

Collateral, Financial Intermediation, and the Distribution of Debt Capacity*

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Abstract

We study whether borrowers optimally conserve debt capacity to take advantage of investment opportunities due to temporarily low asset prices, when financing is subject to collateral constraints due to limited enforcement. We find that borrowers may exhaust their debt capacity and thus may be unable to take advantage of such opportunities, even if they can arrange for loan commitments or contingent financing. The cost of conserving debt capacity is the opportunity cost of foregone investment. This opportunity cost is higher for borrowers with higher productivity and borrowers who are less well capitalized, and such borrowers are hence more likely to exhaust their debt capacity. Borrowers who exhaust their debt capacity may be forced to contract when cash flows are low, and hence capital may be less productively deployed then. Higher collateralizability may make the contraction more severe. We consider the role of financial intermediaries which are better able to collateralize claims, that is, are “securitization specialists,” and study the dynamics of intermediary capital and spreads between intermediated and direct finance. When intermediary capital is scarce and spreads are high, borrowers who exhaust their debt capacity may be forced to contract by even more.

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1 Introduction

When asset prices are temporarily low, investment opportunities arise. In order to be able to take advantage of these, borrowers must either have funds available or be able to raise financing. We study whether borrowers optimally conserve debt capacity to take advantage of such opportunities when financing is subject to collateral constraints due to limited enforcement. We find that borrowers may exhaust their debt capacity and hence be unable to exploit opportunities that arise, even if they can arrange for loan commitments or contingent financing and contracting is constrained efficient. Conserving debt capacity has a cost: it reduces earlier investment. Borrowers who are more productive may exhaust their debt capacity, since the opportunity cost of conserving debt capacity is too high for them, while less productive borrowers conserve debt capacity. More productive borrowers are hence likely more constrained and may contract when asset prices and cash flows are low. In contrast, less productive borrowers are able to use their free debt capacity in such times to expand. Capital may hence be less productively deployed on average in such times. Moreover, borrowers with fewer internal funds exhaust their debt capacity, rendering them unable to seize investment opportunities due to low asset prices, while borrowers with more internal funds conserve some of their debt capacity, allowing them to seize opportunities.

We also consider the role of financial intermediaries, which are better able to collateralize claims but have limited capital. In the model, borrowers are able to obtain collateralized loans from both direct lenders as well as financial intermediaries. When financial intermediary capital is scarce, intermediated finance is more expensive than direct finance, that is, the spread between intermediated finance and direct finance is positive. The cross-sectional capital structure implication is that the more productive and more constrained borrowers borrow from intermediaries. Our model allows the analysis of the dynamics of intermediary capital and the spread between intermediated finance and direct finance.¹ If spreads are high when investment opportunities arise due to temporarily low asset prices, then borrowers which have exhausted their debt capacity may be forced to contract by more than they otherwise would. Indeed, they may contract for two reasons: first, because cash flows are low, and second, because intermediated finance becomes more expensive. Importantly in our model both borrowers and financial intermediaries are able to enter into contracts contingent on all states, that is contracting is complete. The only friction in our model is limited enforcement. Hence, we do not make an assumption that

¹See Holmström and Tirole (1998) for a related model of financial intermediation in a static environment in which there is a spread between the cost of intermediated and direct finance since intermediaries have limited capital.

aggregate states are not contractible.

We endogenize collateral constraints similar to the ones in Kiyotaki and Moore (1997) in an economy with limited contract enforcement in the spirit of Kehoe and Levine (1993, 2001, 2006). We assume that borrowers have limited commitment and can default on their promises to pay and abscond with all cash flows and a fraction of capital. We assume that agents who default can be excluded from neither the market for capital nor from borrowing and lending. Kehoe and Levine and most of the subsequent literature assume instead that agents who default are excluded from intertemporal trade. A notable exception is Lustig (2007) who considers limited enforcement similar to the one in our model in an endowment economy.

Dynamic models with limited commitment are used extensively in the literature to study optimal risk sharing² and asset pricing with heterogeneity,³ for example. Albuquerque and Hopenhayn (2004) and Hopenhayn and Werning (2007) analyze the implications for dynamic firm financing and Cooley, Marimon, and Quadrini (2004) consider the aggregate implications of firm financing with limited commitment.

We show that our model with limited enforcement implies that agents can borrow in a state-contingent way and that borrowing against each state is limited by the collateral value in that state. We also show that attention can be restricted to one period state contingent debt, and there is no additional role for long term debt or loan commitments. Thus we obtain collateral constraints which are similar to the ones in Kiyotaki and Moore (1997), albeit state contingent. Kiyotaki and Moore motivate their collateral constraints with an incomplete contracting model based on Hart and Moore (1994) and do not consider state-contingent borrowing. Several authors study models with similarly motivated collateral constraints. For example, Krishnamurthy (2003) studies a model where both borrowers and lenders have to collateralize their promises.⁴ Most closely related is Lorenzoni and Walentin (2007) who study a model with similar collateral constraints. Their focus is on the relation between investment, Tobin's q , and cash flow, and they do not consider aggregate shocks. Moreover, they restrict attention to the case in which agents always exhaust their debt capacity.

Shleifer and Vishny (1992) study debt capacity and the choice of optimal leverage in a model with aggregate states. They argue that debt may result in forced liquidations in bad times which in turn may limit the leverage that firms choose. They do not consider

²See, e.g., Kocherlakota (1996), Ligon, Thomas, and Worrall (1997), Kehoe and Perri (2002, 2004), and Krueger and Uhlig (2006).

³See, e.g., Alvarez and Jermann (2000, 2001), Lustig (2007), and Lustig and van Nieuwerburgh (2007).

⁴See also, Iacoviello (2005) who studies a business cycle model with collateral constraints; and Eisfeldt and Rampini (2007, 2008) who study firm financing subject to collateral constraints.

contingent financing, which is the focus here.

The role of intermediary capital is studied by Holmström and Tirole (1997). Intermediary capital in their model provides intermediaries with incentives to monitor and the amount of intermediary capital affects the availability of financing. They do not consider the dynamics of intermediary capital as we do here.⁵ The role of financial intermediaries during times where financing is constrained has been studied by Allen and Gale (1998, 2004), Gorton and Huang (2004), and Acharya, Shin, and Yorulmazer (2007), among others.⁶

This paper is also related to the emerging literature on contracting models of dynamic firm financing, most notably Clementi and Hopenhayn (2006) and DeMarzo and Fishman (2007a, 2007b), in addition to the papers mentioned above.

Finally, several other roles of collateral have been considered in the literature. When cash flows are private information, collateral may be used to induce agents to repay loans (see Diamond (1984), Lacker (2001), and Rampini (2005)). It has also been argued that collateral affects the interest rate that borrowers pay (see Barro (1976)), alleviates credit rationing due to adverse selection (see Bester (1985))⁷, reduces underinvestment problems (see Stulz and Johnson (1992)), provides lenders with an incentive to monitor (see Rajan and Winton (1995)), and renders markets more complete (see Dubey, Geanakoplos, and Shubik (2005) and Geanakoplos (1997)).

The paper proceeds as follows: Section 2 provides the model of collateral constraints due to limited enforcement and discusses the role of long term debt and loan commitments. Section 3 studies the distribution of debt capacity and the effect of collateralizability and asset prices on the extent to which agents who exhaust their debt capacity might contract. Section 4 considers the role of financial intermediation and Section 5 concludes.

2 Modeling collateralized borrowing

In this section we provide a dynamic model of collateralized borrowing. We first describe an economy in which lending is subject to collateral constraints, which are plausible but exogenously specified. We then provide a model of an economy with limited enforcement

⁵Cantillo (2004) provides a theory of financial intermediaries as specialized lenders. See also Diamond and Rajan (2000) who provide a model of bank capital which trades off liquidity creation and costs of distress.

⁶Gromb and Vayanos (2002) study the dynamics of a model with financially constrained arbitrageurs.

⁷See also Chan and Kanatas (1985), Besanko and Thakor (1987a, b), and Chan and Thakor (1987), who study the role of collateral in models with adverse selection, and Berger and Udell (1995) and Boot, Thakor, and Udell (1991), who study the role of collateral in models with moral hazard.

which limits agents ability to promise, and show that this economy is equivalent to the economy with collateral constraints. Furthermore, we analyze the role of long term debt and loan commitments and study the dynamics of minimum down payment requirements.

2.1 Environment

There are 3 dates, 0, 1, and 2. There is a continuum of agents of measure 1. Agents are risk neutral, subject to limited liability, and have preferences over (non-negative) dividends given by

$$E \left[\sum_{t=0}^2 d_t \right].$$

There are two goods in the economy, output goods and capital. Each agent is endowed with w_0 units of the output good at time 0 and no capital. Agents also have access to a production technology described below. These agents can be interpreted as entrepreneurs, for example, and will typically have a financing need and hence we refer to them at times as “borrowers.”

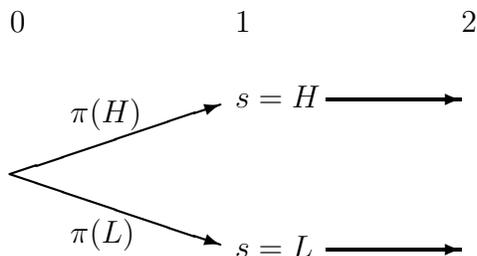
The entrepreneurs’ production technology is as follows. An amount of capital k_0 invested at time 0 returns $A_1(s)f(k_0)$ in output goods at time 1 in state s , where $s \in \mathcal{S}$, as well as the depreciated capital $(1 - \delta)k_0$. Entrepreneurs also have access to a production technology at time 1 which, for an investment of $k_1(s)$, returns $A_2(s)f(k_1(s))$ in output goods at time 2 as well as the depreciated capital $(1 - \delta)k_1(s)$.

In addition to the borrowers described above, there is also a continuum of measure 1 of lenders in the economy which are unconstrained and risk neutral and discount the future at a rate $\beta < 1$. Lenders have a large endowment of funds in all dates and states. Lenders cannot run the production technology. Lenders have a large amount of collateral and hence are not subject to collateral constraints or enforcement problems but rather are able to commit to deliver on their promises. Lenders are thus willing to provide any state contingent loans at an expected rate of return $R = 1/\beta$ subject to borrowers’ collateral or enforcement constraints.

