2}^x\ \varepsilon_{t-1}^x\ \varepsilon_t^x)^\top$ are the last 3 supply innovations. The main difference with respect to our baseline model is that equilibrium prices are now stationary. That is, the coefficients α, β, a, and b do not change over time, whereas the price coefficients in Theorem 1 change as the economy approaches the finite end-point.

We now show that information percolation generates momentum, even though prices are stationary. The random walk specification (14) - (15) helps us isolate the effect of information percolation on prices. In particular, when investors do not have private information, this specification directly implies that stock returns are serially uncorrelated. The solid blue line in Figure 14 then shows that, once investors receive private information, stock returns exhibit momentum, which information percolation further amplifies.[37]

The same intuition applies when the dividend and supply processes are not random walks. Because these processes are now mean-reverting, stock returns exhibit reversals in most cases (without information percolation). Our calculations show that information percolation can also generate momentum, much as in the baseline case. For instance, the dashed red line in

[37]To be consistent with our main model, we compute the serial correlation of returns using ex-dividend prices. Alternatively, one could assume several trading rounds in-between dividend payment dates (which would bring this extension even closer to our baseline case), with similar results. See Makarov and Rytchkov (2009) for a detailed analysis when returns are computed using cum-dividend prices.

Figure 14 shows how information percolation can turn reversals into momentum when the dividend and supply processes are mean-reverting with AR(1) parameters 0.9.

7 Rumors

Social interactions are natural propagators of rumors. In this section, we show that a "rumor" can generate a phase of price over-shooting followed by a phase of price correction. Under certain conditions, this convergence pattern can jointly produce short-term momentum and long-term reversal.

We follow Peterson and Gist (1951) and define a rumor as "*an unverified account or explanation of events circulating from person to person and pertaining to an object, event, or issue in public concern.*" To introduce a rumor in our model, we consider a small modification of the setup of Section 2 and assume that agents receive signals of the form:

$$\tilde{z}_t^i = \widetilde{U} + \widetilde{V} + \tilde{\epsilon}_t^i \tag{16}$$

where $\widetilde{V}$ is normally distributed with zero mean and precision ν.

The *common* noise, $\widetilde{V}$, satisfies two important properties of a rumor (as defined above): *i)* it circulates from person to person and *ii)* it is unverifiable. The first property arises as private signals now contain a rumor that is circulated from one agent to another through word-of-mouth communication. The second property results from the signal specification in (16): on average, private signals only reveal the sum of the fundamental value and the rumor $(\widetilde{U} + \widetilde{V})$. As a result, the rumor is unverifiable as agents cannot distinguish fundamental information from the rumor, either using prices or their private signals.

Rumors do not last forever, but eventually subside. To incorporate this aspect, we assume that each agent receives a signal at time $t = 3$ that is centered on the fundamental:

$$\widetilde{Z}_3^i = \widetilde{U} + \tilde{\epsilon}_3^i.$$

Using this signal, agents can back out the content of the rumor (on average) at time $t = 3$ and the rumor subsides. Overall, agents are aware of the rumor, but cannot learn about its content until time $t = 3$.

In the presence of a rumor, asset prices and investors' asset demands do not have a closed-form solution. Theorem 4 describes a system of recursive equations for the equilibrium price coefficients. We provide the proof of Theorem 4 and we solve this system of equations through an efficient numerical scheme that we describe in Appendix A.6.

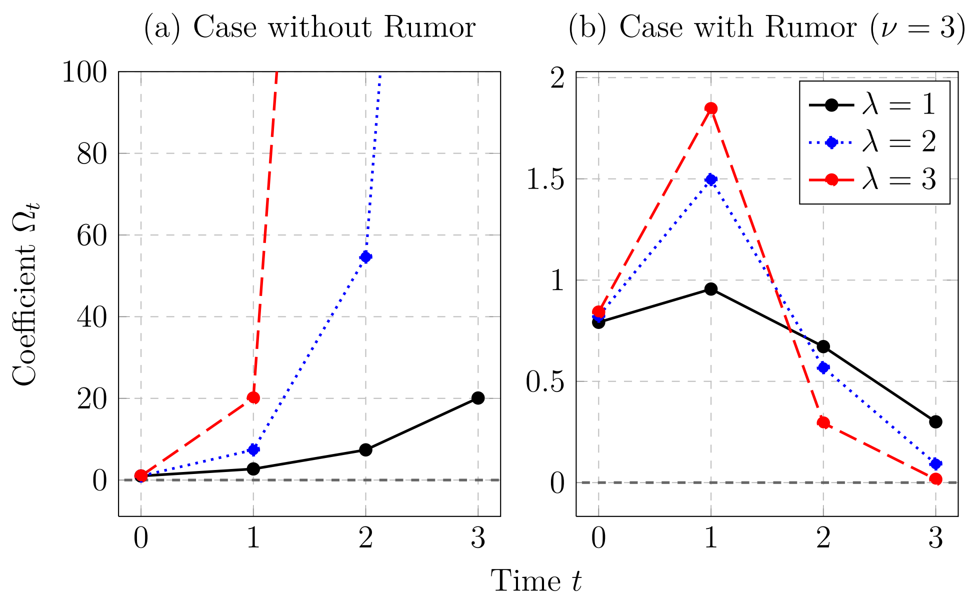

Figure 15: Price Coefficient Ω with and without Rumor

The Figure depicts the same coefficient Ω over time. Each line corresponds to a meeting intensity of $\lambda = 1$, $\lambda = 2$, and $\lambda = 3$. Panel (a) shows the coefficient Ω in the case of no rumor, while panel (b) shows the same coefficient in the case involving a rumor.

Theorem 4. *In the presence of a rumor, the price $\widetilde{P}$ is informationally equivalent to*

$$\widetilde{Q}_t = \widetilde{U} + \frac{\Lambda_t}{rS\Omega_t}\widetilde{V} - \frac{1}{rS\Omega_t}\widetilde{X}_t$$

where the equilibrium coefficients Ω and Λ solve a fixed-point problem given by a system of recursive equations:

$$\Omega_t = \frac{1}{rS}\sum_{j=0}^{t}\bar{\theta}_j - \sum_{j=0}^{t-1}\Omega_j \tag{17}$$

$$\Lambda_t = \sum_{j=0}^{t}\bar{\theta}_j - \sum_{j=0}^{t-1}\Lambda_j$$

in which $\bar{\theta}$ denotes the average coefficients of agents' private signals in their optimal demand.

The rumor has two important effects on prices, the first of which is immediately apparent when looking at the coefficient Ω. We plot this coefficient in Figure 15, both when signals contain a rumor (panel (b) with $\nu = 3$) and when they do not (panel (a)).

When signals do not contain a rumor, the coefficient Ω represents the average number of incremental signals. Panel (a) shows that this average rises as time passes by, all the more so as social interactions intensify. When signals contain a rumor (panel (b)), the coefficient Ω only increases initially. Intuitively, agents know they possess information of lower quality due

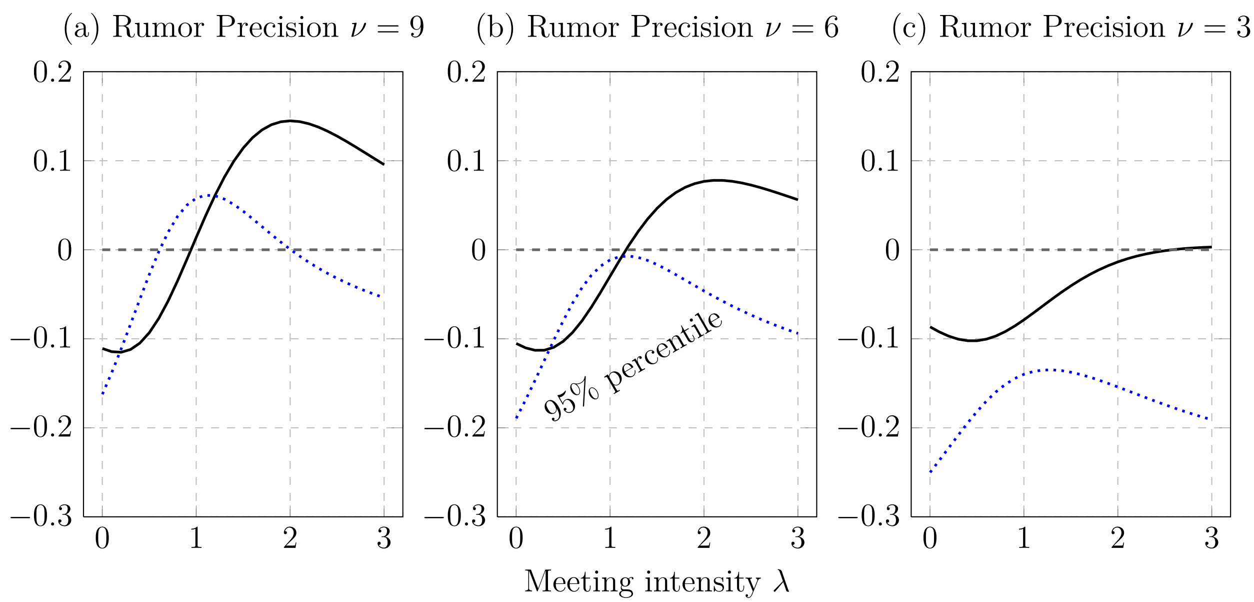

Figure 16: Serial Correlation of Return and Rumors

The Figure depicts the serial correlation of stock returns over the first (the solid black line) and the second period (the blue dotted line) as a function of the meeting intensity. The first period serial correlation is defined as $\mathrm{corr}(\widetilde{P}_1 - \widetilde{P}_0, \widetilde{P}_2 - \widetilde{P}_1)$, whereas the second period serial correlation is defined as $\mathrm{corr}(\widetilde{P}_2 - \widetilde{P}_1, \widetilde{P}_3 - \widetilde{P}_2)$. Each panel corresponds to a different rumor precision ν.

to the presence of the rumor and therefore apply a discount on the actual number of signals they have—the coefficient Ω now represents a *discounted* average of incremental signals.