We will consider two specifications of the financing constraints. First, we assume that agents can borrow, at time t , up to $\theta \in (0, 1)$ times the resale value of capital against each state at time $t + 1$. Second, we assume that markets are complete but there is limited enforcement; borrowers can abscond with the cash flows from the production technology and with fraction $1 - \theta$ of capital. Importantly we assume that entrepreneurs cannot be excluded from future borrowing or the market for capital.

Finally, we assume that output goods can be transformed into capital goods (and vice versa) at a rate ϕ_0 at time 0 and at a rate of $\phi_t(s)$ at time $t \in \mathcal{T} \equiv \{1, 2\}$ in

Figure 1: Time Line



state $s \in \mathcal{S} \equiv \{H, L\}$, where state s has probability $\pi(s)$. Thus, for simplicity, we assume a very simple stochastic structure with two states at time 1 and no further uncertainty as illustrated in Figure 1. We assume that $\phi_1(H) > \phi_1(L)$ and that $A_1(H) > A_1(L)$, that is, we assume that in the state L capital is relatively cheap, but cash flows are low at the same time. This is meant to capture the idea that state L is an economy wide downturn.

2.2 Collateral constraints

Consider the borrower's problem subject to collateral constraints, which we derive endogenously as implications of limited contract enforcement below. The borrower maximizes the expected value by choosing dividends $\{d_0, d_t(s)\}$, investment $\{k_0, k_1(s)\}$, and state contingent borrowing $\{b_{t-1}(s)\}$ for all $(s, t) \in \mathcal{S} \times \mathcal{T}$ subject to budget constraints, collateral constraints, and limited liability and non-negativity constraints:

$$\max_{\{d_0, d_t(s), k_0, k_1(s), b_{t-1}(s)\}_{s \in \mathcal{S}, t \in \mathcal{T}}} d_0 + \sum_{s \in \mathcal{S}} \pi(s) \left\{ \sum_{t \in \mathcal{T}} d_t(s) \right\}$$

subject to the budget constraints for all dates and states,

$$\begin{aligned} w_0 + \sum_{s \in \mathcal{S}} \pi(s) b_0(s) &\geq d_0 + \phi_0 k_0 \\ A_1(s) f(k_0) + \phi_1(s) k_0 (1 - \delta) + b_1(s) &\geq d_1(s) + \phi_1(s) k_1(s) + R b_0(s), \quad \forall s \in \mathcal{S}, \\ A_2(s) f(k_1(s)) + \phi_2(s) k_1(s) (1 - \delta) &\geq d_2(s) + R b_1(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

the collateral constraints,

$$\begin{aligned} \phi_1(s) \theta k_0 (1 - \delta) &\geq R b_0(s), \quad \forall s \in \mathcal{S}, \\ \phi_2(s) \theta k_1(s) (1 - \delta) &\geq R b_1(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

and $d_0 \geq 0$, $d_t(s) \geq 0$, $k_0 \geq 0$, $k_1(s) \geq 0$, $\forall s \in \mathcal{S}$ and $t \in \mathcal{T}$. Note that if the borrower promises to pay $Rb_0(s)$ in state s at time 1, he receives an amount of funds $\pi(s)b_0(s)$ at time 0. This guarantees the lender an expected return of R on the loan. Moreover, note that the amount that the borrower can credibly promise to repay at time t in state s is limited to a fraction θ of the resale value of capital in that state.

This model of collateralized borrowing allows us to be precise about the meaning of the term *debt capacity*. One unit of capital has state s debt capacity equal to a fraction θ of the present value of the resale value of capital, $R^{-1}\phi_1(s)\theta(1 - \delta)$. One unit of capital has (overall) debt capacity equal to a fraction θ of the present value of the expected resale value of capital, $R^{-1}\sum_{s \in \mathcal{S}}\pi(s)\phi_1(s)\theta(1 - \delta)$. The overall debt capacity of a firm, of course, depends on the amount of capital the firm acquires and hence is endogenous. A firm exhausts its state s debt capacity if $R^{-1}\phi_1(s)\theta(1 - \delta) \geq b_0(s)$ holds with equality and has free state s debt capacity otherwise, and analogously for the firm's overall debt capacity.

2.3 Limited enforcement

The economy with collateral constraints described above is equivalent to an economy with limited enforcement of contracts. Specifically, suppose agents can default on their promises, that is walk away from their debt obligations and abscond with all cash flows and fraction $1 - \theta$ of capital, and that lenders can seize only fraction θ of the capital and do not have access to any other enforcement mechanism. In particular, borrowers cannot be excluded from further borrowing or from purchasing capital goods. Thus, enforcement is limited as in Kehoe and Levine (1993) but unlike in their model, agents cannot be excluded from intertemporal markets here.⁸

The problem with limited enforcement is similar to the borrower's problem with collateral constraints, with the following changes: first, we do not require that each state contingent loan breaks even on its own and we instead denote borrowing with $\{l_0, l_1(s)\}$ and state contingent repayments by $\{b_{t-1}(s)\}$; second, we require that the lender breaks even in expected value (see (5)); third, the collateral constraints are replaced by enforcement constraints (equations (6) and (7)); finally, the time 1 enforcement constraints involve the dividends $\{\hat{d}_1(s), \hat{d}_2(s)\}$ that the borrower could attain after default and solve the problem in equations (9)-(13).

⁸If θ were equal to 0, that is, if the borrower could abscond with all cash flows and all capital and would not be excluded from future lending, then borrowers could not borrow at all (see Bulow and Rogoff (1989)).

The contracting problem is thus:

$$\max_{\{d_0, d_t(s), l_0, l_1(s), k_0, k_1(s), b_{t-1}(s)\}_{s \in \mathcal{S}, t \in \mathcal{T}}} d_0 + \sum_{s \in \mathcal{S}} \pi(s) \left\{ \sum_{t \in \mathcal{T}} d_t(s) \right\} \quad (1)$$

subject to the budget constraints,

$$w_0 + l_0 \geq d_0 + \phi_0 k_0 \quad (2)$$

$$A_1(s)f(k_0) + \phi_1(s)k_0(1 - \delta) + l_1(s) \geq d_1(s) + \phi_1(s)k_1(s) + Rb_0(s), \quad \forall s \in \mathcal{S}, \quad (3)$$

$$A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1 - \delta) \geq d_2(s) + Rb_1(s), \quad \forall s \in \mathcal{S}, \quad (4)$$

the lender's ex ante participation constraint

$$\sum_{s \in \mathcal{S}} \pi(s) \left\{ \sum_{t \in \mathcal{T}} R^{-(t-1)} b_{t-1}(s) \right\} \geq l_0 + \sum_{s \in \mathcal{S}} \pi(s) R^{-1} l_1(s) \quad (5)$$

the enforcement constraints

$$d_1(s) + d_2(s) \geq \hat{d}_1(s) + \hat{d}_2(s), \quad \forall s \in \mathcal{S}, \quad (6)$$

$$d_2(s) \geq A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1 - \theta)(1 - \delta), \quad \forall s \in \mathcal{S}, \quad (7)$$

and

$$d_0 \geq 0, \quad d_t(s) \geq 0, \quad k_0 \geq 0, \quad k_1(s) \geq 0, \quad \forall s \in \mathcal{S} \text{ and } t \in \mathcal{T}, \quad (8)$$

where $\{\hat{d}_1(s), \hat{d}_2(s)\}$ are the dividends that the borrower could achieve after default, i.e., solve

$$\{\hat{d}_1(s), \hat{d}_2(s)\} \in \arg \max_{\{d'_t(s), k'_1(s), b'_1(s)\}_{t \in \mathcal{T}}} \sum_{t \in \mathcal{T}} d'_t(s) \quad (9)$$

subject to

$$A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta) + b'_1(s) \geq d'_1(s) + \phi_1(s)k'_1(s), \quad (10)$$

$$A_2(s)f(k'_1(s)) + \phi_2(s)k'_1(s)(1 - \delta) \geq d'_2(s) + Rb'_1(s), \quad (11)$$

and

$$d'_2(s) \geq A_2(s)f(k'_1(s)) + \phi_2(s)k'_1(s)(1 - \theta)(1 - \delta) \quad (12)$$

and

$$d'_t(s) \geq 0, \quad k'_1(s) \geq 0, \quad \forall t \in \mathcal{T}. \quad (13)$$

We choose not to write down the problem recursively since, in principle, long term contracts could add value. Next we show that in fact, we can restrict attention to one period debt without loss of generality.

2.4 Irrelevance of long term debt

Long term debt cannot add value in the problem with limited enforcement. Intuitively, the enforcement constraints that we impose restrict the payments that the borrower can credibly promise to the lender to payments with present value less than or equal to the value of capital that the borrower cannot abscond with. Any long term debt contract which satisfies this restriction can be implemented with a sequence of one period debt contracts. Hence, long term debt is irrelevant.

Lemma 1 *Without loss of generality, $l_0 = \sum_{s \in \mathcal{S}} \pi(s) b_0(s)$ and $l_1(s) = b_1(s)$, $\forall s \in \mathcal{S}$, that is, considering state-contingent one period debt is sufficient.*

Proof of Lemma 1. Note that $Rb_1(s)$ is the total payment from the borrower to the lender at time 2, and there is no need to distinguish payments due to funds lent at time 0 (l_0) and at time 1 in state s ($l_1(s)$). Moreover, the program only determines the net payment $Rb_0(s) - l_1(s)$, $\forall s \in \mathcal{S}$, and thus we are free to set $l_1(s) = b_1(s)$, $\forall s \in \mathcal{S}$. Equation (5) then simplifies to $\sum_{s \in \mathcal{S}} \pi(s) b_0(s) \geq l_0$ and using the fact that this equation holds with equality we can substitute for l_0 . \square

In contrast, when borrowers can be excluded from intertemporal trade, which is the case typically considered in the literature, long term contracts would not be irrelevant.

2.5 Collateral constraints due to limited enforcement

We now show that the model with limited enforcement is equivalent to the model with state-contingent collateral constraints which were specified in an “ad-hoc” way before.

Lemma 2 *The participation constraints (6) and (7) can equivalently be written as*

$$\phi_1(s) \theta k_0 (1 - \delta) \geq Rb_0(s), \quad \forall s \in \mathcal{S}, \quad (14)$$

$$\phi_2(s) \theta k_1(s) (1 - \delta) \geq Rb_1(s), \quad \forall s \in \mathcal{S}. \quad (15)$$

Proof of Lemma 2. Notice that (4) holds with equality. Substituting for $d_2(s)$ in (7) using (4) and canceling terms implies (15). Conversely, (15) together with (4) at equality implies (7).

To obtain (14), assume that $Rb_0(s) > \phi_1(s) \theta k_0 (1 - \delta)$. Let $X(s) \equiv \{d_t(s), k_1(s), b_1(s)\}_{t \in \mathcal{T}}$ be the allocation from time 1 onward in state s . Consider default at time 1 to an allocation $X'(s) = X(s)$. Note that (4) implies

$$\begin{aligned} A_1(s) f(k_0) + \phi_1(s) k_0 (1 - \theta) (1 - \delta) + b_1(s) &> A_1(s) f(k_0) + \phi_1(s) k_0 (1 - \delta) - Rb_0(s) + b_1(s) \\ &\geq d_1(s) + \phi_1(s) k_1(s), \end{aligned}$$

and hence $X'(s)$ is feasible. Moreover $d'_1(s)$ can be increased which violates (6), a contradiction. Conversely, (14) implies that the optimal allocation after default, $\hat{X}(s)$ say, is a feasible allocation and hence the contractual allocation $X(s)$ must attain at least that value, implying that (6) is satisfied. \square

Lustig (2007) considers a similar outside option in an endowment economy and Lorenzoni and Walentin (2007) consider collateral constraints with a similar motivation in an economy with constant returns to scale. The original formulation of the enforcement constraints is in the same spirit as the one used to endogenize debt constraints in Kehoe and Levine (1993), although the limits on enforcement are different here. Kehoe and Levine assume that agents who default are excluded from intertemporal markets whereas we assume that agents cannot be excluded. Lemma 2 shows that, given our assumptions about the limits on enforcement, the constraints can equivalently be formulated as collateral constraints in the spirit of Kiyotaki and Moore (1997), but, importantly, are aggregate state contingent.

One advantage of this equivalent formulation is that the constraint set (2)-(4), (8), and (14)-(15) is convex. We study this problem henceforth. The first order conditions, which are hence necessary and sufficient, are stated in the appendix.

More importantly though, the equivalent formulation has the advantage that the implementation of the optimal dynamic lending contract is rather simple: borrowers have access to state-contingent secured loans only. Such lending arrangements can hence be decentralized relatively easily.

2.6 Dynamics of minimum down payments

This model of collateralized borrowing has the property that the minimum down payment is lower when the price of capital is expected to rise. This property seems empirically plausible and is consistent with anecdotal evidence that down payment requirements (or “lending standards”) vary inversely with expected capital appreciation. To see this, define the minimum down payment \wp_0 and $\wp_1(s)$ as

$$\wp_0 \equiv \phi_0 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s) \phi_1(s) \theta (1 - \delta) \quad \text{and} \quad \wp_1(s) \equiv \phi_1(s) - R^{-1} \phi_2(s) \theta (1 - \delta).$$

The minimum amount that a borrower needs to pay down per unit of the asset is the price of the asset minus the collateralizable fraction of the discounted expected resale value, that is, minus the maximum amount that the borrower can borrow against the asset. The minimum down payment as a fraction of the price of capital at time 0, for

example, is $\varphi_0/\phi_0 \equiv 1 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s) \phi_1(s)/\phi_0 \theta(1 - \delta)$ and thus is decreasing in the expected capital appreciation $\sum_{s \in \mathcal{S}} \pi(s) \phi_1(s)/\phi_0$. Thus, expectations about future asset prices have an important effect on current down payment requirements. We are not aware of other models that predict such variation in down payment requirements.

2.7 The role of loan commitments

Are there gains for borrowers of entering into a loan commitment? Define a *loan commitment* as a binding agreement to provide a loan of a particular size at some future date for a fee paid up front. So far, we have set $l_1(s) = b_1(s)$, $\forall s \in \mathcal{S}$, which is without loss of generality given Lemma 1. Clearly this implies that $NPV_1(s) = -l_1(s) + R^{-1}Rb_1(s) = 0$, $\forall s \in \mathcal{S}$, that is all loans have zero net present value to the lender when extended. Thus, loan commitments are unnecessary and fees are zero.

Now consider a loan commitment $\{c_0(s), l_1(s), b_1(s)\}$ in which for a fee $c_0(s)$ to be paid at time 0, the lender agrees to provide a loan $l_1(s) > b_1(s)$ in state s at time 1 such that

$$c_0(s) + \pi(s)R^{-1}\{-l_1(s) + R^{-1}Rb_1(s)\} = 0,$$

which means that the loan commitment has zero net present value at time 0 due to competition in the market for loan commitments. In contrast, the net present value to the lender of a loan commitment in state s at time 1 is $NPV_1(s) = -l_1(s) + R^{-1}Rb_1(s) < 0$, that is, negative, which is why it is in fact a commitment.

Instead of taking out such a loan commitment, the borrower could take out a loan $\hat{l}_1(s) = b_1(s)$. The borrower could then reduce $b_0(s)$ to keep the net worth in state s at time 1 the same, i.e., set $\hat{b}_0(s) = b_0(s) - \Delta l_1(s)$ where $\Delta l_1(s) = l_1(s) - \hat{l}_1(s) > 0$. Since the borrower borrows less against state s at time 1, the borrower needs additional initial net worth in the amount of $\Delta w_0 = \pi(s)R^{-1}\Delta l_1(s)$. But competitive pricing of loan commitments implies that $c_0(s) = -\pi(s)R^{-1}NPV_1(s) = \pi(s)R^{-1}\{\Delta l_1(s)\}$ and hence $c_0(s) = \Delta w_0$. Thus, loan commitments are equivalent to saving contingent debt capacity. The key insight here is that lining up loan commitments requires internal funds up front and thus has a cost in terms of reduced investment up front. Arranging for loan commitments or contingent financing is akin to conserving contingent debt capacity. Borrowers who choose to exhaust their debt capacity thus do not arrange for loan commitments either.

The fact that loan commitments are equivalent to conserving contingent debt capacity does not mean that loan commitments are irrelevant. In fact, loan commitments are a particularly plausible interpretation of how borrowers conserve debt capacity in a state

contingent way in practice. Thus, loan commitments may be the practical implementation of the state contingent loans determined by the model.

3 The Distribution of Debt Capacity

In this section we use the model of collateralized borrowing developed above to study the distribution of debt capacity and the dynamics of investment by different firms. We also analyze the effect of collateralizability and asset prices on the extent to which constrained firms might contract, that is, scale down their investment. Furthermore, we consider the role of borrower net worth. We obtain two main results. First, more productive borrowers may exhaust their debt capacity since the opportunity cost of conserving debt capacity, which is foregone investment earlier on, is higher for them. Second, in states where asset prices and cash flows are low, capital may hence be less productively deployed on average, since more productive borrowers, who have exhausted their debt capacity, contract relative to less productive borrowers.

3.1 Who conserves and who exhausts debt capacity?

In order to abstract from net worth effects for now, we assume that investment exhibits constant returns to scale, that is, $f(k) = k$ and hence $f'(k) = 1$. Define the returns $R_1(k_0, s)$ and $R_2(k_1(s), s)$ as

$$R_1(k_0, s) \equiv \frac{A_1(s)f'(k_0) + \phi_1(s)(1 - \theta)(1 - \delta)}{\varphi_0} \quad (16)$$

and

$$R_2(k_1(s), s) \equiv \frac{A_2(s)f'(k_1(s)) + \phi_2(s)(1 - \theta)(1 - \delta)}{\varphi_1(s)}, \quad (17)$$

which are the returns on the borrower's internal funds when he invests by making the minimum down payment (that is, by choosing maximal leverage). With constant returns to scale, $R_1(k_0, s)$ and $R_2(k_1(s), s)$ do not depend on k_0 or $k_1(s)$ and we hence simplify the notation to $R_1(s)$ and $R_2(s)$.

Moreover, we assume that investment at time 1 is sufficiently productive, namely that

Assumption 1 $R_2(s) > R, \forall s \in \mathcal{S}$.

This simplifies the analysis by implying that borrowers are constrained at time 1 and do not pay dividends before time 2:

Lemma 3 *Given Assumption 1, $\lambda_1(s) > 0$ and $d_0 = d_1(s) = 0, \forall s \in \mathcal{S}$.*

Proof of Lemma 3. Using (26), (28), (32), and Assumption 1, $R\mu_2(s) + R\lambda_1(s) = \mu_1(s) \geq R_2(s)\mu_2(s) > R\mu_2(s)$ and thus $\lambda_1(s) > 0, \forall s \in \mathcal{S}$. Moreover, $\mu_0 \geq \mu_1(s) \geq \mu_2(s) + \lambda_2(s) > \mu_2(s) \geq 1$. Then (25) and (26) imply $\nu_0^d > 0$ and $\nu_1^d(s) > 0, \forall s \in \mathcal{S}$. \square

This in turn enables us to solve the borrower's problem at time 1 in state s explicitly. Define the net worth at time 1 in state s as

$$w_1(s) \equiv A_1(s)k_0 + \phi_1(s)k_0(1 - \delta) - Rb_0(s)$$

and the value attained by an agent at time 1 in state s with that net worth as $V_1(w_1(s), s)$. Lemma 3 implies the following corollary:

Corollary 1 $k_1(s) = \frac{1}{\varphi_1(s)}w_1(s)$ and $V_1(w_1(s), s) = R_2(s)w_1(s), \forall s \in \mathcal{S}$.

Proof of Corollary 1. Since $d_1(s) = 0$ and using (3) and (15) at equality we have $k_1(s) = \frac{1}{\varphi_1(s)}w_1(s)$. Moreover, (4) and (15) at equality imply that $d_2(s) = (A_2(s) + \phi_2(s)(1 - \theta)(1 - \delta))k_1(s)$ and hence $V_1(w_1(s), s) = d_1(s) + d_2(s) = R_2(s)w_1(s), \forall s \in \mathcal{S}$. \square

Having solved the time 1 problem, we can now solve the borrower's time 0 problem. This leads to our first main result. Depending on how productive investment is in the first period, that is at time 0, agents will either invest as much as they can and exhaust their debt capacity with respect to all states at time 1 or conserve all their net worth and debt capacity for state s' at time 1, where they will invest the maximal amount. The state s' is the state where the return is the highest, that is, $s' \in \arg \max_{s \in \mathcal{S}} R_2(s')$.