At time $t = 2$, the discounted average Ω declines: agents anticipate that they will get better information at time $t = 3$ and apply a stronger discount on their number of signals. At time $t = 3$, the discounted average number of signals reaches zero for $\lambda = 3$: when agents have collected a vast number of signals, they can accurately forecast $\widetilde{U} + \widetilde{V}$. Hence, when they get the signal that is centered on the fundamental, they ignore their other signals. Overall, the rumor induces agents to interpret their information with some caution.

We now investigate how this convergence pattern relates to the serial correlation of stock returns. Intuitively, the first phase of price "over-shooting" generates short-term momentum and the second phase of price correction generates long-term reversal. To show this, we proceed as in Section 3.1 and plot returns' serial correlation in Figure 16.

When the rumor is fairly precise (panel (a)), returns mostly exhibit momentum: despite the presence of the rumor, agents' precision rises over time, generating momentum. As the precision of the rumor decreases (panel (b)), agents discount their actual number of signals more strongly. As a result, agents progressively cut back their positions—they adjust their trades to reflect that their information is of lower quality. While these portfolio adjustments do not prevent returns to exhibit momentum in the first-period, they induce reversal in the second period as the price gradually corrects. Finally, when the rumor's precision is low (panel (c)), agents become extremely cautious about their information and the improvement in their precision is not sufficient to generate momentum.

Overall, our model can jointly generate short-term momentum—consistent with the empirical finding of Jegadeesh and Titman (1993)—and long-term reversal—consistent with the over-reaction phenomenon of De Bondt and Thaler (1985).

8 Conclusion

We show that the diffusion of information in financial markets generates momentum in asset returns. When investors accumulate information through word-of-mouth communication, the price convergence towards fundamentals is accelerated, producing a trend. All investors observe this trend and some decide to trade on it, but, due to a rich heterogeneity in investment strategies generated by word-of-mouth communication, other investors decide to be contrarians and bet against the trend. As a result, momentum is not completely eliminated and appears to the econometrician as a profitable empirical anomaly. We also show that word-of-mouth can spread rumors and generate reversals.

In our theoretical model, momentum returns are positively related to past extreme returns, past trading volume, and disagreement (Moskowitz et al. 2012; Bandarchuk and Hilscher 2013; Lee and Swaminathan 2000; Zhang 2006; Verardo 2009). We show that information percolation generates a hump-shaped relation between firm size and momentum returns (Hong et al., 2000) and we also relate our results with findings on the term-structure of time-series momentum from Moskowitz et al. (2012) and Jegadeesh (1990). Finally, we show that our results hold in a infinite horizon economy and that momentum is not completely eliminated by a large, unconstrained, risk-neutral arbitrageur.

A legitimate question is what empirical exercise would validate our model. We believe that natural experiments capturing an exogenous increase or decrease in the intensity of word-of-mouth communication could make a worthwhile empirical point. For example, Shiller (2000) relates the obvious increase in the word-of-mouth communication intensity once the telephone became effective during the 1920s with the steady increase of volatility during the same period. Another option is to study the consequences of the Regulation Fair Disclosure, promulgated by the U.S. Securities and Exchange Commission in August 2000. This regulation forbids firms and their insiders to provide information to some investors (often large institutional investors). Hence, after August 2000 there should be less information propagated through the word-of-mouth communication channel.

Even though we abstract from individual behavioral biases, we believe that adding behavioral biases as in Daniel et al. (1998) or Barberis et al. (1998) would amplify the effects analyzed in this paper. Other questions are worthwhile investigating, such as extending the setup to multiple assets, where information percolation could generate rich dynamics of the conditional correlation among assets. It is also interesting to study precisely the mechanism of information transmission and find conditions under which investors find it beneficial to tell the truth (Stein, 2008).

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A Appendix

A.1 Information Percolation

A.1.1 Distribution of Incremental Signals: Closed-Form Solution

To obtain the closed-form solution for the distribution π of incremental signals, we first derive the equation for its dynamics.

Lemma 1. *The probability density function π over the additional number of signals collected by each agent satisfies*

$$\frac{d}{dt}\pi_t(n) = -\lambda\pi_t(n) + \lambda(\pi_t * \mu_t)(n), \quad \pi_0 = \delta_{n=1}. \tag{18}$$

Proof. We compute the distribution of new signals that have been gathered between time 0 and time T. Denote by X_{t_i} the number of new signals gathered if a meeting occurs at time t_i and observe that it is distributed as

$$X_{t_i} \sim \mu\left(t_i, \cdot\right)$$

where the distribution $\mu\left(t, x\right)$ satisfies the differential equation in (1). Furthermore, the number $N\left(T\right)$ of meetings that took place between time 0 and T is a Poisson counter with intensity λ; accordingly, the total number Y_T of new signals gathered between time 0 and T is given by $\sum_{i=1}^{N(T)} X_{t_i}$. We now characterize its distribution. First, observe that Y_T, conditional on the set of times $\{0 \leq t_1 \leq t_2 \leq ... \leq t_{N(T)} \leq T\}$ at which a meeting occurs (up to time T) and the total number of meetings $N\left(T\right)$ (that is, conditioning on the whole trajectory A_T^N of the Poisson process), is distributed as

$$\begin{aligned} Y_T|A_T^N &\sim \int_{\mathbb{R}^{N-1}} \mu\left(Y_{t_N} - Y_{t_{N-1}}, t_N\right) \mathrm{d}\mu\left(Y_{t_{N-1}} - Y_{t_{N-2}}, t_{N-1}\right) ...\mathrm{d}\mu\left(Y_{t_1} - 0, t_1\right) \\ &\equiv \int_{\mathbb{R}^{N-1}} \mu\left(X_{t_N}, t_N\right) \mathrm{d}\mu\left(X_{t_{N-1}}, t_{N-1}\right) ...\mathrm{d}\mu\left(X_{t_1}, t_1\right). \end{aligned} \tag{19}$$

Second, observe that the distribution $\mu\left(X_{t_i}, t_i\right)$ of increment can be expressed as a translation $\mathcal{T}$ of the type measure μ and the increment x. Hence, the distribution in (19) may be written as

$$Y_T|A_T^N \sim \Gamma_{i=1}^{N(T)} \mu_{t_i}$$

where, for any probability measures $\alpha_1, ..., \alpha_k$, we write $\Gamma_{i=1}^k = \alpha_1 * \alpha_2 * ... * \alpha_k$.

Now, observe that each t_i in the sequence of meetings $\{0 \leq t_1 \leq t_2 \leq ... \leq t_{N(\tau)} \leq T\}$ conditional on $N\left(T\right)$ is uniformly distributed over T; accordingly, we have that

$$Y_T|N\left(T\right) \sim \Gamma_{i=1}^{N(T)} \frac{1}{T}\int_0^T \mu_{t_i}\mathrm{d}t_i = \left(\frac{1}{T^{N(T)}}\left(\int_0^T \mu_s \mathrm{d}s\right)^{*N(T)}\right)$$

where $*n$ denotes the $n-$fold convolution.

Finally, since $N\left(T\right)$ is a Poisson(λ) counter, we have

$$Y_T \sim \sum_{k=0}^{\infty} e^{-\lambda T}\frac{(\lambda T)^k}{k!}\frac{1}{T^k}\left(\int_0^T \mu_s \mathrm{d}s\right)^{*k} = \sum_{k=0}^{\infty} e^{-\lambda T}\frac{\lambda^k}{k!}\left(\int_0^T \mu_s \mathrm{d}s\right)^{*k}.$$

Using the fact that, by Taylor expansion, e^x is equivalently written as $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, we can write

$$\pi_T = e^{\lambda\left(\int_0^T \mu_s \mathrm{d}s - T\right)}.$$

Differentiating this expression, we obtain (18). $\square$

A.1.2 Proof of Proposition 1

To obtain the closed form, it is convenient to think of individual signals received at times $t = 0, 1, 2$ separately. That is, even though all are signals about the same fundamental, we will treat them separately. Assume that signals received at time $t = 0, 1, 2$ are of type s_0, s_1, and s_2 respectively.

Let us fix some notation:

0. $n_0 \equiv$ the total number of type s_0 signals received between time 0 and t.
1. $n_1 \equiv$ the total number of type s_1 signals received between time 1 and t.
2. $n_2 \equiv$ the total number of type s_2 signals received between time 2 and t.

and:

0. $m_0 \equiv$ the incremental number of type s_0 signals received between *the last trading date* and t.
1. $m_1 \equiv$ the incremental number of type s_1 signals received between *the last trading date* and t.
2. $m_2 \equiv$ the incremental number of type s_2 signals received between *the last trading date* and t.