Proposition 1 *If $\sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s) > \max_s \{RR_2(s)\}$, then $k_0 = \frac{1}{\varphi_0}w_0$ and $V_0(w_0) = \sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s)w_0$. Otherwise, $k_0 = 0$, $w_1(s') = \frac{R}{\pi(s')}w_0$, and $V_0(w_0) = RR_2(s')w_0$, where s' such that $R_2(s') = \max_s \{R_2(s)\}$.*

Proof of Proposition 1. Suppose $k_0 = 0$. Then $w_1(s) = -Rb_0(s)$ and using Corollary 1 we have

$$V_0(w_0) \equiv \max_{\{b_0(s)\}_{s \in \mathcal{S}}} \sum_{s \in \mathcal{S}} \pi(s)(-RR_2(s)b_0(s))$$

subject to $w_0 \geq -\sum_{s \in \mathcal{S}} \pi(s)b_0(s)$ and $-Rb_0(s) \geq 0, \forall s \in \mathcal{S}$. If s' such that $R_2(s') = \max_s \{R_2(s)\}$, then $b_0(s') = -\frac{1}{\pi(s')}w_0$ and $V_0(w_0) = RR_2(s')w_0$.

Suppose $k_0 > 0$. Then $w_1(s) \geq (A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta))k_0 > 0$, which implies that $k_1(s) > 0$ (and $\nu_1^k(s) = 0$) and $d_2(s) > 0$ (and $\mu_2(s) = 1$). From (32), $\mu_1(s) = R_2(s)$, and (27) and (31) can be written as

$$\mu_0 = RR_2(s) + R\lambda_0(s), \quad \forall s \in \mathcal{S}, \quad (18)$$

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s)R_1(s)R_2(s). \quad (19)$$

Note that this is only possible if $\sum_{s \in \mathcal{S}} \pi(s) R_1(s) R_2(s) \geq \max_s \{R R_2(s)\}$. Moreover, the case where the inequality is an equality is not generic and hence generically $\lambda_0(s) > 0$, $\forall s \in \mathcal{S}$. But then (14) implies $b_0(s) = R^{-1} \phi_1(s) \theta k_0 (1 - \delta)$ and (2) implies $k_0 = \frac{1}{\varphi_0} w_0$. Using Corollary 1 we get $V_0(w_0) = \sum_{s \in \mathcal{S}} \pi(s) R_1(s) R_2(s) w_0$.

Thus, if $\sum_{s \in \mathcal{S}} \pi(s) R_1(s) R_2(s) > \max_s \{R R_2(s)\}$, $k_0 > 0$ attains a higher value and the optimal k_0 and value attained are as stated in the proposition. Otherwise, $k_0 = 0$ attains a higher value and is hence optimal. \square

The condition for investment is $\sum_{s \in \mathcal{S}} \pi(s) R_1(s) R_2(s) > \max_s \{R R_2(s)\}$ and thus borrowers with higher productivity in the first period, say higher $\sum_{s \in \mathcal{S}} \pi(s) R_1(s)$, are more likely to invest and exhaust their debt capacity, all else equal. Moreover, the covariance between returns in the first period and returns in the second period, that is, investment opportunities, of course also matters. But we do not explicitly explore variation in that covariance here.

3.2 Relative contraction of productive firms

Now consider a borrower who invests at time 0 and who exhausts his debt capacity by Proposition 2. It is possible that such a borrower may not be able to deploy as much capital at time 1 as he deploys at time 0. Thus it is possible that borrowers are “forced to” contract. This may occur in a state s in which cash flows $A_1(s)$ are sufficiently low. Importantly, this occurs despite the fact that the borrower could arrange for contingent financing.

Proposition 2 *For an open set of parameters, $\exists s \in \mathcal{S}$ such that $k_1(s) < k_0$.*

Proof of Proposition 2. Suppose $\sum_{s \in \mathcal{S}} \pi(s) R_1(s) R_2(s) > \max_s \{R R_2(s)\}$. Then, by Proposition 1, $k_0 = \frac{1}{\varphi_0} w_0 > 0$ and $w_1(s) = (A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta)) k_0$. Moreover, $k_1(s) = \frac{1}{\varphi_1(s)} w_1(s)$ by Corollary 1. Thus,

$$k_1(s) = \left(\frac{A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta)}{\phi_1(s) - R^{-1} \phi_2(s) \theta (1 - \delta)} \right) k_0$$

and the statement is true as long as $\exists s \in \mathcal{S}$ such that the term in parenthesis on the right hand side is less than 1. Now note that when $\theta = 1$, the term in parenthesis is less than 1 for $A_1(s)$ sufficiently small. Similarly, when $\theta = 0$, this is again the case for $A_1(s)$ sufficiently small. Moreover, this must be the case in a neighborhood of these parameter values by continuity. \square

Proposition 2 implies our second main result, that productive agents may contract when less productive agents, who did not previously invest, expand. If agents' productivity is persistent, average productivity may hence decline in such states.

3.3 Effect of collateralizability on contraction

When the collateralizability θ increases, agents who invest at time 0 may contract by more. Thus, financial innovation, which increases the collateralizability, may result in more severe contractions of borrowers who exhaust their debt capacity.

Proposition 3 *Suppose the parameters are as in Proposition 2 such that $\frac{k_1(s)}{k_0} < 1$. Then $\frac{\partial}{\partial \theta} \left(\frac{k_1(s)}{k_0} \right) < 0$ as long as $\frac{\phi_1(s)}{\phi_2(s)} > \frac{1}{R} \frac{k_1(s)}{k_0}$.*

Proof of Proposition 3. Note that

$$\frac{\partial}{\partial \theta} \left(\frac{k_1(s)}{k_0} \right) = \frac{\phi_1(s)(1-\delta)}{\varphi_1(s)} \left(\frac{\phi_2(s)}{\phi_1(s)} \frac{1}{R} \frac{k_1(s)}{k_0} - 1 \right) < 0$$

as long as the condition in the statement of the proposition is satisfied. \square

This condition is satisfied for example when $\phi_1(s) = \phi_2(s)$. A higher θ has two effects. First, the agent is able to pledge more funds at time 0 and hence has less "free net worth" left. Second, the agent has a greater ability to borrow going forward and hence requires a smaller "down payment requirement" in terms of net worth. The two effects go in opposite directions, but as long as the price of capital is not too much higher at time 2, the first effect dominates: higher leverage due to higher pledgeability leads to a more severe contraction in capital.

3.4 Effect of asset prices on contraction

How does the extent of the contraction vary with the price of capital $\phi_1(s)$? That is, if the price drops by less in state s at time 1, will borrowers who exhausted their debt capacity contract by more or by less? The following result shows that borrowers contract by more when asset prices fall by less:

Proposition 4 [i] $\frac{\partial}{\partial \phi_1(s)} \left(\frac{k_1(s)}{k_0} \right) < 0$. [ii] *If $\phi_1(s) = \phi_2(s) \equiv \phi(s)$, then $\frac{\partial}{\partial \phi(s)} \left(\frac{k_1(s)}{k_0} \right) < 0$.*

Proof of Proposition 4. For part [i]:

$$\frac{\partial}{\partial \phi_1(s)} \left(\frac{k_1(s)}{k_0} \right) = \frac{(1-\theta)(1-\delta)}{\varphi_1(s)} \left(1 - \frac{\frac{A_1(s)}{(1-\theta)(1-\delta)} + \phi_1(s)}{\phi_1(s) - R^{-1}\phi_2(s)\theta(1-\delta)} \right) < 0$$

and for part [ii]:

$$\frac{\partial}{\partial \phi(s)} \left(\frac{k_1(s)}{k_0} \right) = \frac{(1-\theta)(1-\delta)}{\wp_1(s)} \left(1 - \frac{\frac{A_1(s)}{(1-\theta)(1-\delta)} + \phi(s)}{\phi(s)} \right) < 0. \quad \square$$

A higher price of capital at time 1 in state s has again two effects, raising the “free net worth” while at the same time raising the “down payment requirement,” with the second effect dominating the first. The higher the price of capital, the more capital contracts.

3.5 Role of borrower net worth

To study the effect of borrower net worth, we drop the assumption of constant returns to scale and instead assume that $f(k)$ is strictly concave and satisfies $\lim_{k \rightarrow 0} f'(k) = +\infty$. Then $k_0 > 0$ and $k_1(s) > 0$, $s \in \mathcal{S}$, and thus (31) and (32) simplify to

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) \mu_1(s) \quad (20)$$

$$\mu_1(s) = R_2(k_1(s), s) \mu_2(s). \quad (21)$$

Moreover, we again assume that productivity at time 1 is sufficiently high such that

Assumption 2 $R_2(k_1(s), s) > R$, $\forall s \in \mathcal{S}$.

With these assumptions, agents are again constrained at time 1 in state s and dividends at time 0 and 1 are zero.

Lemma 4 *Given Assumption 2, $\lambda_1(s) > 0$ and $d_0 = d_1(s) = 0$, $\forall s \in \mathcal{S}$.*

The proof of Lemma 4 is analogous to the proof of Lemma 3 and is hence omitted.