Let us focus first on the total number of signals, n_0, n_1, and n_2. From time 0 to 1, the distribution of n_0 has the support $[1, \infty)$. Recursive computations show that this distribution is

$$\mu_{\text{total},n_0} = e^{-n_0 \lambda \tau} \left(e^{\lambda \tau} - 1\right)^{n_0 - 1}$$

where $0 \leq \tau \leq 1$.

From time 1 to 2, the distribution of $\{n_0, n_1\}$ has the support $[1, \infty) \times [1, \infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{total},n_0,n_1} = \begin{cases} \binom{n_0-1}{n_1-1} e^{-n_0\lambda - n_1 \lambda \tau} \left(e^{\lambda} - 1\right)^{n_0 - n_1} \left(e^{\lambda\tau} - 1\right)^{n_1 - 1}, & \text{if } n_0 \geq n_1 \\ 0, & \text{otherwise} \end{cases}$$

From time 2 to 3, the distribution of $\{n_0, n_1, n_2\}$ has the support $[1, \infty) \times [1, \infty) \times [1, \infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{total},n_0,n_1,n_2} = \begin{cases} \binom{n_0-1}{n_1-1}\binom{n_1-1}{n_2-1} e^{-n_0\lambda - n_1\lambda - n_2 \lambda \tau} \left(e^{\lambda} - 1\right)^{n_0 - n_2} \left(e^{\lambda\tau} - 1\right)^{n_2 - 1}, & \text{if } n_0 \geq n_1 \geq n_2 \\ 0, & \text{otherwise} \end{cases}$$

Focus now on the distribution of increments, m_0, m_1, and m_2. From time 0 to 1, the distribution of m_0 has the support $[0, \infty)$. Recursive computations show that this distribution is

$$\mu_{\text{incr},n_0} = e^{(-m_0+1)\lambda\tau} \left(e^{\lambda\tau} - 1\right)^{m_0} \tag{20}$$

From time 1 to 2, the distribution of $\{m_0, m_1\}$ has the support $[0,\infty) \times [0,\infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{incr},m_0,m_1} = \begin{cases} \binom{m_0-1}{m_1-1} e^{-m_0\lambda-(m_1+1)\lambda\tau} \left(e^{\lambda}-1\right)^{m_0-m_1} \left(e^{\lambda\tau}-1\right)^{m_1}, & \text{if } m_0 \geq m_1 \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

From time 2 to 3, the distribution of $\{m_0, m_1, m_2\}$ has the support $[0,\infty) \times [0,\infty) \times [0,\infty)$. Further recursive computations show that this distribution is

$$\mu_{\text{incr},m_0,m_1,m_2} = \begin{cases} \binom{m_0-1}{m_1-1}\binom{m_1-1}{m_2-1} e^{-m_0\lambda-m_1\lambda-(m_2+1)\lambda\tau} \left(e^{\lambda}-1\right)^{m_0-m_2} \left(e^{\lambda\tau}-1\right)^{m_2}, & \text{if } m_0 \geq m_1 \geq m_2 \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

We can now group the signals of the same type, since they are all informative about the same fundamental. From time 0 to 1, signals are only of type s_0 and thus the probability density function over the additional number of signals follows from (20) with $\tau = 1$, with $m_0 = n-1$:

$$\pi_1(n) = e^{-n\lambda}\left(e^{\lambda}-1\right)^{n-1}$$

From time 1 to 2, given a number n of additional signals one has to find all the combinations of m_0 and m_1 for which $m_0 + m_1 = n-1$, and then make the sum of all the corresponding terms in (21) with $\tau = 1$:

$$\pi_2(n) = \sum_{i=0}^{n-1}\sum_{j=0}^{i}\left[\binom{i-1}{j-1}\mathbb{1}_{\{i+j=n-1\}} e^{-i\lambda-(j+1)\lambda}\left(e^{\lambda}-1\right)^{i}\right]$$

From time 2 to 3, given a number n of additional signals one has to find all the combinations of m_0, m_1, and m_2 for which $m_0 + m_1 + m_2 = n-1$, and then make the sum of all the corresponding terms in (22) with $\tau = 1$:

$$\pi_3(n) = \sum_{i=0}^{n-1}\sum_{j=0}^{i}\sum_{k=0}^{j}\left[\binom{i-1}{j-1}\binom{j-1}{k-1}\mathbb{1}_{\{i+j+k=n-1\}} e^{-(i+j)\lambda-(k+1)\lambda}\left(e^{\lambda}-1\right)^{i}\right]$$

A.1.3 Numerical Approach: Fourier Transform

To obtain the distribution of incremental signals numerically, we proceed through discrete Fourier transforms. Denote the Fourier transform of μ and π by $\widehat{\mu}_t(z) := \int_{\mathbb{R}} e^{izn}\mathrm{d}\mu_t(n)$ and $\widehat{\pi}_t(z) := \int_{\mathbb{R}} e^{izn}\mathrm{d}\pi_t(n)$, respectively, where $i = \sqrt{-1}$ and $z \in \mathbb{R}$. As in Duffie and Manso (2007), $\widehat{\mu}$ is given in closed form:

$$\widehat{\mu}_t(z) = \frac{\widehat{\mu}_0(z)}{e^{\lambda t}(1-\widehat{\mu}_0(z)) + \widehat{\mu}_0(z)}$$

where $\widehat{\mu}_0(z) = e^{iz}$ since $\mu_0(n)$ is a Dirac mass at 1.

To obtain $\widehat{\pi}$, we integrate (18) and get the following equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\pi}_t(z) = -\lambda\widehat{\pi}_t(z) + \lambda\widehat{\mu}_t(z)\widehat{\pi}_t(z),$$

the solution of which is also available in closed form:

$$\widehat{\pi}_t(z) = \frac{\widehat{\pi}_0(z)e^{i\pi}}{e^{\lambda t}(\widehat{\mu}_0(z) - 1) - \widehat{\mu}_0(z)}$$

where $\widehat{\pi}_0(z) = e^{iz}$ since $\pi_0(n)$ is a Dirac mass at 1.

We can now recover π numerically by using the inverse Fourier formula. To do so, notice that both distributions $\mu(n)$ and $\pi(n)$ are so-called *lattice* distributions for which every possible realization of n can be represented as $a + bk$ where k only takes integral values—in our case, $n \in \mathbb{N}$, $a = \frac{N}{2}$, and $b = \frac{1}{2}$. For this class of discrete distributions, the inverse Fourier formula writes

$$P\left[X = x_k\right] = \frac{b}{2\pi} \int_{-\pi/b}^{\pi/b} e^{-izx_k} \widehat{\mu}_t\left(z\right) \mathrm{d}z. \tag{23}$$

To compute the integral in (23), we use fast Fourier Transform: we rewrite (23) as

$$P\left[X = x_k\right] = \frac{b}{\pi} \int_{0}^{\pi/b} e^{-izx_k} \widehat{\mu}_t\left(z\right) \mathrm{d}z.$$

We then use the trapezoid rule rule to discretize the integral:

$$P\left[X = x_k\right] \approx \frac{b}{\pi} \frac{e^{-i\frac{\pi}{b}x_k} \widehat{\mu}_t\left(\frac{\pi}{b}\right) \Delta z}{2} + \frac{b}{\pi} \sum_{j=0}^{M-1} \delta_j e^{-ij\Delta z x_k} \widehat{\mu}_t\left(j\Delta z\right) \Delta z$$

with $\delta_j = \frac{1}{2}$ if $j = 1$ and $\delta_j = 1$ otherwise. We need to choose Δz such that the upper integration bound $\frac{\pi}{b} = 2\pi$ is reached by z; accordingly, we set $\Delta z \equiv \frac{2\pi}{M}$. Furthermore, the grid points for x_k are $x_k = -d + \lambda k,\ j \in \mathbb{N}$ and the fast Fourier transform method imposes that $\lambda \Delta z = \frac{2\pi}{M}$, i.e., $\lambda = 1$. Since $x_k \in \mathbb{N}$, we must have that $d = 0$. As a result, $P\left[X = x_k\right]$ takes the form of a discrete Fourier transform:

$$P\left[X = x_k\right] \approx \frac{1}{M} \sum_{j=0}^{M-1} g_j e^{-jk\frac{2\pi}{M}i}$$

with

$$g_j = \delta_j \frac{M}{2\pi} \widehat{\mu}_t\left(j\frac{2\pi}{M}\right) \frac{2\pi}{M}.$$

Finally, since each agent gets an additional signal every period, we must use the discrete Fourier transform sequentially: to derive the distribution π at time $t+1$, we use the numerical expression for $\mu_t(n)$ that we computed at time t and compute $\widehat{\mu}_0(z) = \sum_{k=1}^{N} e^{izk} f_0(k)$ where

$$f_0(n) = \begin{cases} \mu_t(n-1) & \text{if } n \geq t+1 \\ 0 & \text{otherwise} \end{cases}.$$

We then substitute $\widehat{\mu}_0(n)$ into $\widehat{\pi}_t(n)|_{t=1}$ and apply the inverse Fourier formula, which yields π_{t+1}.

A.2 Proof of Theorem 1

We provide the proof for a two trading session economy, that is, we eliminate for ease of exposition dates $t = 2$ and $t = 3$. Once the equilibrium quantities are written in a recursive form, as in Brennan and Cao (1997), or in He and Wang (1995), it is straightforward to derive the full recursive equilibrium solution.

The model is solved backwards, starting from date 1 and then going back to date 0. First, conjecture that prices in period 0 and period 1 are

$$\widetilde{P}_0 = \beta_0 \widetilde{U} - \alpha_{0,0} \widetilde{X}_0 \tag{24}$$

$$\widetilde{P}_1 = \beta_1 \widetilde{U} - \alpha_{1,0} \widetilde{X}_0 - \alpha_{1,1} \widetilde{X}_1 \tag{25}$$

Consider the normalized price signal in period zero (which is informationally equivalent to $\widetilde{P}_0$):

$$\widetilde{Q}_0 = \frac{1}{\beta_0} \widetilde{P}_0 = \widetilde{U} - \frac{\alpha_{0,0}}{\beta_0} \widetilde{X}_0 \tag{26}$$

Replace $\widetilde{X}_0$ from (26) into (25) to obtain

$$\widetilde{P}_1 = \varphi_1 \widetilde{U} + \xi_1 \widetilde{Q}_0 - \alpha_{1,1} \widetilde{X}_1 \tag{27}$$

where $\varphi_1 = \beta_1 - \alpha_{1,0} \frac{\beta_0}{\alpha_{0,0}}$ and $\xi_1 = \alpha_{1,0} \frac{\beta_0}{\alpha_{0,0}}$. These coefficients are to be determined in equilibrium. We normalize the price signal in period $t = 1$ and obtain $\widetilde{Q}_1$:

$$\widetilde{Q}_1 = \frac{1}{\varphi_1} \left(\widetilde{P}_1 - \xi_1 \widetilde{Q}_0 \right) = \widetilde{U} - \frac{\alpha_{1,1}}{\varphi_1} \widetilde{X}_1$$

Observing $\{\widetilde{Q}_0, \widetilde{Q}_1\}$ is equivalent with observing $\{\widetilde{P}_0, \widetilde{P}_1\}$. We conjecture the following relationships (see Admati 1985):

$$\frac{\alpha_{0,0}}{\beta_0} = \frac{1}{\gamma S \Omega_0} \tag{28}$$

$$\frac{\alpha_{1,1}}{\varphi_1} = \frac{1}{\gamma S \Omega_1} \tag{29}$$

where Ω_t is the cross-sectional average of the number of additional signals at time $t = 0, 1$, $\Omega_t = \sum_{\omega \in \mathbb{N}} \omega \pi_t(\omega)$. In our setup, $\Omega_0 = 1\ \forall \lambda$, $\Omega_1 = 1$ if $\lambda = 0$, and $\Omega_1 > 1$ if $\lambda > 0$. Relationships (28) and (29), which make the calculations that follow straightforward, are to be verified once the solution is obtained. Thus, the normalized price signals, informationally equivalent with prices, are:

$$\widetilde{Q}_0 = \widetilde{U} - \frac{1}{\gamma S \Omega_0} \widetilde{X}_0 \tag{30}$$

$$\widetilde{Q}_1 = \widetilde{U} - \frac{1}{\gamma S \Omega_1} \widetilde{X}_1 \tag{31}$$

A.2.1 Period 1

Consider an investor i who, at date $t=1$, collects $\omega_1^i \geq 1$ additional signals. At date $t=1$, investor i chooses $\widetilde{D}_1^i$ to maximize expected utility of final wealth:

$$\max_{\widetilde{D}_1^i} \mathbb{E}\left[-e^{-\frac{1}{\gamma}\widetilde{W}_2^i}\middle|\widetilde{F}_1^i\right]$$

where the final wealth at date $t=2$ (at liquidation) is

$$\widetilde{W}_2^i = X^i\widetilde{P}_0 + \widetilde{D}_0^i\left(\widetilde{P}_1 - \widetilde{P}_0\right) + \widetilde{D}_1^i\left(\widetilde{U} - \widetilde{P}_1\right) \tag{32}$$

and $\widetilde{F}_1^i$ represents the total information available at date $t=1$. This information is given by $\widetilde{Z}_1^i$, $\widetilde{Z}_0^i$ (private signals) and $\widetilde{Q}_1$, $\widetilde{Q}_0$ (public signals, informationally equivalent with prices and defined in (30) and (31)). Note that $\widetilde{Z}_0^i$ represent only one signal of precision S, but $\widetilde{Z}_1^i$ represent the average of the ω_1^i additional signals collected by the investor at date $t=1$ (ω_1^i signals of equal precision S are informationally equivalent with a signal equal to their average and having precision $\omega_1^i S$). With this information at hand at date $t=1$, investor i will try to forecast $\widetilde{U}$. The state variables corresponding to investor i are therefore

Precision →	$\frac{1}{H}$	$\frac{1}{\omega_1^i S}$	$\frac{1}{S}$	$\frac{1}{\Phi}$	$\frac{1}{\Phi}$	
Variable ↓	$\widetilde{U}$	$\widetilde{\varepsilon}_1^i$	$\widetilde{\varepsilon}_0^i$	$\widetilde{X}_1$	$\widetilde{X}_0$	$\mathbb{E}[\cdot]$
$\widetilde{U}$	1	0	0	0	0	0
$\widetilde{Z}_1^i$	1	1	0	0	0	0
$\widetilde{Z}_0^i$	1	0	1	0	0	0
$\widetilde{Q}_1$	1	0	0	$-\frac{1}{\gamma S\Omega_1}$	0	0
$\widetilde{Q}_0$	1	0	0	0	$-\frac{1}{\gamma S\Omega_0}$	0

It is straightforward to calculate

$$K_1^i = Var^{-1}\left[\widetilde{U}\middle|\widetilde{Z}_1^i, \widetilde{Z}_0^i, \widetilde{Q}_1, \widetilde{Q}_0\right]$$
$$\widetilde{\mu}_1^i = \mathbb{E}\left[\widetilde{U}\middle|\widetilde{Z}_1^i, \widetilde{Z}_0^i, \widetilde{Q}_1, \widetilde{Q}_0\right]$$

by using the projection theorem:

Theorem 5 (Projection Theorem). *Consider a n-dimensional normal random variable*

$$(\theta, s) \sim N\left(\left[\begin{array}{c}\mu_\theta \\ \mu_s\end{array}\right], \left[\begin{array}{cc}\Sigma_{\theta,\theta} & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,s}\end{array}\right]\right)$$

The conditional density of θ given s is normal with conditional mean

$$\mu_\theta + \Sigma_{\theta,s}\Sigma_{s,s}^{-1}(s-\mu_s)$$

and variance-covariance matrix

$$\Sigma_{\theta,\theta} - \Sigma_{\theta,s}\Sigma_{s,s}^{-1}\Sigma_{s,\theta}$$

provided $\Sigma_{s,s}$ is non-singular.

We get

$$
\begin{aligned}
K_1^i &= H + S\left(1+\omega_1^i\right) + \gamma^2 S^2 \Phi \left(\Omega_0^2 + \Omega_1^2\right) \\
\widetilde{\mu}_1^i &= \frac{1}{K_1^i}\left[S\widetilde{Z}_0^i + S\omega_1^i \widetilde{Z}_1^i + \gamma^2 S^2 \Phi \left(\Omega_0^2 \widetilde{Q}_0 + \Omega_1^2 \widetilde{Q}_1\right)\right]
\end{aligned} \tag{33}
$$

The optimal demand of trader i in period 1 has a standard form (from the normality of distribution assumption in conjunction with the exponential utility function):

$$
\widetilde{D}_1^i = \gamma K_1^i \left(\widetilde{\mu}_1^i - \widetilde{P}_1\right) \tag{34}
$$

Replace (33) in (34) to obtain

$$
\widetilde{D}_1^i = \gamma \left[S\widetilde{Z}_0^i + S\omega_1^i \widetilde{Z}_1^i + \gamma^2 S^2 \Phi \left(\Omega_0^2 \widetilde{Q}_0 + \Omega_1^2 \widetilde{Q}_1\right) - K_1^i \widetilde{P}_1\right] \tag{35}
$$

We can now integrate the optimal demands to get the total demand. We follow the convention used by Admati (1985) that implies $\int_0^1 \widetilde{Z}_j^i = \widetilde{U}$, a.s.. More important, we have now to keep track of the heterogeneity in information endowments when aggregating all individual demands. In particular, at time $t=1$ there is an infinity of types of investors with respect to their number of signals, and in each such type there is a continuum of investors. Consequently, the total demand at time $t=1$ is

$$
\widetilde{D}_1 = \int_0^1 \widetilde{D}_1^i = \sum_{\omega_1^i=1}^{\infty} \left[\pi_1(\omega_1^i) \int_{\omega_1^i} \widetilde{D}_1^i\right]
$$

which yields

$$
\widetilde{D}_1 = \gamma \left[S\left(\Omega_0 + \Omega_1\right)\widetilde{U} + \gamma^2 S^2 \Phi \left(\Omega_0^2 \widetilde{Q}_0 + \Omega_1^2 \widetilde{Q}_1\right) - K_1 \widetilde{P}_1\right] \tag{36}
$$

where K_1 is the average precision across the entire population of agents:

$$
K_1 \equiv \sum_{\omega_1^i=1}^{\infty} K_1^i(\omega_1^i)\pi_1(\omega_1^i) = H + S\left(\Omega_0 + \Omega_1\right) + \gamma^2 S^2 \Phi \left(\Omega_0^2 + \Omega_1^2\right)
$$

Replace (31) in (36) to obtain

$$
\widetilde{D}_1 = \gamma \left[\left(S\Omega_0 + S\Omega_1 + \gamma^2 S^2 \Phi \Omega_1^2\right)\widetilde{U} + \gamma^2 S^2 \Phi \Omega_0^2 \widetilde{Q}_0 - \gamma \Phi S \Omega_1 \widetilde{X}_1 - K_1 \widetilde{P}_1\right]
$$