Defining net worth at time 1 in state s as $w_1(s) \equiv A_1(s)f(k_0) + \phi_1(s)k_0(1-\delta) - Rb_0(s)$ we have the following corollary, which characterizes the solution to the time 1 problem and is proved as before:

Corollary 2 $\forall s \in \mathcal{S}$, $k_1(s) = \frac{1}{\wp_1(s)} w_1(s)$ and

$$V_1(w_1(s), s) = \left(A_2(s) f' \left(\frac{w_1(s)}{\wp_1(s)} \right) + \phi_2(s)(1-\theta)(1-\delta) - Rb_0(s) \right) \frac{w_1(s)}{\wp_1(s)}.$$

Notice that since $k_1(s) > 0$, $\forall s \in \mathcal{S}$, we have

$$d_2(s) \geq A_2(s)f(k_1(s)) + \phi_2(s)k_1(s)(1-\theta)(1-\delta) > 0,$$

and thus $\mu_2(s) = 1$. Therefore (21) implies that $\mu_1(s) = R_2(k_1(s), s)$ and (27) and (20) simplify to

$$\mu_0 = RR_2(k_1(s), s) + R\lambda_0(s), \quad \forall s \in \mathcal{S}, \quad (22)$$

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) R_2(k_1(s), s). \quad (23)$$

Suppose that the parameters satisfy the following assumption:

Assumption 3 (i) $R_2(k, H) < R_2(k, L)$, for k in the relevant range; and (ii) $k_1(H) > k_1(L)$, where $k_1(s) \equiv (A_s(s)f(w_0/\wp_0) + \phi_1(s)w_0/\wp_0(1 - \theta)(1 - \delta))/\wp_1(s)$ for w_0 in the relevant range.

This assumption is satisfied, for example, when $A_2(H) = A_2(L)$ and $\phi_2(H) = \phi_2(L)$ and $A_1(H) \gg A_1(L)$. Intuitively, the assumption requires that the return on investment is higher in the low state at time 1, but that cash flows are sufficiently higher in the high state so that a borrower who invest his entire net worth in the technology will have more capital in the high state than the low state. Given this assumption, the borrower will exhaust his total debt capacity when net worth is very low, will conserve debt capacity for the low state only when net worth is in an intermediate range, and will be unconstrained in terms of first period investment when net worth is high enough. Thus, whether or not an agent conserves debt capacity for state L now depends on the borrower's net worth. The following proposition, which is proved in the appendix, summarizes this result:

Proposition 5 Suppose Assumption 3 holds. Then there exist $\underline{w}_0 < \bar{w}_0$ such that (i) for $w_0 \leq \underline{w}_0$, $\lambda_0(s) > 0$, $\forall s \in \mathcal{S}$, $k_0 = \frac{1}{\wp_0}w_0$, and $k_1(s) = \frac{1}{\wp_1(s)}(A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta))$; (ii) for $\underline{w}_0 < w_0 < \bar{w}_0$, $\lambda_0(H) > 0$ and $\lambda_0(L) = 0$; and (iii) for $\bar{w}_0 \leq w_0$, $\lambda_0(s) = 0, \forall s \in \mathcal{S}$, $R_2(k_1(H), H) = R_2(k_1(L), L)$, and $R = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s)$.

The borrower conserves debt capacity for the low state only if he is not too constrained.

4 Financial intermediation

In this section we study how financial intermediaries affect the distribution of debt capacity as well as how collateralized borrowing in turn affects the dynamics of intermediary capital. In addition to the lenders considered above, which we henceforth refer to as providing *direct finance*, we introduce a second class of lenders, *financial intermediaries*. We model financial intermediaries as lenders which are able to collateralize a larger fraction of capital, that is, are able to enforce their claims better, but have limited internal

funds. Thus, intermediaries in our model are “securitization specialists.” Our model is related to Holmström and Tirole (1998), but our focus is on the dynamics of intermediated financing. We provide conditions for intermediary capital to be scarce when asset prices and cash flows are low, implying higher spreads between the cost of intermediated finance and direct finance. Moreover, we show that in that case borrowers who exhaust their debt capacity may contract for two reasons, on the one hand because they have low cash flow and hence low net worth as before, and on the other hand because the cost of intermediated funds increases.

4.1 A model of financial intermediaries

Suppose a representative financial intermediary with capital w_0^i is able to collateralize up to fraction $\theta^i > \theta$ of the resale value of capital.⁹ In other words, a borrower who borrows from a financial intermediary can abscond with only $1 - \theta^i$ of capital that is pledged to an intermediary as well as all cash flows (as before).

To simplify the exposition, we start by considering a one period problem only and study the capital structure implications in the cross section of borrowers. The intermediary lends at a state contingent interest rate $R_0^i(s)$, $\forall s \in \mathcal{S}$, which we will determine in equilibrium. The intermediary solves

$$\max_{\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}} d_0^i + \sum_{s \in \mathcal{S}} \pi(s) R^{-1} d_1^i(s)$$

subject to

$$w_0^i \geq d_0^i + \sum_{s \in \mathcal{S}} \pi(s) l_0^i(s)$$

and

$$R_0^i(s) l_0^i(s) \geq d_1^i(s), \quad \forall s \in \mathcal{S},$$

as well as $d_0^i \geq 0$, $d_1^i(s) \geq 0$, $l_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$, where $l_0^i(s)$ is the amount that the intermediary lends against state s . The intermediary’s problem does not explicitly involve collateral constraints since the intermediary is in fact lending, and collateral constraints are instead imposed on the borrowers for both direct as well as intermediated finance. Moreover, we can assume that $R_0^i(s) \geq R$, $\forall s \in \mathcal{S}$, since the intermediary could always lend to the direct lenders at an expected return of R . Importantly, the intermediary

⁹We consider a representative financial intermediary since intermediaries have constant returns to scale in our model and hence aggregation in the intermediation sector is straightforward. The distribution of intermediaries’ net worth is hence irrelevant and only the aggregate capital of the intermediation sector (w_0^i) matters.

cannot borrow at an expected rate R since the intermediary does not have collateral himself.

To simplify the analysis we assume that lenders provide direct finance to the borrowers directly, hence the name, which is without loss of generality. We could alternatively and equivalently assume that lenders lend to the intermediary who in turn lends to the borrowers. The assumption is that lenders can only seize a fraction θ of the underlying capital, whether or not the lending is intermediated. In particular, lenders cannot seize any of the additional capital that the intermediary is able to seize; that is, of 1 unit of capital, lenders can seize θ , whether they lend through the intermediary or not, while intermediaries can seize an extra $\theta^i - \theta$, but the lenders cannot in turn collateralize this additional amount.

Let us index agents by their types $n \in \mathcal{N}$ and denote the density of agents of type n by $\psi(n)$ (and the distribution by $\Psi(n)$). We have suppressed agents' types thus far, and will continue to do so whenever possible, but, to define an equilibrium, it is useful to make the dependence on type explicit. For example, we assume that both agents' initial endowment $w_0(n)$ and productivity $A_t(s|n)$ may depend on n . An *equilibrium* consists of state contingent interest rates on intermediated funds $R_0^i(s)$, $\forall s \in \mathcal{S}$, and an allocation such that $\{d_0(n), d_1(s|n), k_0(n), b_0(s|n), b_0^i(s|n)\}_{s \in \mathcal{S}}$ solves agent n 's problem, $\forall n \in \mathcal{N}$, and $\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}$ solves the representative intermediary's problem, and such that the market for intermediated finance clears, that is,

$$\int_{\mathcal{N}} b_0^i(s|n) d\Psi(n) \leq l_0^i(s), \quad \forall s \in \mathcal{S},$$

with equality if $R_0^i(s) > R$.¹⁰

In the one period problem, intermediaries charge the same interest rate on intermediated loans for both states:

Lemma 5 $R_0^i(H) = R_0^i(L) \equiv R_0^i$ without loss of generality.

Proof of Lemma 5. First, $l_0^i(s) \geq 0$ is implied by $R_0^i(s)l_0^i(s) \geq d_1^i(s) \geq 0$ and hence redundant. The first order conditions of the intermediary's problem are $\mu_0 = 1 + \nu_0^d$, $\mu_1(s) = R^{-1} + \nu_1^d(s)$, and $\mu_0 = R_0^i(s)\mu_1(s)$, $\forall s \in \mathcal{S}$. Thus, $R_0^i(H)(R^{-1} + \nu_1^d(H)) = R_0^i(L)(R^{-1} + \nu_1^d(L))$. Since $R_0^i(s) \geq R$ we can set $d_0^i = 0$ w.l.o.g., and hence at most one of $\nu_1^d(s)$ can be strictly positive. Now suppose $R_0^i(s) > R_0^i(s')$, $s \neq s'$. Then $\nu_1^d(s') > 0$

¹⁰The markets for output goods, capital goods, and direct finance do not impose additional restrictions due to Walras' law, the fact that capital goods can be transformed into output goods with a linear and reversible technology, and the fact that direct lenders are risk neutral and have plenty of funds at all dates and in all states.

and hence $l_0^i(s') = 0$, that is, there is no intermediated lending against state s' . But for the intermediary to be willing to lend against state s' , he would require an expected return of $R_0^i(s)$, so we can set $R_0^i(H) = R_0^i(L) \equiv R_0^i$. \square

Thus, the borrower can borrow using direct finance at an expected rate of R as before and from financial intermediaries at a rate R_0^i as determined above, stated formally:

$$\max_{\{d_0, d_1(s), k_0, b_0(s), b_0^i(s)\}_{s \in \mathcal{S}}} d_0 + \sum_{s \in \mathcal{S}} \pi(s) d_1(s)$$

subject to the budget constraints,

$$\begin{aligned} w_0 + \sum_{s \in \mathcal{S}} \pi(s) \{b_0(s) + b_0^i(s)\} &\geq d_0 + \phi_0 k_0 \\ A_1(s) f(k_0) + \phi_1(s) k_0 (1 - \delta) &\geq d_1(s) + R b_0(s) + R_0^i b_0^i(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

two sets of collateral constraints,

$$\begin{aligned} \phi_1(s) \theta k_0 (1 - \delta) &\geq R b_0(s), \quad \forall s \in \mathcal{S}, \\ \phi_1(s) \theta^i k_0 (1 - \delta) &\geq R b_0(s) + R_0^i b_0^i(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

and $d_0 \geq 0$, $d_1(s) \geq 0$, $k_0 \geq 0$, $b_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$ and $t \in \mathcal{T}$. There are now two collateral constraints for each state: the first constraint restricts direct finance and is as before; the second constraint restricts the total promises the borrower makes against state s , which cannot exceed the amount that the intermediary can collateralize.