The market clearing condition is $\widetilde{D}_1 = \widetilde{X}_0 + \widetilde{X}_1$. Once we impose market clearing, we can use the conjectured $\widetilde{P}_1$ equation (27) to get the undetermined coefficients φ_1, ξ_1, and $\alpha_{1,1}$:

$$
\begin{aligned}
\varphi_1 &= \frac{S\Omega_1\left(1+\gamma^2 S\Phi\Omega_1\right)}{K_1}, \\
\xi_1 &= \frac{S\Omega_0\left(1+\gamma^2 S\Phi\Omega_0\right)}{K_1} \\
\alpha_{1,1} &= \frac{1+\gamma^2 S\Phi\Omega_1}{\gamma K_1}
\end{aligned}
$$

From these solutions, we can verify that, indeed, $\frac{\alpha_{1,1}}{\varphi_1} = \frac{1}{\gamma S \Omega_1}$. Hence, (29) is now verified. The undetermined coefficients of $\widetilde{P}_1$ from the conjectured form (25) are

$$\begin{aligned}
\beta_1 &= \frac{K_1 - H}{K_1} \\
\alpha_{1,0} &= \frac{1 + \gamma^2 S \Phi \Omega_0}{\gamma K_1} \\
\alpha_{1,1} &= \frac{1 + \gamma^2 S \Phi \Omega_1}{\gamma K_1}
\end{aligned}$$

and thus

$$\widetilde{P}_1 = \frac{K_1 - H}{K_1} \widetilde{U} - \frac{1 + \gamma^2 S \Phi \Omega_0}{\gamma K_1} \widetilde{X}_0 - \frac{1 + \gamma^2 S \Phi \Omega_1}{\gamma K_1} \widetilde{X}_1 \tag{37}$$

which can also be written as

$$\widetilde{P}_1 = \frac{S \Omega_0 + \gamma^2 S^2 \Phi \Omega_0^2}{\gamma K_1} \widetilde{Q}_0 + \frac{S \Omega_1 + \gamma^2 S^2 \Phi \Omega_1^2}{\gamma K_1} \widetilde{Q}_1 \tag{38}$$

Furthermore, replacing $\widetilde{P}_1$ written under the form (38) in the optimal demand written under the form (35) gives the following result:

$$\widetilde{D}_1^i = \gamma \left[S \Omega_0 \left(\widetilde{Z}_0^i - \widetilde{Q}_0 \right) + S \omega_1^i \widetilde{Z}_1^i - S \Omega_1 \widetilde{Q}_1 - \left(K_1^i - K_1 \right) \widetilde{P}_1 \right] \tag{39}$$

The coefficient of $\widetilde{P}_1$ in (39) represents the cumulative informational (dis)advantage of investor i, which we denote thereafter by $A_1^i \equiv K_1^i - K_1$. Thus, the optimal demand of investor i at time $t = 1$ is

$$\widetilde{D}_1^i = \gamma \left(S \omega_0^i \widetilde{Z}_0^i - S \Omega_0 \widetilde{Q}_0 + S \omega_1^i \widetilde{Z}_1^i - S \Omega_1 \widetilde{Q}_1 - A_1^i \widetilde{P}_1 \right) \tag{40}$$

where $\omega_0^i = \Omega_0 = 1$, but we have included them here to highlight the recursive form of the optimal demand.

A.2.2 Period 0

The problem of investor i at time $t = 0$ is

$$\max_{\widetilde{D}_0^i} \mathbb{E} \left[-e^{-\frac{1}{\gamma} \widetilde{W}_2^i} \middle| \widetilde{Z}_0^i, \widetilde{Q}_0 \right]$$

where the final wealth is given in (32). Observe that, at time $t = 0$, investor i needs to estimate $\widetilde{U}$, $\widetilde{P}_1$ and $\widetilde{D}_1^i$, after observing $\widetilde{Z}_0^i$ and $\widetilde{Q}_0$. $\widetilde{P}_1$ and $\widetilde{D}_1^i$ are given by (37) and (39). The maximization problem then becomes:

$$\max_{\widetilde{D}_0^i} \mathbb{E} \left[-e^{-\frac{1}{\gamma} \left[X^i \widetilde{P}_0 + \widetilde{D}_0^i \left(\widetilde{P}_1 - \widetilde{P}_0 \right) + \widetilde{D}_1^i \left(\widetilde{U} - \widetilde{P}_1 \right) \right]} \middle| \widetilde{Z}_0^i, \widetilde{Q}_0 \right]$$

Lemma 2. *When an agent builds her portfolio, her future number of signals is irrelevant.*

Proof. We prove the claim in a slightly more general context: for the proof only, suppose agent i

starts with an arbitrary number n_0 of signals at time $t=0$. The value function V^i of agent i is then given by

$$
\begin{aligned}
V^i(n_0, W_0) &= e^{-\frac{1}{\gamma}W_0} \max_{\widetilde{D}_0^i(n_0)} \mathbb{E}\left[-e^{-\frac{1}{\gamma}\left(\widetilde{D}_0^i(n_0)\left(\widetilde{P}_1-\widetilde{P}_0\right)+\widetilde{D}_1^i(n_1)\left(\widetilde{U}-\widetilde{P}_1\right)\right)}\middle|\widetilde{Z}_0^i, \widetilde{Q}_0\right] \\
&= e^{-\frac{1}{\gamma}W_0} \max_{\widetilde{D}_0^i(n_0)} \sum_{k=1}^{\infty} \mu_1(k) \underbrace{\mathbb{E}\left[-e^{-\frac{1}{\gamma}\left(\widetilde{D}_0^i(n_0)\left(\widetilde{P}_1-\widetilde{P}_0\right)+\widetilde{D}_1^i(k)\left(\widetilde{U}-\widetilde{P}_1\right)\right)}\middle|\widetilde{Z}_0^i, \widetilde{Q}_0; n_1 = k\right]}_{g\left(n_0,k,\widetilde{D}_0^i\right)}.
\end{aligned}
$$

The function g represents an expectation of an exponential affine quadratic normal variable. To derive its explicit form, we use the following theorem.

Theorem 6. *Consider a random vector $z \sim N(0,\Sigma)$. Then,*

$$
E\left[e^{z'Fz+G'z+H}\right] = |I-2\Sigma F|^{-\frac{1}{2}}\, e^{\frac{1}{2}G'(I-2\Sigma F)^{-1}\Sigma G+H}.
$$

Tedious computations then show that

$$
g\left(n_0,k,\widetilde{D}_0^i\right) = -\left|I-2\Sigma(n_0,k)F(k)\right|^{-\frac{1}{2}}\, e^{\frac{1}{2}G\left(n_0,k,\widetilde{D}_0^i\right)'(I-2\Sigma(n_0,k)F(k))^{-1}\Sigma(n_0,k)G\left(n_0,\widetilde{D}_0^i\right)+H\left(n_0,k,\widetilde{D}_0^i\right)} \tag{41}
$$

where

$$
\Sigma(n_0,k) = \begin{pmatrix}
\frac{1}{K_0^i(n_0)} & \frac{K_1-H}{K_1K_0^i(n_0)} & \frac{1}{K_0^i(n_0)}-\frac{1}{K_1^i(k)} \\
\frac{K_1-H}{K_1K_0^i(n_0)} & \frac{(K_1^i(k)-S(k-n_0))(1+\gamma^2S\Phi\Omega_{t+1})^2}{K_1^2K_0^i(n_0)\gamma^2\Phi} & \frac{K_1-H}{K_1K_0^i(n_0)} \\
\frac{1}{K_0^i(n_0)}-\frac{1}{K_1^i(k)} & \frac{K_1-H}{K_1K_0^i(n_0)} & \frac{1}{K_0^i(n_0)}-\frac{1}{K_1^i(k)}
\end{pmatrix},
$$

$$
F(k) = \begin{pmatrix}
0 & \frac{1}{2}K^i(k) & -\frac{1}{2}K^i(k) \\
\frac{1}{2}K^i(k) & -K^i(k) & \frac{1}{2}K^i(k) \\
-\frac{1}{2}K^i(k) & \frac{1}{2}K^i(k) & 0
\end{pmatrix},
$$

$$
G\left(n_0,k,\widetilde{D}_0^i\right) = \begin{pmatrix}
\frac{K_1^i(k)S(\Omega_0(H+n_0S(1+\gamma^2S\Omega_0\Phi))\widetilde{Q}_0-K_0n_0\widetilde{Z}_0^i)}{K_1K_0^i(n_0)} \\
-\frac{\widetilde{D}_0^i}{\gamma}-\frac{2K_1^i(k)S(\Omega_0(H+n_0S(1+\gamma^2S\Phi\Omega_0))\widetilde{Q}_0-K_0n_0\widetilde{Z}_0^i)}{K_1K_0^i(n_0)} \\
\frac{K_1^i(k)S(\Omega_0(H+n_0S(1+\gamma^2S\Omega_0\Phi))\widetilde{Q}_0-K_0n_0\widetilde{Z}_0^i)}{K_1K_0^i(n_0)}
\end{pmatrix},
$$

$$
H\left(n_0,k,\widetilde{D}_0^i\right) = -\frac{\widetilde{P}_0}{\gamma}\widetilde{D}_0^i - \frac{S\begin{pmatrix} K_1^i(k)\gamma S((K_0n_0+H(\Omega_0-n_0))\widetilde{Q}_0-K_0n_0\widetilde{Z}_0^i)^2 \\ +K_1K_0^i(n_0)\widetilde{D}_0^i\begin{pmatrix}(K_0n_0+K_1\gamma^2S\Phi\Omega_0^2+H(\Omega_0-n_0))\widetilde{Q}_0 \\ +n_0S\Omega_1(1+\gamma^2S\Phi\Omega_1)\widetilde{Z}_0^i\end{pmatrix}\end{pmatrix}}{\gamma K_1^2K_0^i(n_0)^2}.
$$