4.2 Capital structure: intermediated vs. direct finance

In the cross section, the capital structure of firms varies as follows: the least productive firms do not invest; more productive firms invest and exhaust the direct financing capacity; and the most productive firms exhaust both their direct financing as well as their intermediated financing capacity. The next proposition states this formally:

Proposition 6 *Suppose $R_0^i > R$. If $R \geq \sum_{s \in \mathcal{S}} \pi(s) (A_1(s) + \phi_1(s) (1 - \delta)) / \phi_0$, then $k_0 = 0$ and $V(w_0) = R w_0$; otherwise, if $R_0^i \geq \mu_0^* \equiv \sum_{s \in \mathcal{S}} \pi(s) R_1(s)$, then $k_0 = (1/\varphi_0) w_0$ and $V(w_0) = \mu_0^* w_0$, and if $R_0^i < \mu_0^*$, then $k_0 = (1/\bar{\varphi}_0) w_0$ and $V(w_0) = \bar{\mu}_0^* w_0$ where $\bar{\varphi}_0$ and $\bar{\mu}_0^*$ are defined in the proof.*

The proof is in the appendix. The value of internal funds is $\mu_0 \geq R$ and thus exceeds the value of external funds when the borrower is constrained. Moreover, the more

productive the borrower is, the higher the value of internal funds is, and the more constrained the borrower is. Thus, it is the more constrained borrowers which borrow from the financial intermediary in our model.

Similarly, if investment were subject to decreasing returns to scale and all borrowers had the same productivity but differed in their initial endowment, then it would be the borrowers with less internal funds which would be more constrained and would borrow from the financial intermediary. The cross sectional capital structure implications would hence be similar to the ones in Holmström and Tirole (1998) in this case.

4.3 Dynamics of intermediary capital

We now analyze the dynamics of intermediary capital in the environment introduced in Section 2. Consider the intermediary's dynamic problem. We start with the intermediary's problem at time 1 in state s , given intermediary capital $w_1^i(s)$:

$$\max_{\{d_1^i(s), d_2^i(s), l_1^i(s)\}} d_1^i(s) + R^{-1}d_2^i(s)$$

subject to the budget constraints

$$\begin{aligned} w_1^i(s) &\geq d_1^i(s) + l_1^i(s), \\ R_1^i(s)l_1^i(s) &\geq d_2^i(s), \end{aligned}$$

as well as $d_1^i(s) \geq 0$, $d_2^i(s) \geq 0$, $l_1^i(s) \geq 0$, where $l_1^i(s)$ is the amount that the intermediary lends against time 2. Assuming that $R_1^i(s) \geq R$, we have $d_1^i(s) = 0$, $l_1^i(s) = w_1^i(s)$, $d_2^i(s) = R_1^i(s)l_1^i(s)$, and hence $V_1^i(w_1^i(s)) = R^{-1}R_1^i(s)w_1^i(s)$.

At time 0 the intermediary solves

$$\max_{\{d_0^i, d_1^i(s), l_0^i(s)\}_{s \in \mathcal{S}}} d_0^i + \sum_{s \in \mathcal{S}} \pi(s)R^{-2}R_1^i(s)w_1^i(s)$$

subject to the budget constraints

$$\begin{aligned} w_0^i &\geq d_0^i + \sum_{s \in \mathcal{S}} \pi(s)l_0^i(s) \\ R_0^i(s)l_0^i(s) &\geq w_1^i(s), \quad \forall s \in \mathcal{S}, \end{aligned}$$

as well as $d_0^i \geq 0$, $w_1^i(s) \geq 0$, $l_0^i(s) \geq 0$, $\forall s \in \mathcal{S}$, where $l_0^i(s)$ is the amount that the intermediary lends against state s .¹¹ The first order conditions are $\mu_0 = 1 + \nu_0^d$, $\mu_1(s) =$

¹¹The intermediary may choose to lend some funds to the direct lenders as well, in order to conserve debt capacity for state s , for example, but there is no need to keep track of such lending separately from lending to the borrowers. The reason is that whenever the intermediary lends funds to the direct lenders, the interest rate on intermediated funds for that state equals R .

$R^{-2}R_1^i(s) + \nu_1^w(s)$, and $\mu_0 = R_0^i(s)\mu_1(s)$. As long as $R_1^i(s) > R$ for some s, t , $d_0^i = 0$. Moreover, we have

$$R^{-2}R_0^i(s)R_1^i(s) + R_0^i(s)\nu_1^w(s) = R^{-2}R_0^i(s')R_1^i(s') + R_0^i(s')\nu_1^w(s').$$

If the intermediary has positive net wealth in both states at time 1 we have

$$R_0^i(s)R_1^i(s) = R_0^i(s')R_1^i(s').$$

To characterize the dynamics of intermediary capital and the spread between intermediated finance and direct finance, we first study the case in which intermediaries have plenty of capital and then consider the case in which intermediaries have limited capital.

4.4 Well capitalized intermediaries

Suppose that the representative intermediary is well capitalized, that is, w_0^i is sufficiently large such that the intermediary has excess funds at time 0 and at time 1 in all states and $R_0^i(s) = R = R_1^i(s)$, $\forall s \in \mathcal{S}$. The borrower's problem is then equivalent to the problem without intermediation studied in Sections 2 and 3 except that θ is replaced by θ^i since borrowers are able to borrow up to fraction θ^i of capital in total. Assume that there are two types of borrowers, more productive, "good" borrowers with measure $\psi(g)$, and less productive, "bad" borrowers with measure $\psi(b)$ ($= 1 - \psi(g)$). Assume that the more productive entrepreneurs (type g) optimally choose positive investment at time 0, $k_0(g) > 0$, which means that parameters are such that $\sum_{s \in \mathcal{S}} \pi(s)R_1(s|g)R_2(s|g) > \max_{s \in \mathcal{S}} RR_2(s|g)$ evaluated at θ^i instead of θ . We furthermore assume that here there is a maximum scale \bar{k} at which the technology can be operated. When \bar{k} is sufficiently high, the model is as before. But we will consider the case where \bar{k} binds in state H , which implies that borrowers will use their high cash flows in that state to partially pay down their loans from the intermediaries.¹² For the less productive entrepreneurs (type b) assume that the inequality is reversed and that $L = \arg \max_{s \in \mathcal{S}} RR_1(s|b)$ again evaluating all expressions at θ^i , so that the less productive entrepreneurs conserve their net worth at time 0 and invest at time 1 in state L only.¹³ From the solution to the equivalent problem we can determine the minimum amount of financing that intermediaries must provide to

¹²This requires a slight modification of the condition for investment at time 0 to be optimal, namely that $\pi(H)R_1(H|g)R + \pi(L)R_1(L|g)R_2(L|g) > \max_{s \in \mathcal{S}} RR_2(s|g)$. Note that the return in state H in the second period is now R since additional funds are simply used to pay down debt.

¹³This is the case as long as investment by type b is sufficiently unproductive at time 0 and $\frac{A_2(L|b) + \phi_2(L)(1-\theta_i)(1-\delta)}{\phi_1(L) - R^{-1}\phi_2(L)\theta^i(1-\delta)} > \frac{A_2(H|b) + \phi_2(H)(1-\theta_i)(1-\delta)}{\phi_1(H) - R^{-1}\phi_2(H)\theta^i(1-\delta)}$.

implement the solution. The intermediary extends loans in the amount of

$$\underline{l}_1^i(s) = R^{-1}\phi_2(s)(\theta^i - \theta) \left(\sum_{n \in \{g, b\}} \psi(n)k_1(s|n) \right) (1 - \delta)$$

at time 1 in state s , where $k_1(s|n) = 1/\varphi_1^i(s)w_1(s|n)$, $\forall n \in \{g, b\}$, and $\varphi_1^i(s)$ is $\varphi_1(s)$ with θ replaced by θ^i , if investment is less than maximum scale, and extends loans of $\max\{\bar{k}(\phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta)) - w_1(s|n), 0\}$ otherwise. Borrowers' net worth in turn is $w_1(s|g) = (A_1(s|g) + \phi_1(s)(1 - \theta^i)(1 - \delta))k_0(g)$, $w_1(H|b) = 0$, and $w_1(L|b) = R/\pi(L)w_0(b)$. The total loan repayments to the intermediary at time 1 in state s are

$$R\underline{l}_0^i(s) = \phi_1(s)(\theta^i - \theta)\psi(g)k_0(g)(1 - \delta),$$

where $k_0(g) = 1/\varphi_0^i w_0(g)$ and φ_0^i is φ_0 with θ replaced by θ^i . Thus, the net lending of the financial intermediary at time 1 in state s is

$$\begin{aligned} \underline{nl}_1^i(s) &\equiv \underline{l}_1^i(s) - R\underline{l}_0^i(s) \\ &= \left(R^{-1}\phi_2(s) \left(\frac{A_1(s) + \phi_1(s)(1 - \theta^i)(1 - \delta)}{\varphi_1^i(s)} \right) - \phi_1(s) \right) (\theta^i - \theta)(1 - \delta)\psi(g)k_0(g) \\ &\quad + R^{-1}\phi_2(s)(\theta^i - \theta)(1 - \delta)\psi(b)\frac{1}{\varphi_1^i(s)}w_1(s|b), \end{aligned} \quad (24)$$

as long as investment is below maximum scale. Net lending will thus be higher in state s at time 1 when cash flows of productive borrowers are high which allows them to expand. It will also be higher when there are more less productive borrowers entering. If in state H the productive borrowers have sufficient capital to reach maximum scale, then net lending will be

$$\underline{nl}_1^i(H) = \max\{\bar{k}(\phi_1(H) - R^{-1}\phi_2(H)\theta(1 - \delta)) - w_1(H|n), 0\} - \phi_1(H)(\theta^i - \theta)(1 - \delta)\psi(g)k_0(g).$$

In this case, net lending will be lower when cash flows of productive borrowers are high since they will repay loans rather than expand capital further.

When the aggregate net worth of financial intermediaries is sufficiently high, intermediaries are well capitalized and the spreads between intermediated finance and direct finance are zero, as the next results shows.