Further computations show that

$$
h(n_0,k) \equiv |I-2\Sigma(n_0,k)F(k)| = \frac{K_1^i(k)}{K_1^2K_0^i(n_0)\gamma^2\Phi}\begin{pmatrix} H^2\gamma^2\Phi+H(1+\gamma^2\Phi(K_0+K_1-2H+S\Omega_1)) \\ +S\begin{pmatrix} n_0(1+\gamma^2S\Phi\Omega_1)^2 \\ +\gamma^2S\Phi\Omega_0^2(2+\gamma^2\Phi(K_1-H+S(\Omega_0+\Omega_1))) \end{pmatrix}\end{pmatrix}
$$

and

$$
q\left(n_0,\widetilde{D}_0^i\right) \equiv \frac{1}{2}G\left(n_0,k,\widetilde{D}_0^i\right)'\left(I-2\Sigma(n_0,k)F(k)\right)^{-1}\Sigma(n_0,k)G\left(n_0,\widetilde{D}_0^i\right)+H\left(n_0,k,\widetilde{D}_0^i\right)
$$

$$
= \frac{1}{2K_0^i(n_0)\gamma^2\left(H^2\gamma^2\Phi + HX + S\left(n_0 Z_1^2 + \gamma^2 S\Phi Y\Omega_0^2\right)\right)}
$$

$$
\times \left(\begin{array}{c} 2K_0^i(n_0)\gamma \widetilde{D}_0^i \left(\begin{array}{c} \widetilde{P}_0\left(H^2\gamma^2\Phi + HX + S\left(n_0 Z_1^2 + \gamma^2 S\Phi Y\Omega_0^2\right)\right) \\ -S\left(\gamma^2\Phi\Omega_0\widetilde{Q}_0\left(H + S\Omega_0\left(\gamma^2\Phi(K_1 + S(\Omega_0+\Omega_1)) + 2\right)\right) + n_0 Z_1^2 \widetilde{Z}_0^i\right) \end{array} \right) \\ + K_0^i(n_0) Z_1^2 (\widetilde{D}_0^i)^2 - \gamma^4\Phi\left(S\Omega_0\widetilde{Q}_0(H + n_0 S Z_0) - K_0 n_0 S \widetilde{Z}_0^i\right)^2 \end{array} \right)
$$

with

$$
\begin{aligned}
X &= 1 + \gamma^2\Phi(K_0 + K_1 - 2H + S\Omega_1), \\
Y &= 2 + \gamma^2\Phi(K_1 - H + S(\Omega_0 + \Omega_1)), \\
Z_t &= 1 + \gamma^2 S\Phi\Omega_t.
\end{aligned}
$$

Plugging these expressions into (41), agent i solves

$$
V^i(n_0, W_0) = e^{-\frac{1}{\gamma}W_0} \underbrace{\left(\sum_{k=1}^{\infty}\mu_1(k)h(n_0,k)\right)}_{\text{anticipation of future signals}} \max_{\widetilde{D}_0^i(n_0)} -e^{q\left(n_0, \widetilde{D}_0^i\right)} \tag{42}
$$

and it follows that her portfolio decision is independent of her expectation regarding her future number of signals. □

To obtain agent i's optimal demand, we solve the problem in (42) and impose the first-order condition

$$
\frac{\partial}{\partial \widetilde{D}_0^i} q\left(n_0, \widetilde{D}_0^i\right) = 0.
$$

We integrate the resulting optimal demand and impose market clearing in order to solve for the undetermined coefficients of $\widetilde{P}_0$, i.e., β_0 and $\alpha_{0,0}$. The solutions for these coefficients are:

$$
\begin{aligned}
\beta_0 &= \frac{K_0 - H}{K_0} \\
\alpha_{0,0} &= \frac{1 + \gamma^2 S\Phi\Omega_0}{\gamma K_0}
\end{aligned} \tag{43}
$$

where

$$
\begin{aligned}
K_0 = K_0^i &= H + S + \gamma^2 S^2\Phi\Omega_0^2 \\
\widetilde{\mu}_0^i &= \frac{1}{K_0}\left(S\widetilde{Z}_0^i + \gamma^2 S^2\Phi\Omega_0^2\widetilde{Q}_0\right)
\end{aligned} \tag{44}
$$

We have $K_0 = K_0^i$ in (44) because investors start with homogeneous information endowments at time 0, i.e., all have one signal. In other words, the cumulative informational (dis)advantage is zero for all investors: $A_0^i = 0$, $\forall i$. The optimal demand of investor i at time $t = 0$ is

$$
\widetilde{D}_0^i = \gamma\left(S\omega_0^i\widetilde{Z}_0^i - S\Omega_0\widetilde{Q}_0 - A_0^i\widetilde{P}_0\right) \tag{45}
$$

where we have added the last term to show that the demand at time $t = 0$ takes a similar form with the demand at time $t = 1$, expressed in equation (40). At this point, we can use (43) and (44) to verify that, indeed, $\frac{\alpha_{0,0}}{\beta_0} = \frac{1}{\gamma S\Omega_0}$. Hence, (28) is now verified. The solution can then be written in a recursive form and extended to more than 2 trading periods, as done in Theorem 1.

A.2.3 Optimal Trading Strategy

The optimal trading strategy of investor i at time $t = 1$ follows from (40) and (45):

$$\begin{aligned}\Delta\widetilde{D}_1^i \equiv \widetilde{D}_1^i - \widetilde{D}_0^i &= \gamma\left(S\omega_1^i \widetilde{Z}_1^i - S\Omega_1 \widetilde{Q}_1 - A_1^i \widetilde{P}_1 + A_0^i \widetilde{P}_0\right) \\ &= \gamma\left[S\omega_1^i \widetilde{Z}_1^i - S\Omega_1 \widetilde{Q}_1 - A_1^i\left(\widetilde{P}_1 - \widetilde{P}_0\right) - \left(A_1^i - A_0^i\right)\widetilde{P}_0\right]\end{aligned}$$

The cumulative informational (dis)advantage at time $t = 1$, A_1^i, has a recursive form:

$$A_1^i = A_0^i + S\omega_1^i - S\Omega_1 = A_0^i + a_1^i$$

where $a_1^i \equiv S\omega_1^i - S\Omega_1$ is the marginal informational (dis)advantage of trader i at time $t = 1$.

Thus, the optimal trading strategy becomes:

$$\begin{aligned}\Delta\widetilde{D}_1^i &= \gamma\left[S\omega_1^i\left(\widetilde{Z}_1^i - \widetilde{P}_0\right) - S\Omega_1\left(\widetilde{Q}_1 - \widetilde{P}_0\right) - A_1^i\left(\widetilde{P}_1 - \widetilde{P}_0\right)\right] \\ &= \gamma\left[\left(S\omega_1^i - S\Omega_1\right)\left(\widetilde{Z}_1^i - \widetilde{P}_0\right) + S\Omega_1\left(\widetilde{Z}_1^i - \widetilde{P}_0\right) - S\Omega_1\left(\widetilde{Q}_1 - \widetilde{P}_0\right) - A_1^i\left(\widetilde{P}_1 - \widetilde{P}_0\right)\right] \quad (46)\end{aligned}$$

Furthermore, it can be verified that

$$\begin{aligned}\widetilde{Q}_0 &= \frac{K_0\widetilde{P}_0}{S\Omega_0 + \gamma^2 S^2 \Phi \Omega_0^2} \\ \widetilde{Q}_1 &= \frac{K_1\widetilde{P}_1 - K_0\widetilde{P}_0}{S\Omega_1 + \gamma^2 S^2 \Phi \Omega_1^2}\end{aligned}$$

Replacing $\widetilde{Q}_1$ in (46) yields

$$\Delta\widetilde{D}_1^i = \gamma\left[a_1^i\left(\widetilde{Z}_1^i - \widetilde{P}_0\right) + S\Omega_1\left(\widetilde{Z}_1^i - \widetilde{P}_0\right) - \frac{K_1}{1 + \gamma^2 S\Phi\Omega_1}\left(\widetilde{P}_1 - \widetilde{P}_0\right) - A_1^i\left(\widetilde{P}_1 - \widetilde{P}_0\right)\right]$$

This is the same recursive form as in (9) and thus the proof of Theorem 1 is complete. □

A.3 Formula for ρ_t and Proof of Proposition 2

$$\rho_t = \frac{\left(\frac{1}{K_t} - \frac{1}{K_{t+1}}\right)\left(\left(\frac{1}{K_{t-1}} - \frac{1}{K_t}\right)\left(H + \sum_{j=0}^{t-1}(1+\gamma^2 S\Omega_j\Phi)^2\right) - \frac{1}{\gamma^2 K_t \Phi}(1+\gamma^2 S\Omega_t\Phi)^2\right)}{\left(\frac{1}{K_{t-1}} - \frac{1}{K_t}\right)^2\left(H + \frac{1}{\gamma^2\Phi}\sum_{j=0}^{t-1}(1+\gamma^2 S\Omega_j\Phi)^2\right) + \frac{1}{\gamma^2 K_t^2\Phi}(1+\gamma^2 S\Omega_t\Phi)^2}. \tag{47}$$

Proof of Proposition 2

Shutting down social interactions, prices are obtained as a special case of (2) when the average incremental number of signal satisfies $\Omega_t \equiv 1$; accordingly, the numerator in (47) simplifies to

$$-\frac{HS\left(\gamma^2 S\Phi + 1\right)^2}{\gamma^2\Phi\left(H + St\left(\gamma^2 S\Phi + 1\right)\right)\left(H + S(t+1)\left(\gamma^2 S\Phi + 1\right)\right)^2\left(H + S(t+2)\left(\gamma^2 S\Phi + 1\right)\right)} < 0.$$

This expression is negative for any time t.

A.4 Proof of Theorem 3

We provide the proof for a two trading session economy, that is, we eliminate for ease of exposition dates $t = 2$ and $t = 3$.

The model is solved backwards, starting from date 1 and then going back to date 0. First, conjecture that prices in period 0 and period 1 are

$$\widetilde{P}_0 = \beta_0 \widetilde{U} - \gamma_{0,0} \widetilde{X}_0 + \lambda_0 \widetilde{x}_0 \tag{48}$$

$$\widetilde{P}_1 = \beta_1 \widetilde{U} - \gamma_{1,0} \widetilde{X}_0 - \gamma_{1,1} \widetilde{X}_1 + \lambda_1 \widetilde{x}_1 \tag{49}$$

Consider the normalized price signal in period zero (which is informationally equivalent to $\widetilde{P}_0$):

$$\widetilde{Q}_0 = \frac{1}{\beta_0}(\widetilde{P}_0 - \lambda_0 \widetilde{x}_0) = \widetilde{U} - \frac{\gamma_{0,0}}{\beta_0} \widetilde{X}_0 \tag{50}$$

where the demand of the risk-neutral trader is observable because she only trades on public information, i.e., prices.

Replace $\widetilde{X}_0$ from (50) into (49) to obtain

$$\widetilde{P}_1 = \varphi_1 \widetilde{U} + \xi_1 \widetilde{Q}_0 - \gamma_{1,1} \widetilde{X}_1 + \lambda_1 \widetilde{x}_1 \tag{51}$$

where $\varphi_1 = \beta_1 - \gamma_{1,0} \frac{\beta_0}{\gamma_{0,0}}$ and $\xi_1 = \gamma_{1,0} \frac{\beta_0}{\gamma_{0,0}}$. We normalize the price signal in period $t = 1$ and obtain $\widetilde{Q}_1$:

$$\widetilde{Q}_1 = \frac{1}{\varphi_1} \left(\widetilde{P}_1 - \xi_1 \widetilde{Q}_0 - \lambda_1 \widetilde{x}_1 \right) = \widetilde{U} - \frac{\gamma_{1,1}}{\varphi_1} \widetilde{X}_1$$

Observing $\{\widetilde{Q}_0, \widetilde{Q}_1\}$ is equivalent with observing $\{\widetilde{P}_0, \widetilde{P}_1\}$. As in the setup of Section 2.2, we conjecture the following relationships:

$$\widetilde{Q}_0 = \widetilde{U} - \frac{1}{\gamma S \Omega_0} \widetilde{X}_0$$

$$\widetilde{Q}_1 = \widetilde{U} - \frac{1}{\gamma S \Omega_1} \widetilde{X}_1$$

A.4.1 Period 1

At time $t = 1$, both the precision and the posterior mean of an investor i remain identical to those of Section 2.2 in (33) along with her demand in (34). Integrating informed agents' demand again yields (36). The risk-neutral agent solves

$$\max_{\widetilde{x}_1} \widetilde{x}_1 E[\widetilde{U} - \widetilde{P}_t | \widetilde{Q}_0, \widetilde{Q}_1] = \max_{\widetilde{x}_1} \widetilde{x}_1 E[\widetilde{U} - \varphi_1 \widetilde{Q}_1 - \xi_1 \widetilde{Q_0} - \lambda_1 \widetilde{x}_1 | \widetilde{Q}_0, \widetilde{Q}_1].$$

and her optimal demand satisfies

$$\widetilde{x}_1 = \frac{1}{2\lambda_1} \left(E[\widetilde{U} | \widetilde{Q}_0, \widetilde{Q}_1] - \varphi_1 \widetilde{Q}_1 - \xi_1 \widetilde{Q_0} \right)$$

where

$$E[\widetilde{U}|\widetilde{Q}_0,\widetilde{Q}_1] = \frac{\gamma^2 S^2 \Phi\left(\Omega_0^2\widetilde{Q}_0 + \Omega_1^2\widetilde{Q}_1\right)}{H + \gamma^2 S^2 \Phi(\Omega_0^2 + \Omega_1^2)}.$$

The market clearing condition is $\widetilde{D}_1 + \widetilde{x}_1 = \widetilde{X}_0 + \widetilde{X}_1$. Once we impose market clearing, we can use the conjectured equation (51) to get the undetermined coefficients φ_1, ξ_1, $\gamma_{1,1}$, and λ_1:

$$\begin{aligned}
\varphi_1 &= \frac{S\Omega_1\left(1+\gamma^2 S\Phi\Omega_1\right)}{K_1}, \\
\xi_1 &= \frac{S\Omega_0\left(1+\gamma^2 S\Phi\Omega_0\right)}{K_1} \\
\gamma_{1,1} &= \frac{1+\gamma^2 S\Phi\Omega_1}{\gamma K_1} \\
\lambda_1 &= \frac{1}{\gamma K_1}
\end{aligned}$$

From these solutions, we can verify that, indeed, $\frac{\gamma_{1,1}}{\varphi_1} = \frac{1}{\gamma S\Omega_1}$. Hence, (29) is also verified in the presence of the risk-neutral agent.

A.4.2 Period 0

The problem of investor i at time $t = 0$ is, as in Section 2.2,

$$\max_{\widetilde{D}_0^i} \mathbb{E}\left[-e^{-\frac{1}{\gamma}\widetilde{W}_2^i}\middle|\widetilde{Z}_0^i,\widetilde{Q}_0\right]$$

where the expectation now takes into account the new price function in (51). Importantly, Lemma 2 still holds and informed agents' portfolio remains independent of the expected number of signals they will get in the future. The risk-neutral agent solves

$$\max_{\widetilde{x}_0} \widetilde{x}_0 E[\widetilde{P}_1 - \widetilde{P}_0|\widetilde{Q}_0] = \max_{\widetilde{x}_0} \widetilde{x}_0 E[\widetilde{P}_1 - \beta_0\widetilde{Q}_0 - \lambda_0\widetilde{x}_0|\widetilde{Q}_0]. \tag{52}$$

The optimization problem in (52) only involves the profits of period 0 because we assume that the risk-neutral agent does not take into account that a deviation from her strategy will affect current and future price signals for informed agents who cannot detect a deviation in her strategy; in that sense, the risk-neutral agent is myopic. As a result, her optimal demand satisfies

$$\widetilde{x}_0 = \frac{1}{\lambda_0}\left(E[\widetilde{P}_1|\widetilde{Q}_0] - \beta_0\widetilde{Q}_0\right)$$

where

$$E[\widetilde{P}_1|\widetilde{Q}_0] = \frac{S\left(2\gamma^2 S\Phi + \frac{H}{H+S(\Omega_0+\Omega_1+\gamma^2 S\Phi(\Omega_0^2+\Omega_1^2))}\right)}{2(H+\gamma^2 S^2\Phi\Omega_0^2)}$$

Integrating informed investors' optimal demand and imposing market clearing $D_0 + \widetilde{x}_0 = \widetilde{X}_0$, we obtain β_0, $\gamma_{0,0}$, and λ_0. We can then verify that, indeed, $\frac{\gamma_{0,0}}{\beta_0} = \frac{1}{\gamma S\Omega_0}$. By induction, the solution of the equilibrium for the for trading dates takes the form in Theorem 3.

A.5 Appendix for Section 6.2

This Appendix mainly follows Andrei (2013). Consider the following processes for dividends and noisy supply:

$$D_t = \kappa_d D_{t-1} + \varepsilon_t^d \tag{53}$$

$$X_t = \kappa_x X_{t-1} + \varepsilon_t^x \tag{54}$$

where $0 \leq \kappa_d \leq 1$ and $0 \leq \kappa_x \leq 1$. The dividend and supply innovations are *i.i.d.* with normal distributions: $\varepsilon_t^d \sim \mathcal{N}(0, 1/H)$ and $\varepsilon_t^x \sim \mathcal{N}(0, 1/\Phi)$. There is one riskless bond assumed to have an infinitely elastic supply at positive constant gross interest rate R.