Proposition 7 *If $w_0^i \geq \underline{w}_0^i \equiv \sum_{s \in \mathcal{S}} \pi(s) (\underline{l}_0^i + R^{-1} \max\{\underline{nl}_1^i(s), 0\})$, banks are well capitalized and $R_0^i(s) = R = R_1^i(s)$, $\forall s \in \mathcal{S}$.*

Proof of Proposition 7. If $w_0^i \geq \underline{w}_0^i$, the intermediary has sufficient net worth at time 0 to fund the loans that borrowers demand at time 0 at a cost of intermediary funds of R

for all dates and states. Moreover, the intermediary has sufficient funds to fund the net lending borrowers require at time 1 in all states at this cost of intermediary funds (by lending to the direct lenders at rate R). Moreover, the lender is indifferent at the margin between consuming a dividend at time 0 and lending to the direct lenders at rate R . \square

4.5 Limited intermediary capital

Suppose instead that the intermediary is not well capitalized, that is, that $w_0^i < \underline{w}_0^i$. Clearly, $R_0^i(s) = R = R_1^i(s)$, $\forall s \in \mathcal{S}$ is then not an equilibrium, and there will be a spread between intermediated funds in some dates and states. Consider the case in which

Assumption 4 $\underline{nl}_1^i(L) > 0 > \underline{nl}_1^i(H)$.

that is, the intermediary requires additional capital in the low state only. This is the case if loan repayments fall short of the net lending demand by borrowers in state L at a cost of intermediated loans of R . We discuss conditions under which this is the case below. The next result characterizes the dynamics of the spread between intermediated and direct financing:

Proposition 8 *Suppose Assumption 4 holds. Then $\exists \varepsilon > 0$ such that $\forall w_0^i < \underline{w}_0^i$ and $\varepsilon > \underline{w}_0^i - w_0^i$, $R^i \equiv R_0^i(H) = R_1^i(L) > R$, and $R_0^i(L) = R_1^i(H) = R$.*

Denoting the time 0 spread on a loan requiring the payment of 1 unit in all states at date 1 by $\varsigma_0 \equiv \sum_{s \in \mathcal{S}} \pi(s) R_0^i(s) - R$ and the time 1 spread in state s on a loan requiring the payment of 1 unit at date 2 by $\varsigma_1(s) \equiv R_1^i(s) - R$, we have the following immediate corollary of this proposition:

Corollary 3 *Under the conditions of Proposition 8, $\varsigma_1(L) > \varsigma_0 > \varsigma_1(H) = 0$.*

Proposition 8 and Corollary 3 say that there is a positive spread between intermediated finance both at time 0 as well as in state L at time 1. The spread is highest in state L at time 1, however. The fact that intermediary capital is expected to be scarce in some future dates and states, implies that it is scarce at time 0 as well, and that spreads are positive then, too. Moreover, spreads are positive at time 0 even if the intermediaries are able to fund all current loans, because intermediaries optimally conserve some of their funds for future states with positive net loan demand.

We now provide the proof of Proposition 8.

Proof of Proposition 8. The borrowers' problem is the maximization of a concave function on a convex set defined by the constraints. By the theorem of the maximum the

solution is hence continuous. Thus aggregate loan demand is continuous as well. This is also true for the lender's problem. Now, to be able to provide the required loans, a well capitalized intermediary needs to conserve a strictly positive amount of net worth for state L and, if $w_0^i = \underline{w}_0^i$ will conserve no net worth for state H . By continuity then, for w_0^i less than but sufficiently close to \underline{w}_0^i , the financial intermediary will continue to conserve net worth for state L . But then $R_0^i(L) = R$. Moreover, again by continuity, the financial intermediary will continue to have excess funds in state H at time 1 implying $R_1^i(H) = R$. Since $R_0^i(H)R_1^i(H) = R_0^i(L)R_1^i(L)$, $R^i \equiv R_0^i(H) = R_1^i(L)$. Moreover, $R^i > R$ since otherwise there would be excess demand for intermediary loans. Thus, $\varsigma_1(L) = R^i - R > \varsigma_0 = \pi(L)(R^i - R) > \varsigma_1(H) = 0$. \square

When will Assumption 4 be satisfied? Consider first state H . Since we assume that the less productive borrowers do not invest in this state, the demand for loans is determined by the more productive borrowers. Given the high cash flows, their net worth increases in this state and hence their investment expands. This in turn raises the loan demand. Thus, it is possible for net loan demand to be positive in this state. However, if the productive borrowers reach maximum scale, then they use their net worth to pay down intermediated debt and net loan demand is negative. Second, in state L the less productive borrowers enter. The larger their aggregate net worth, $\psi(b)w_0(b)$, the higher is net loan demand. Moreover, net loan demand of the more productive borrowers depends again on their cash flows. If cash flows are sufficiently high, net loan demand by these borrowers may still be positive, but when cash flows are low enough, such that the more productive borrowers are forced to contract, net loan demand by these borrowers is negative. Aggregate net loan demand in state L is still positive, as long as the demand for intermediated loans from the less productive borrowers who are investing is sufficiently high.

The intermediary responds to the positive net loan demand in state L by conserving net worth for state L , but not to the point where spreads between intermediated finance and direct finance are zero. Intermediary capital is scarce and hence earns a higher return.

4.6 Impact of limited intermediary capital on borrowers

When financial intermediary capital is scarce, then, the scarcer intermediary capital, the more borrowers will contract (or the less they will expand) in the state where intermediary capital is scarce.

Proposition 9 *Suppose w_0^i is as in Proposition 8. If s such that $\underline{nl}_1^i(s) > 0 > \underline{nl}_1^i(s')$, $s' \neq s$, then $\frac{d}{dw_0^i} \frac{k_1^g(s)}{k_0^g} > 0$.*

Proof of Proposition 9. When the intermediary is almost well capitalized we have by continuity that

$$\frac{k_1^g(s)}{k_0^g} = \frac{A_1(s) + \phi_1(s)(1 - \theta^i)(1 - \delta)}{\phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta) - R_1^i(s)^{-1}\phi_2(s)(\theta^i - \theta)(1 - \delta)}$$

and thus $\frac{d}{dw_0^i} \frac{k_1^g(s)}{k_0^g} = \frac{\partial}{\partial R_1^i(s)} \frac{k_1^g(s)}{k_0^g} \frac{d}{dw_0^i} R_1^i(s) > 0 \square$

Thus, productive borrowers may now contract for two reasons: first, because they have low cash flow and hence low net worth in state L , and second, because the cost of intermediated funds increases in state L . Moreover, scarce intermediary capital increases the down payment requirement, $\varphi_1^i(s) = \phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta) - R_1^i(s)^{-1}\phi_2(s)(\theta^i - \theta)(1 - \delta)$, and, as a fraction of total debt, intermediated finance becomes less important.

4.7 An example with limited intermediary capital

To illustrate the dynamics of the spread between intermediated funds and direct finance in our model, we provide an example. The parameters of the example are provided in Panel A of Table 1. The parameters satisfy the assumptions in this section. In Panel B we consider the case of a well capitalized intermediary. The more productive borrowers invest at time 0 and the less productive borrowers invest at time 1 in state L only. Moreover, the more productive borrowers contract at time 1 in state L , that is, $k_1(L|g) < k_0(g)$. Since intermediaries are well capitalized the spreads are zero and hence not reported. In Panel C we consider the case in which intermediaries have 2.5% less capital than they would require to be well capitalized. Since intermediary capital is scarce, investment is reduced. Note that the more productive borrowers now contract by more at time 1 in state L . Moreover, spreads between intermediated finance and direct finance are now positive. Indeed, the spread at time 0 is almost 1% and the spread at time 1 in state L almost 2%. Of course, this example is illustrative only and is not calibrated. Nevertheless, this suggests that a relatively modest reduction in intermediary capital might have a considerable impact on spreads.

5 Conclusion

We provide a dynamic model of collateralized lending, allowing for both direct lending as well as lending by financial intermediaries. We endogenously derive the collateral constraints based on limited enforcement. We show that considering one period state-contingent debt is sufficient, and that long term debt and loan commitments are redun-

dant, that is, do not increase debt capacity. Taking out loan commitments is equivalent to conserving debt capacity. The cross-sectional distribution of debt capacity in our model is endogenous. In particular, we show that more productive agents may be more constrained when asset prices and cash flows are low, and may hence not be able to seize investment opportunities that arise due to low asset prices. Similarly, agents with less internal funds may exhaust their debt capacity as well, while agents with more internal funds conserve some debt capacity to take advantage of such investment opportunities. More productive agents may in fact be forced to scale down investment in such times, and they may be forced to scale down investment by more, the more collateralizable the assets. The reason is that higher collateralizability allows them to borrow more ex ante, but leaves them with less net worth ex post when cash flows are low. Moreover, capital may hence be less productively deployed in such times.

We model financial intermediaries as lenders which are able to collateralize a larger fraction of capital but have limited funds. Such financial intermediaries finance borrowers with higher leverage. We study the dynamics of intermediation capital and spreads between intermediated finance and direct finance. Spreads on intermediated finance are high when the demand for intermediated finance is high. In states where there are investment opportunities due to low asset prices, spreads are high when the demand from borrowers trying to take advantage of the investment opportunities is high. These higher spreads may force borrowers who previously invested to contract by more in such states, consistent with anecdotal evidence.