The economy is populated by a continuum of rational agents, indexed by i, with CARA utilities and common risk aversion $1/\gamma$. Each agent lives for two periods, while the economy goes on forever (overlapping generations). All investors observe the past and current realizations of dividends and of the stock prices. Additionally, each investor observe an information signal about the dividend innovation 3-steps ahead:

$$\widetilde{z}_t^i = \varepsilon_{t+3}^d + \widetilde{\epsilon}_t^i$$

As time goes by, investors share their private information at random meetings. The information structure and the probability density function over the number of private signals is described in Andrei (2013). As usual in noisy rational expectations, we conjecture a linear function of model innovations for the equilibrium price:

$$P_t = \alpha D_t + \beta X_{t-3} + (a_3 \ a_2 \ a_1)\epsilon_t^d + (b_3 \ b_2 \ b_1)\epsilon_t^x$$

Proposition 1 in Andrei (2013) describes the rational expectations equilibrium, which is found by solving a fixed point problem provided by the market clearing condition. Infinite horizon models with overlapping generations have multiple equilibria (there are 2^N equilibria for a model with N assets). The model studied here has 2 equilibria, one low volatility equilibrium and one high volatility equilibrium. We focus on the low volatility equilibrium, which is the limit of the unique equilibrium in the finite version of the model.

To understand how the two equilibria arise, let's assume that there is no private information. In this case, the equilibrium price has a closed form solution:

$$P_t = \frac{\kappa_d}{R - \kappa_d} D_t - \frac{\Sigma}{\gamma}\frac{\kappa_x^3}{R - \kappa_x} X_{t-3} - \frac{\Sigma}{\gamma}\frac{\kappa_x^2}{R - \kappa_x}\varepsilon_{t-2}^x - \frac{\Sigma}{\gamma}\frac{\kappa_x}{R - \kappa_x}\varepsilon_{t-1}^x - \frac{\Sigma}{\gamma}\frac{1}{R - \kappa_x}\varepsilon_t^x$$

where $\Sigma \equiv (\alpha + 1)^2\sigma_d^2 + b_1^2\sigma_x^2$. Thus, the coefficient b_1 has to solve a quadratic equation:

$$b_1 = -\frac{\gamma}{R - \kappa_x}\left[\left(\frac{R}{R - \kappa_d}\right)^2 \sigma_d^2 + b_1^2\sigma_x^2\right]$$

For different parameter values, the above quadratic equation can have two solutions, one solution, or none. In this particular example (no private information), the autocovariance of stock returns,

$\text{Cov}\left(P_{t+1}-P_t, P_{t+2}-P_{t+1}\right)$, is

$$\begin{aligned}\text{Cov}\left(P_{t+1}-P_t, P_{t+2}-P_{t+1}\right) = &-\alpha^2\sigma_d^2\frac{1-\kappa_d}{1+\kappa_d}+\beta^2(\kappa_x-1)^2\kappa_x\frac{\sigma_x^2}{1-\kappa_x^2}+\\ &\begin{pmatrix}\beta-b_3 & b_3-b_2 & b_2-b_1 & b_1\end{pmatrix}\begin{pmatrix}-\beta(1-\kappa_x)\\ \beta-b_3\\ b_3-b_2\\ b_2-b_1\end{pmatrix}\end{aligned}$$

It can be shown numerically that this covariance is generally negative when $\kappa_d<1$ and $\kappa_x<1$. In the random walk specification (53) - (54), the covariance is zero.

Now, if agents receive private information, the model has to be solved numerically using the methodology described in Andrei (2013). More precisely, α, β, a, and b solve the following equations:

$$\begin{aligned}(\alpha+1)\kappa_d-R\alpha&=0\\ \bar{K}_t\beta\kappa_x-\bar{K}_tR\beta-\frac{1}{\gamma}\kappa_x^3&=0\\ \bar{K}_tb^*\mathbb{B}^{-1}\mathbb{A}+\bar{L}_t\mathbb{H}-\bar{K}_tRa&=\mathbf{0}_{1\times3}\\ \bar{K}_tb^*+\bar{L}_t\mathbb{B}^*-\bar{K}_tRb-\frac{1}{\gamma}\begin{pmatrix}\kappa_x^2 & \kappa_x & 1\end{pmatrix}&=\mathbf{0}_{1\times3}\end{aligned}$$

where $\bar{K}_t$, b^*, $\mathbb{B}$, $\mathbb{A}$, $\bar{L}_t$, $\mathbb{H}$, and $\mathbb{B}^*$ are defined in Appendix A.3 of Andrei (2013).

A.6 Appendix for Section 7

To prove Theorem 4, we adapt the expression for the price $\widetilde{P}$ in Brennan and Cao (1997) and write

$$\widetilde{P}_t = \beta_t \widetilde{U} + \alpha_t \widetilde{V} + \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j - \gamma_t \widetilde{X}_t.$$

The price is informationally equivalent to

$$\widetilde{Q}_t = \frac{1}{\beta_t} \widetilde{P}_t - \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j = \widetilde{U} + \frac{\alpha_t}{\beta_t} \widetilde{V} - \frac{\gamma_t}{\beta_t} \widetilde{X}_t.$$

Furthermore, we can write agent i's individual demand as

$$\widetilde{D}_t^i = \omega_t^i \widetilde{P}_t + \sum_{k=0}^{t} \lambda_k^i \widetilde{Q}_k + \sum_{k=0}^{t} \theta_k^i \widetilde{Z}_k^i.$$

By the law of large numbers, we have that $\int_{i\in[0,1]} \widetilde{Z}_k^i \mathrm{d}\mu(i) = \widetilde{U} + \widetilde{V}$. As a result, when we aggregate individual demands, we obtain

$$\int_{i\in[0,1]} \widetilde{D}_t^i \mathrm{d}\mu(i) = \bar{\omega}_t \widetilde{P}_t + \sum_{k=0}^{t} \bar{\lambda}_k \widetilde{Q}_k + \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} + \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V}$$

where $\bar{\omega}_t = \sum_{\mathbb{N}} \pi_t(k) \omega_t^i(k)$, $\bar{\lambda}_t = \sum_{\mathbb{N}} \pi_t(k) \lambda_t^i(k)$, and $\bar{\theta}_t = \sum_{\mathbb{N}} \pi_t(k) \theta_t^i(k)$.

Imposing market clearing, we have

$$\sum_{k=0}^{t} \widetilde{X}_k - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V} = \bar{\omega}_t \widetilde{P}_t + \sum_{k=0}^{t} \bar{\lambda}_k \widetilde{Q}_k.$$

Substituting

$$\widetilde{P}_t = \beta_t \widetilde{Q}_t + \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j$$

into the above equation, we obtain

$$\sum_{k=0}^{t} \widetilde{X}_k - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V} = \bar{\omega}_t (\beta_t \widetilde{Q}_t + \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j) + \sum_{k=0}^{t} \bar{\lambda}_k \widetilde{Q}_k.$$

Furthermore, notice that

$$\widetilde{X}_k = \frac{\beta_k}{\gamma_k} \left(\widetilde{U} + \frac{\alpha_k}{\beta_k} \widetilde{V} - \widetilde{Q}_k \right).$$

Substituting and regrouping, we obtain

$$\widetilde{X}_t + \sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \left(\widetilde{U} + \frac{\alpha_k}{\beta_k} \widetilde{V} - \widetilde{Q}_k \right) - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V} = (\bar{\omega}_t \beta_t + \bar{\lambda}_t) \widetilde{Q}_t + \sum_{j=0}^{t-1} (\xi_{j,t} \bar{\omega}_t + \bar{\lambda}_j \widetilde{Q}_j),$$

or, equivalently,

$$\widetilde{X}_t + \left(\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k\right) \widetilde{U} + \left(\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \frac{\alpha_k}{\beta_k} - \sum_{k=0}^{t} \bar{\theta}_k\right) \widetilde{V} - \sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \widetilde{Q}_k \equiv f(\{\widetilde{Q}_j\}_{j=0}^t).$$

The right-hand side of this equation is only a function of $\{\widetilde{Q}_j\}_{j=0}^t$. By separation of variables, the left-hand side must also be a function of $\{\widetilde{Q}_j\}_{j=0}^t$ only. Hence, it must be that

$$-\left(\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k\right) \left(\widetilde{U} + \frac{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \frac{\alpha_k}{\beta_k} - \sum_{k=0}^{t} \bar{\theta}_k}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k^1} \widetilde{V} - \frac{1}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} \widetilde{X}_t\right) \equiv \widetilde{Q}_t.$$

This equality holds if and only if

$$\frac{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \frac{\alpha_k}{\beta_k} - \sum_{k=0}^{t} \bar{\theta}_k}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} = \frac{\alpha_t}{\beta_t}$$

and

$$\frac{1}{\sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} - \bar{\theta}_k} = \frac{\gamma_t}{\beta_t}.$$

Without loss of generality, we set

$$\frac{\gamma_t}{\beta_t} = \frac{1}{rS\Omega_t}$$

so that

$$\sum_{k=0}^{t} (\bar{\theta}_k) = \sum_{k=0}^{t} \frac{\beta_k}{\gamma_k} = rS \sum_{k=0}^{t} \Omega_k.$$

and

$$\frac{\alpha_t}{\beta_t} = \frac{\eta \Lambda_t}{rS\Omega_t}$$

so that:

$$\eta \sum_{k=0}^{t} \bar{\theta}_k = \sum_{k=0}^{t} \frac{\beta_k}{\gamma_k} \frac{\alpha_k}{\beta_k} = \sum_{k=0}^{t} \Lambda_k.$$

The system of equations in (17) follows. This system of equations is a fixed point: to solve it, we solve the problem recursively (as in Appendix A.2, except accounting for the rumor) over 4 periods. We then start with guess values for $\{\Omega_j\}_{j=0}^3$ and $\{\Lambda_j\}_{j=0}^3$ and get, through the fixed point in (17), new values for these coefficients. Iterating and invoking the Contraction-Mapping Theorem, we obtain the equilibrium coefficients. □