Appendix

First Order Conditions of the Problem subject to Collateral Constraints. The first order conditions, which are necessary and sufficient, are

$$\mu_0 = 1 + \nu_0^d, \quad (25)$$

$$\mu_t(s) = 1 + \nu_t^d(s), \quad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \quad (26)$$

$$\mu_0 = R\mu_1(s) + R\lambda_0(s), \quad \forall s \in \mathcal{S}, \quad (27)$$

$$\mu_1(s) = R\mu_2(s) + R\lambda_1(s), \quad \forall s \in \mathcal{S}, \quad (28)$$

$$\phi_0\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) \{ (A_1(s)f'(k_0) + \phi_1(s)(1 - \delta))\mu_1(s) + \phi_1(s)\theta(1 - \delta)\lambda_0(s) \} + \nu_0^k \quad (29)$$

$$\phi_1(s)\mu_1(s) = (A_2(s)f'(k_1(s)) + \phi_2(s)(1 - \delta))\mu_2(s) + \phi_2(s)\theta(1 - \delta)\lambda_1(s) + \nu_1^k(s), \quad \forall s. \quad (30)$$

Using the return definitions (16) and (17) and equations (27) and (28), (29) and (30) can be written as

$$\mu_0 = \sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) \mu_1(s) + \frac{1}{\wp_0} \nu_0^k \quad (31)$$

$$\mu_1(s) = R_2(k_1(s), s) \mu_2(s) + \frac{1}{\wp_1(s)} \nu_1^k(s). \quad (32)$$

Proof of Proposition 5. Assumption 3 together with equation (22) implies that there are three cases to consider ($\lambda_0(s)$ positive for both states, for the high state only, and for neither state) since $R_2(k_1(H), H) + \lambda_0(H) = R_2(k_1(L), L) + \lambda_0(L)$. When $\lambda_0(s) > 0$, $\forall s \in \mathcal{S}$, $k_0 = w_0/\wp_0$, $k_1(s) = (A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta))/\wp_1(s)$. When moreover $\lambda_0(L) = 0$, then $\mu_0 = RR_2(k_1(L), L)$. Thus, there exists \underline{w}_0 such that the collateral constraint for state L is just satisfied, and

$$\sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s) R_2(\underline{k}_1(s), s) = RR_2(\underline{k}_1(L), L)$$

where $\underline{k}_0 = \underline{w}_0/\wp_0$ and $\underline{k}_1(s) = (A_1(s)f(\underline{k}_0) + \phi_1(s)\underline{k}_0(1 - \theta)(1 - \delta))/\wp_1(s)$. Furthermore, $RR_2(\underline{k}_1(L), L) = \sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s) R_2(\underline{k}_1(s), s) < (\sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s)) R_2(\underline{k}_1(L), L)$ and thus $R < \sum_{s \in \mathcal{S}} \pi(s) R_1(\underline{k}_0, s)$.

When $\lambda_0(s) = 0$, $\forall s \in \mathcal{S}$, $R_2(k_1(H), H) = R_2(k_1(L), L)$, and $\sum_{s \in \mathcal{S}} \pi(s) R_1(k_0, s) = R$. Thus, there exists \bar{w}_0 such that the collateral constraint against the high state is just satisfied, $\bar{k}_1(H) = (A_1(H)f(\bar{w}_0/\wp_0) + \phi_1(H)\bar{w}_0/\wp_0(1 - \theta)(1 - \delta))/\wp_1(H)$, $R_2(\bar{k}_1(H), H) = R_2(\bar{k}_1(L), L)$ and $\sum_{s \in \mathcal{S}} \pi(s) R_1(\bar{k}_0, s) = R$, where $\bar{k}_1(L)$ is determined using the budget constraints for time 0 and state L at time 1. Finally, $\bar{w}_0 \geq \bar{k}_0\wp_0 > \underline{k}_0\wp_0 = \underline{w}_0$. \square

Proof of Proposition 6. The first order conditions are $\mu_0 = 1 + \nu_0^d$, $\mu_1(s) = 1 + \nu_1^d(s)$,

and

$$\mu_0 = R(\mu_1(s) + \lambda_0^i(s)) + R\lambda_0(s) \quad (33)$$

$$\mu_0 = R_0^i(\mu_1(s) + \lambda_0^i(s)) - \nu_0^i(s) \quad (34)$$

$$\begin{aligned} \phi_0\mu_0 = & \sum_{s \in \mathcal{S}} \pi(s) \{A_1(s)f'(k_0) + \phi_1(s)(1 - \delta)\mu_1(s) + \phi_1(s)\theta(1 - \delta)\lambda_0(s) \\ & + \phi_1(s)\theta^i(1 - \delta)\lambda_0^i(s)\} + \nu_0^k, \end{aligned} \quad (35)$$

where (33), (34), and (35) are the first order conditions with respect to direct finance, intermediated finance, and capital, respectively, and $\lambda_0(s)$ and $\lambda_0^i(s)$ are the Kuhn-Tucker multipliers on the collateral constraints for direct finance and total promises, respectively.

Suppose $\nu_0^k > 0$ and hence $k_0 = 0$. Then $b_0^i(s) = 0, \forall s \in \mathcal{S}$, and $V(w_0) = Rw_0$. Thus, henceforth assume that $\nu_0^k = 0$ and $k_0 > 0$. When $k_0 > 0$, the time 1 budget constraints together with the collateral constraints imply that $d_1(s) > 0$ and hence $\mu_1(s) = 1, \forall s \in \mathcal{S}$.

Suppose $\lambda_0(s) = 0$, for some s . Then (33) and (34) imply that $\nu_0^i(s) > 0$ and hence $\lambda_0^i(s) = 0$. Using (33) for s and s' we have $\mu_0 = R = R + R(\lambda_0^i(s') + \lambda_0(s'))$ and thus $\lambda_0^i(s') = 0 = \lambda_0(s')$. Substituting into (35) we conclude that $R = \sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1 - \delta))/\phi_0$ which is not generically true. Hence, $\lambda_0(s) > 0$ for some s . Indeed, since $R + R\lambda_0^i(s) + R\lambda_0(s) = R + R\lambda_0^i(s') + R\lambda_0(s')$, $\lambda_0(s') > 0$ as well, since otherwise the right hand side would equal R (due to the fact that $\lambda_0(s') = 0$ implies $\lambda_0^i(s') = 0$), a contradiction. Hence, $\lambda_0(s) > 0, \forall s \in \mathcal{S}$.

There are three cases to consider. First, consider the case where $\nu_0^i(s) > 0$ and $\lambda_0^i(s) = 0, \forall s \in \mathcal{S}$. Then $\lambda_0(s) = R^{-1}\mu_0 - 1$ and (35) implies

$$\mu_0^* \equiv \frac{\sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta))}{\phi_0 - R^{-1} \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)\theta(1 - \delta)}$$

and $V(w_0) = \mu_0^*w_0$. Suppose instead that $\nu_0^i(s) = 0, \forall s \in \mathcal{S}$. Then $\lambda_0^i(s) = (R_0^i)^{-1}\mu_0 - 1$ and $\lambda_0(s) = (R^{-1} - (R_0^i)^{-1})\mu_0, \forall s \in \mathcal{S}$. Substituting into (35) implies

$$\bar{\mu}_0^* \equiv \frac{\sum_{s \in \mathcal{S}} \pi(s)(A_1(s) + \phi_1(s)(1 - \theta^i)(1 - \delta))}{\phi_0 - \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(R^{-1}\theta(1 - \delta) + (R_0^i)^{-1}(\theta^i - \theta)(1 - \delta))}$$

and $V(w_0) = \bar{\mu}_0^*w_0$. Also, let $\bar{\varphi}_0$ denote the denominator in $\bar{\mu}_0^*$. Let C denote the numerator in μ_0^* such that $\mu_0^* = C/\varphi_0$ and note that

$$\bar{\mu}_0^* = \frac{C - \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}{\bar{\varphi}_0 - (R_0^i)^{-1} \sum_{s \in \mathcal{S}} \pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}.$$

Hence, $\bar{\mu}_0^* > \mu_0^*$ iff $R_0^i < \mu_0^*$.

Finally, suppose $\nu_0^i(s) = 0$ and $\nu_0^i(s') > 0$. Proceeding analogously we obtain

$$\bar{\mu}_0^*(s) = \frac{C - \pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}{\bar{\varphi}_0 - (R_0^i)^{-1}\pi(s)\phi_1(s)(\theta^i - \theta)(1 - \delta)}.$$

Thus, $\bar{\mu}_0^*(s) > \mu_0^*$ iff $R_0^i < \mu_0^*$. Let $\bar{\mu}_0^* = \bar{C}/\bar{\varphi}_0$ and note that

$$\bar{\mu}_0^*(s) = \frac{\bar{C} + \pi(s')\phi_1(s')(\theta^i - \theta)(1 - \delta)}{\bar{\varphi}_0 + (R_0^i)^{-1}\pi(s')\phi_1(s')(\theta^i - \theta)(1 - \delta)}.$$

Now, $\bar{\mu}_0^* > \bar{\mu}_0^*(s)$ iff $R_0^i < \bar{\mu}_0^*$. But then, whenever $R_0^i < \mu_0^*$ then $\bar{\mu}_0^* > \bar{\mu}_0^*(s) > \mu_0^*$. \square

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Table 1: Limited Intermediary Capital: An Example

Panel A: Parameters			
Type distribution	$\psi(g) = 0.50$	$\psi(b) = 0.50$	
Endowments	$w_0(g) = 1$	$w_0(b) = 0.50$	
Lenders' time preference	$R = 1/\beta = 1.05$		
Technology	$\delta = 0.10$	$\theta = 0.80$	$\theta^i = 0.90$
Maximum scale	$\bar{k} = 7$		
Distribution of states	$\pi(H) = 0.5$	$\pi(L) = 0.5$	
Capital prices	$\phi_0 = 1$		
	$\phi_1(H) = 1$	$\phi_1(L) = 0.965$	$\phi_2(H) = \phi_2(L) = 1$
Productivity type g	$A_1(H g) = 0.45$	$A_1(L g) = 0.10$	$A_2(H g) = A_2(L g) = 0.20$
Productivity type b	$A_1(H b) = 0.35$	$A_1(L b) = 0.00$	$A_2(H b) = A_2(L b) = 0.15$
Panel B: Well Capitalized Intermediary ($\underline{w}_0^i = 0.281$)			
Net worth type g	$w_1(H g) = 2.231$	$w_1(L g) = 0.772$	
Net worth type b	$w_1(H b) = 0.000$	$w_1(L b) = 1.050$	
Investment type g	$k_0(g) = 4.131$		
	$k_1(H g) = 7.000$	$k_1(L g) = 3.988$	
Investment type b	$k_0(b) = 0.000$		
	$k_1(H b) = 0.000$	$k_1(L b) = 5.424$	
Panel C: Intermediary with Limited Capital (w_0^i 2.5% less than \underline{w}_0^i)			
Net worth type g	$w_1(H g) = 2.224$	$w_1(L g) = 0.770$	
Net worth type b	$w_1(H b) = 0.000$	$w_1(L b) = 1.050$	
Investment type g	$k_0(g) = 4.118$		
	$k_1(H g) = 7.000$	$k_1(L g) = 3.944$	
Investment type b	$k_0(b) = 0.000$		
	$k_1(H b) = 0.000$	$k_1(L b) = 5.382$	
Spreads	$\varsigma_0 = 0.95\%$		
	$\varsigma_1(H) = 0.00\%$	$\varsigma_1(L) = 1.90\%$	